Problem Set #3

Quantum Error Correction
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Due Thursday, Jan. 25, 2018

Problem #1. Quantum Hamming bound for qudit codes

The quantum Hamming bound for qudits of dimension p becomes

$$\sum_{s=0}^{t} \binom{n}{s} (p^2 - 1)^s \leq p^{n-k}, \tag{1}$$

which must hold for non-degenerate $((n, p^k, 2t+1))_p$ codes.

a) For what values of p does a $[[5,1,3]]_p$ code saturate the quantum Hamming bound?

b) For what values of p would a $[[9,1,5]]_p$ code saturate the quantum Hamming bound? For which values of p would the code violate the quantum Hamming bound? (Note that such a code is only known to exist for prime power p with $p \geq 9$.)

c) For $p = 3$, find the smallest integer values of n and k such that an $[[n,k,3]]_3$ code saturates the quantum Hamming bound or show that no integer n and k work.

Problem #2. Logical operations for qudit code

Consider the following stabilizer code for qutrits (qudits with dimension $p = 3$):

$$\begin{matrix} X & X & Z & Z \\ Z & Z & X & X \end{matrix}$$

a) What are its parameters as a QECC?

b) Find a generating set for the logical Pauli group. (I.e., coset representatives for $\overline{X}_i$ and $\overline{Z}_i$).

c) For your choice of logical Pauli operators, write down the codeword with all logical qubits 0 expanded in the standard basis for the physical qubits.