The AdS/CFT Correspondence
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1 Lecture 3

Q1. Consider the 'AdS brachistochrone curve' in planar Schwarzschild-AdS_5, which you can think of as a string dangling between two fixed boundary points, say at $t = 0$. What is the profile of the string? By comparing the length of this curve in Schwarzschild-AdS_5 relative to its value in AdS_5 obtain the expectation value of a boundary Wilson loop operator.

Q2. Let us get some intuition for holographic entanglement entropy in a variety of situations. Most of these exercises are designed to be done analytically, though you can also attempt to do them numerically (and thence generalize). The task is to compute $S_{\mathcal{A}}(|\,\psi\rangle)$ where I will prescribe the global state $|\,\psi\rangle$ of the CFT, the boundary geometry and the region $\mathcal{A}$ below:

(i) $|\,\psi\rangle = |\,0\rangle$ on $\mathbb{R}^{d-1,1}$ and the region is a disc at $t = 0$.

$$\mathcal{A}_\circ = \{(t, \xi, \Omega_{d-2}) | t = 0\,,\ \Omega_{d-2} : \text{arbitrary}\,,\ 0 \leq \xi \leq R\}$$

(ii) $|\,\psi\rangle = |\,0\rangle$ on $\mathbb{R}^{d-1,1}$ and the region is a strip at $t = 0$.

$$\mathcal{A}_\parallel = \{(t, x, \mathbf{y}_{d-2}) | t = 0\,,\ -L \leq y_i \leq L\,,\ -w \leq x \leq w\}$$

(iii) $|\,\psi\rangle = |\,0\rangle$ on $\mathbb{R} \times \mathbf{S}^{d-1}$ and the region is the polar-cap cutting the sphere $\mathbf{S}^{d-1}$ at a latitude around the north pole (which is set to be $\theta = 0$)

$$\mathcal{A}_{polar} = \{(t, \theta, \Omega_{d-2}) | -\theta_0 < \theta < \theta_0\} \ \text{ where } \ d\Omega_{d-1}^2 = d\theta^2 + \sin^2\theta\, d\Omega_{d-2}^2$$

(iv) Thermal state on $\mathbb{R}^{d-1,1}$ and the region is a disc at $t = 0$, $\mathcal{A}_\circ$.

(v) Thermal state on $\mathbb{R}^{d-1,1}$ and the region is a strip at $t = 0$, $\mathcal{A}_\parallel$.

(vi) Get explicit results for the answers in $d = 2$ in various cases and compare with CFT_2 computations described in other lectures.

(vii) **Bonus 1:** As a more interesting situation, consider the thermofield double representation of the thermal state. The dual geometry is the eternal Schwarzschild-AdS_{d+1} black hole. Take the region $\mathcal{A}$ to be the union of half-spaces on both CFTs, i.e.,

$$\mathcal{A} = \{(t, x, \mathbf{y}_{d-2})_{\text{L}} \cup (t, x, \mathbf{y}_{d-2})_{\text{R}} \mid x_{\text{R}} \geq 0\,, x_{\text{L}} \geq 0\}$$

(viii) **Bonus 2:** Take a CFT_2 at finite temperature on a circle. Let us give a can consider a Gibbs state where in addition to finite T we also include a finite chemical potential for angular momentum. Can you work out the answer for the entanglement for $\mathcal{A}$ being an arc of the spatial circle.