Quantum Error Correction: Problem Set #2

It for Qubit
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Problem #1. Error Syndromes and Cosets for Stabilizer Codes

Recall that $N(S) = \{P | [P, M] = 0 \, \forall M \in S\}$, that S is a normal subgroup of $N(S)$, and that the elements of $N(S)/S$ (i.e., cosets of S in $N(S)$) correspond to logical $\overline{X}$ and $\overline{Z}$ operators.

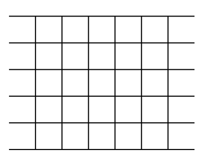
a) Show that two Pauli errors E and F have the same error syndrome for a stabilizer code S iff they are in the same coset of $N(S)$ in the Pauli group.

b) Suppose that for each coset of $N(S)$ we pick some particular coset representative E and perform E whenever syndrome measurement indicates that coset. Suppose, however, a different error F had actually occurred. Relate the overall action on the codespace to an element of $N(S)/S$.

c) For the 5-qubit code, we choose the coset representatives to be the single-qubit errors (and the identity for the 0 syndrome), as there is exactly one in each coset. Use the result of part b to find the actions resulting from the errors $X_1 Z_3$ and $Y_2 X_4 Z_5$.

Problem #2. Surface Codes with Boundary

For this problem, consider a code with qubits located on the edges of the following graph, extended to a square grid of $L \times L$ vertices:



Note that is this case, we are *not* identifying top and bottom or left and right. For each face or vertex in the interior of the graph, have Z_f or X_v in the stabilizer as for the toric code. On the rough edges, we have a three-qubit stabilizer element $Z_f = Z \otimes Z \otimes Z$ for the three edges around each incomplete face. On the smooth edges, we have a three-qubit stabilizer element $X_v = X \otimes X \otimes X$ for the three edges incident at each vertex on the boundary.

a) How many physical qubits does this code have?

b) How many logical qubits does this code have?

c) Consider a path that starts and ends on edges that are part of a rough boundary. Show that the tensor product of Zs along the path is an element of $N(S)$.

d) Consider a dual path starting and ending on smooth boundaries. Show that the tensor product of Xs along the path is an element of $N(S)$.

e) Characterize the non-trivial logical $\overline{X}$ and $\overline{Z}$ operators. What is the distance of this code?