What Shape Dynamics Is

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... Shape Dynamics is the world one shape after another.

Motivation

Why $g_{ab}^{(3)} \rightarrow e^{\phi(x)} g_{ab}^{(3)}$?

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Shape Dynamics Take Home Message

Old Observation (1970s)

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- New scenarios for quantum theory (theory space, observable/evolution split,...).

Canonical GR

$3 + 1$ decomposition

$g_{\mu\nu} \rightarrow$

- $(g_{ab}, \dot{g}_{ab})$: dynamic variables
- (N, N^a): embedding variables

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$\Rightarrow$ Hamilton vector field: $v_g(f) = \{f, g\}$ (directional derivative).

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Non-invertible Legendre transform

Constraints:

- Diff: $g_{ab}\mathcal{L}_{N^c}\pi^{ab} = 0 \ \forall \ N^c$,
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- Ham: $K(\pi^{ab}, g_{ab}) + V(g_{ab}, \nabla g, ...) = 0$
  $\Rightarrow v_{\text{Ham}} =$ hypersurface deformations AND evolution

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- Hamiltonian: $H_{\text{SD}}(\tau) = \int \mathrm{d}^3x \, e^{6\phi(g_{ab},\pi^{ab},\tau)}\sqrt{g}$
  where $\phi(g_{ab}, \pi^{ab}, \tau) =$ solution to elliptic PDE.
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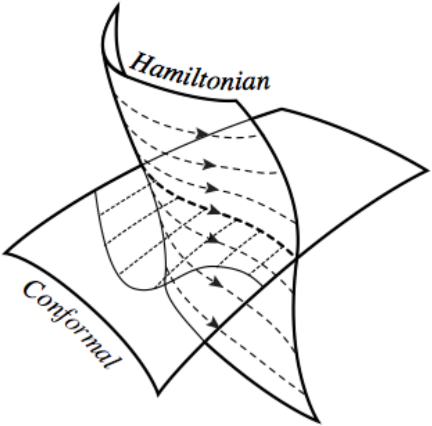
Duality: When $\phi = 0$ and $N = N_{\text{CMC}}$, then $v_{H_{\text{SD}}} = v_{\text{Ham}}$.

Iconic Diagram



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Quantum (?)

- New theory space.
- Split between observables and evolution.

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- Ambitions for quantum theory.

Thank You!