Some String Theory Technology

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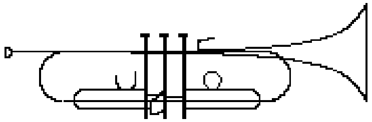


Goal

To introduce/remind:

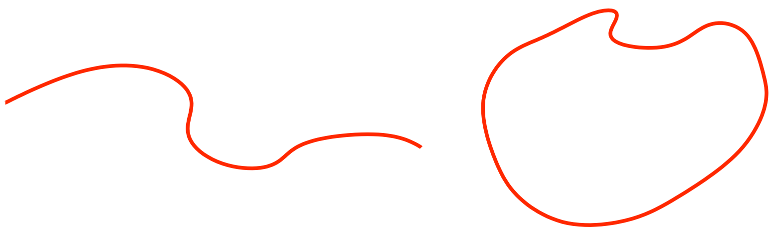
- the language
- the setting and tools
- the scope and limitations

To set the scene for:

- various special topics
- various current events....

Describing Strings

Strings come in two broad varieties, open and closed:

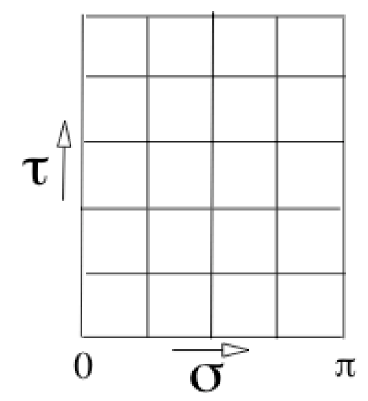


As they move in spacetime, they sweep out a two dimensional surface known as a “worldsheet”.

A starting point: tackle the description of the string’s allowed motion = allowed shapes of worldsheet.

Describing Strings

First parameterize the worldsheet with some coordinates: $\{\sigma, \tau\}$



Describing Strings

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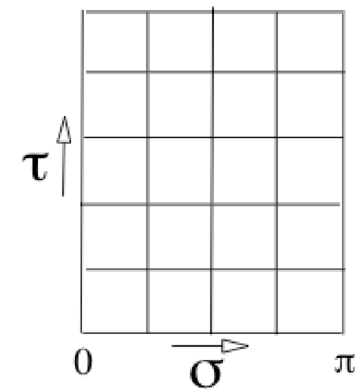
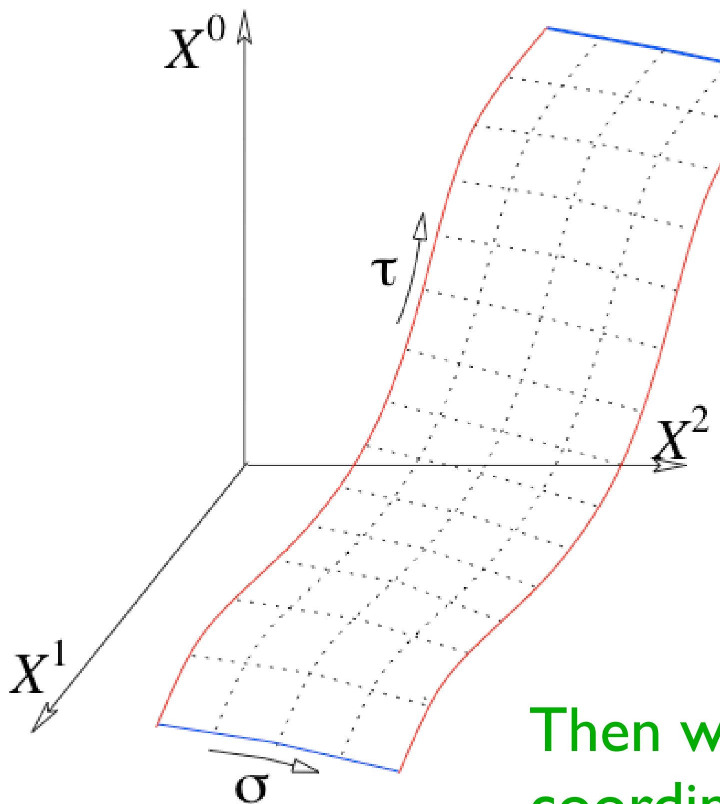


Then we say where the string is in spacetime, which has coordinates: $\{X^{\mu}\}, \mu = 0, 1, \ldots, D-1$

Describing Strings



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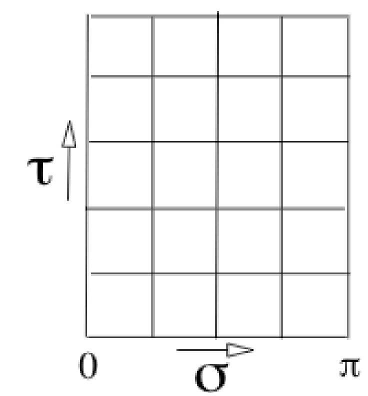


Then we say where the string is in spacetime, which has coordinates: $\{X^\mu\}, \mu = 0, 1, \ldots, D-1$

This implies some map(s): $X^\mu(\tau, \sigma)$

What are the allowed ones?

Describing Strings

Determine the allowed shapes by an action principle:

$$S = -T \int dA = -T \int d\tau d\sigma \, (-\det h_{ab})^{1/2}$$

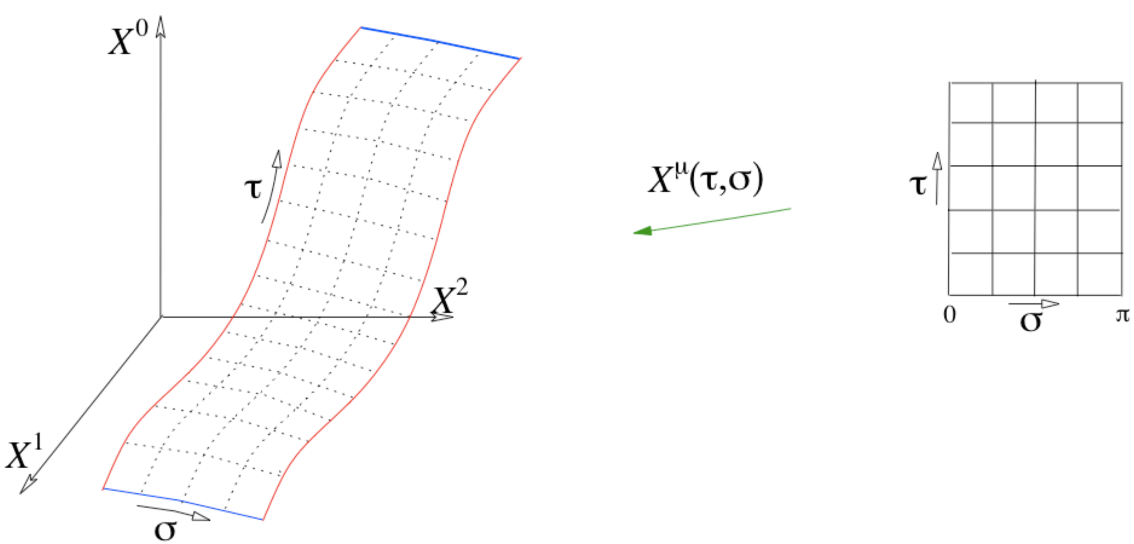
extremize the area swept out by worldsheet

$$h_{ab} = \partial_a X^\mu \partial_b X^\nu g_{\mu\nu}$$

induced metric

$$(\sigma^1, \sigma^2) \equiv (\tau, \sigma)$$

$$(\partial_a \equiv \partial / \partial \sigma^a)$$



Tension: $T = \dfrac{1}{2\pi\alpha'}$

Sets a basic length scale: $\ell_s \sim \sqrt{\alpha'}$

Comparable to Planck length?

Aside: Planck Scale?

Where does the Planck scale come from?

Simple argument:

Quantum effects set a natural length scale: $$\ell_c = \frac{\hbar}{mc}$$

Gravity sets a natural length scale: $$\ell_s = \frac{Gm}{c^2}$$

At what mass do these length scales coincide?

$$m_p = \sqrt{\frac{\hbar c}{G}}$$

This length scale is the Planck length: $$\ell_p = \sqrt{\frac{\hbar G}{c^3}} \sim 1.6 \times 10^{-35}\mathrm{m}$$

Describing Strings

An equivalent action:

$$S = -\frac{1}{4\pi\alpha'}\int d\tau d\sigma (-\gamma)^{1/2}\gamma^{ab}\partial_a X^\mu \partial_b X^\nu \eta_{\mu\nu}$$

$\gamma_{ab}(\sigma, \tau)$ independent metric on worldsheet

This action is more pleasant to work with than the other one.

But they are equivalent!

Describing Strings

The Equivalence:

$$S = -\frac{1}{4\pi\alpha'}\int d\tau d\sigma(-\gamma)^{1/2}\gamma^{ab}\partial_a X^\mu \partial_b X^\nu \eta_{\mu\nu}$$
$$= -\frac{1}{4\pi\alpha'}\int d^2\sigma\,(-\gamma)^{1/2}\gamma^{ab}h_{ab}.$$

Now vary metric:

$$\delta S = -\frac{1}{4\pi\alpha'}\int d^2\sigma \left\{-\frac{1}{2}(-\gamma)^{1/2}\delta\gamma\gamma^{ab}h_{ab} + (-\gamma)^{1/2}\delta\gamma^{ab}h_{ab}\right\}$$

Use: $\delta\gamma = \gamma\gamma^{ab}\delta\gamma_{ab} = -\gamma\gamma_{ab}\delta\gamma^{ab}$

$$\delta S = -\frac{1}{4\pi\alpha'}\int d^2\sigma\,(-\gamma)^{1/2}\delta\gamma^{ab}\left\{h_{ab} - \frac{1}{2}\gamma_{ab}\gamma^{cd}h_{cd}\right\}$$

Describing Strings

The Equivalence:

$$\delta S = -\frac{1}{4\pi\alpha'}\int d^2\sigma\,(-\gamma)^{1/2}\delta\gamma^{ab}\left\{h_{ab} - \frac{1}{2}\gamma_{ab}\gamma^{cd}h_{cd}\right\}$$

solve: $$h_{ab} - \frac{1}{2}\gamma_{ab}\gamma^{cd}h_{cd} = 0$$

or: $$\gamma^{ab}h_{ab} = 2(-h)^{1/2}(-\gamma)^{-1/2}$$

Substitute this in and find:

$$S_{\mathrm{P}} \longrightarrow S_{\mathrm{NG}}$$

Describing Strings

Spacetime Equations of Motion:

$$S = -\frac{1}{4\pi\alpha'}\int d\tau d\sigma (-\gamma)^{1/2}\gamma^{ab}\partial_a X^\mu \partial_b X^\nu \eta_{\mu\nu}$$

Now vary X-fields:

$$\delta S = \frac{1}{2\pi\alpha'}\int d^2\sigma\, \partial_a \left\{(-\gamma)^{1/2}\gamma^{ab}\partial_b X_\mu\right\}\delta X^\mu \\ - \frac{1}{2\pi\alpha'}\int d\tau\, (-\gamma)^{1/2}\partial_\sigma X_\mu \delta X^\mu \Big|_{\sigma=0}^{\sigma=\pi}$$

...and so:

$$\partial_a\left((-\gamma)^{1/2}\gamma^{ab}\partial_b X^\mu\right) \equiv (-\gamma)^{1/2}\nabla^2 X^\mu = 0$$

with boundary conditions...

Describing Strings

with boundary conditions...

$$\left.\begin{array}{l} X'^{\mu}(\tau,0)=0 \\ X'^{\mu}(\tau,\pi)=0 \end{array}\right\}$$

Open:
Neumann (or Dirichlet; more later).

$$\left.\begin{array}{c} X'^{\mu}(\tau,0)=X'^{\mu}(\tau,\pi) \\ X^{\mu}(\tau,0)=X^{\mu}(\tau,\pi) \\ \gamma_{ab}(\tau,0)=\gamma_{ab}(\tau,\pi) \end{array}\right\}$$

Closed: Periodic

Describing Strings

$$S = -\frac{1}{4\pi\alpha'}\int d\tau d\sigma (-\gamma)^{1/2}\gamma^{ab}\partial_a X^\mu \partial_b X^\nu \eta_{\mu\nu}$$

Action has some interesting local ("gauge") invariances:

World-sheet reparameterization invariance: $\sigma, \tau \longrightarrow \tilde{\sigma}(\sigma,\tau), \tilde{\tau}(\sigma,\tau)$

Weyl invariance: $\gamma_{ab} \rightarrow \gamma'_{ab} = e^{2\omega}\gamma_{ab}$

$\omega(\tau,\sigma)$ is function

So three functions altogether

Describing Strings

This allows us to choose a gauge:

"Conformal" gauge

$$\gamma_{ab} = \eta_{ab} e^{\phi} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} e^{\phi}$$

Theory is conformally invariant
(remnant of reparams and Weyl.)

ϕ drops out of the action, classically.

Equations of motion become:

$$\left(\frac{\partial^2}{\partial \sigma^2} - \frac{\partial^2}{\partial \tau^2} \right) X^{\mu}(\tau, \sigma) = 0$$

The two dimensional wave equation!

So the problem of the string's motion is just
solutions to this, with boundary conditions.

Describing Strings

Solutions:

$$X^\mu(\tau,\sigma) = x^\mu + 2\alpha' p^\mu \tau + i(2\alpha')^{1/2} \sum_{n\neq 0} \frac{1}{n} \alpha_n^\mu e^{-in\tau} \cos n\sigma$$

(can also have DD and ND strings. See later.)

Open: Neumann - Neumann (NN)

standing waves

$$\begin{aligned} X^\mu(\tau,\sigma) &= X_R^\mu(\sigma^-) + X_L^\mu(\sigma^+) \\ X_R^\mu(\sigma^-) &= \frac{1}{2}x^\mu + \alpha' p^\mu \sigma^- + i\left(\frac{\alpha'}{2}\right)^{1/2} \sum_{n\neq 0} \frac{1}{n} \alpha_n^\mu e^{-2in\sigma^-} \\ X_L^\mu(\sigma^+) &= \frac{1}{2}x^\mu + \alpha' p^\mu \sigma^+ + i\left(\frac{\alpha'}{2}\right)^{1/2} \sum_{n\neq 0} \frac{1}{n} \tilde{\alpha}_n^\mu e^{-2in\sigma^+} \end{aligned}$$

Closed: Periodic

Traveling waves, independent left- and right-moving

$\tilde{\alpha}_{-n}^\mu = (\tilde{\alpha}_n^\mu)^*$ etc...

Describing Strings

Solutions:

$$X^\mu(\tau,\sigma) = x^\mu + 2\alpha' p^\mu \tau + i(2\alpha')^{1/2} \sum_{n\neq 0} \frac{1}{n} \alpha_n^\mu e^{-in\tau} \cos n\sigma$$

(can also have DD and ND strings. See later.)

Open: Neumann - Neumann (NN)

$$X^\mu(\tau,\sigma) = X_R^\mu(\sigma^-) + X_L^\mu(\sigma^+)$$

$$X_R^\mu(\sigma^-) = \frac{1}{2}x^\mu + \alpha' p^\mu \sigma^- + i\left(\frac{\alpha'}{2}\right)^{1/2} \sum_{n\neq 0} \frac{1}{n} \alpha_n^\mu e^{-2in\sigma^-}$$

$$X_L^\mu(\sigma^+) = \frac{1}{2}x^\mu + \alpha' p^\mu \sigma^+ + i\left(\frac{\alpha'}{2}\right)^{1/2} \sum_{n\neq 0} \frac{1}{n} \tilde{\alpha}_n^\mu e^{-2in\sigma^+}$$

Closed: Periodic

Zero modes:

open string: $\alpha_0^\mu = (2\alpha')^{1/2} p^\mu;$

closed string: $\alpha_0^\mu = \left(\frac{\alpha'}{2}\right)^{1/2} p^\mu$

Back To World-Sheet

Polyakov looks like a dynamical (2d) metric + fields.

So write Einstein-Hilbert action:

$$\chi = \frac{1}{4\pi} \int_{\mathcal{M}} d^2\sigma \, (-\gamma)^{1/2} R$$

With a boundary term (extrinsic curvature)

Cosmological term not included....

$$\Theta = \frac{1}{4\pi\alpha'} \int_{\mathcal{M}} d^2\sigma \, (-\gamma)^{1/2}$$

....since it is not Weyl invariant.

Back To World-Sheet

Full 2d equations of motion:

$$R_{ab} - \frac{1}{2}\gamma_{ab}R = T_{ab}$$

Einstein sourced by stress tensor

Vanishes in 2d (but see later)

So we have: $T_{ab} = 0$

This will be implemented at the quantum level as a constraint, later...

Back To World-Sheet

Stress tensor:

$$T^{ab}(\tau,\sigma) \equiv -\frac{2\pi}{\sqrt{-\gamma}}\frac{\delta S}{\delta\gamma_{ab}} = -\frac{1}{\alpha'}\left\{\partial^a X_\mu \partial^b X^\mu - \frac{1}{2}\gamma^{ab}\gamma_{cd}\partial^c X_\mu \partial^d X^\mu\right\}$$

$$T^a_a \equiv \gamma_{ab}T^{ab} = 0$$

Identity (Weyl)

$$T^{ab} = 0$$

Reparameterizations

Back To World-Sheet

In conformal gauge
our constraints are:

$$T_{\tau\sigma} = T_{\sigma\tau} \equiv \frac{1}{\alpha'} \dot{X}^\mu X'_\mu = 0$$

$$T_{\sigma\sigma} = T_{\tau\tau} = \frac{1}{2\alpha'} \left(\dot{X}^\mu \dot{X}_\mu + X'^\mu X'_\mu \right) = 0$$

$$\left(\frac{\partial^2}{\partial \sigma^2} - \frac{\partial^2}{\partial \tau^2} \right) X^\mu(\tau, \sigma) = 0$$

$$X^\mu(\sigma, \tau) = X^\mu_L(\sigma^+) + X^\mu_R(\sigma^-)$$

$$\sigma^\pm \equiv \tau \pm \sigma$$

$$T_{++} = \frac{1}{2}(T_{\tau\tau} + T_{\tau\sigma}) = \frac{1}{\alpha'} \partial_+ X^\mu \partial_+ X_\mu \equiv \frac{1}{\alpha'} \dot{X}_L^2 = 0$$

$$T_{--} = \frac{1}{2}(T_{\tau\tau} - T_{\tau\sigma}) = \frac{1}{\alpha'} \partial_- X^\mu \partial_- X_\mu \equiv \frac{1}{\alpha'} \dot{X}_R^2 = 0$$

Write everything in terms of Fourier modes as well:

$$L_m = \frac{T}{2} \int_0^\pi e^{-2im\sigma} T_{--} d\sigma = \frac{1}{2} \sum_{-\infty}^{\infty} \alpha_{m-n} \cdot \alpha_n$$

etc....

Connecting Spacetime and WorldSheet

Our action is:

$$\mathcal{L} = -\frac{1}{4\pi\alpha'}\left(\partial_\sigma X^\mu \partial_\sigma X_\mu - \partial_\tau X^\mu \partial_\tau X_\mu\right)$$

So the canonical partner of the field is:

$$\Pi^\mu = \frac{\delta\mathcal{L}}{\delta(\partial_\tau X^\mu)} = \frac{1}{2\pi\alpha'}\dot{X}^\mu$$

So Poisson Brackets:

$$\begin{aligned}
\left[X^\mu(\sigma), \Pi^\nu(\sigma')\right]_{\text{P.B.}} &= \eta^{\mu\nu}\delta(\sigma - \sigma') \\
\left[\Pi^\mu(\sigma), \Pi^\nu(\sigma')\right]_{\text{P.B.}} &= 0,
\end{aligned}$$

Resulting in:

$$\begin{gathered}
[\alpha_m^\mu, \alpha_n^\nu]_{\text{P.B.}} = [\tilde{\alpha}_m^\mu, \tilde{\alpha}_n^\nu]_{\text{P.B.}} = im\delta_{m+n}\eta^{\mu\nu} \\
[p^\mu, x^\nu]_{\text{P.B.}} = \eta^{\mu\nu}; \quad [\alpha_m^\mu, \tilde{\alpha}_n^\nu]_{\text{P.B.}} = 0.
\end{gathered}$$

Connecting Spacetime and WorldSheet

Also:

$$[L_m, L_n]_{\text{P.B.}} = i(m-n)L_{m+n}; \quad [\bar{L}_m, \bar{L}_n]_{\text{P.B.}} = i(m-n)\bar{L}_{m+n}$$
$$[\bar{L}_m, L_n]_{\text{P.B.}} = 0.$$

"Virasoro algebra"

Will impose our constraints
mode by mode...

$$L_m = 0 \qquad \bar{L}_m = 0$$

Connecting Spacetime and WorldSheet

Hamiltonian Density:

$$\mathcal{H} = \dot{X}^\mu \Pi_\mu - \mathcal{L} = \frac{1}{4\pi\alpha'} \left(\partial_\sigma X^\mu \partial_\sigma X_\mu + \partial_\tau X^\mu \partial_\tau X_\mu\right)$$

Hamiltonian:

$$H = \int_0^\pi d\sigma\, \mathcal{H}(\sigma) = \frac{1}{2}\sum_{-\infty}^{\infty} \alpha_{-n}\cdot\alpha_n \qquad \text{(open)}$$

$$H = \int_0^{2\pi} d\sigma\, \mathcal{H}(\sigma) = \frac{1}{2}\sum_{-\infty}^{\infty} \left(\alpha_{-n}\cdot\alpha_n + \tilde{\alpha}_{-n}\cdot\tilde{\alpha}_n\right) \qquad \text{(closed)}$$

$$H = L_0 \qquad \text{(open)}; \qquad H = L_0 + \bar{L}_0 \qquad \text{(closed)}$$

Connecting Spacetime and WorldSheet

Look:

$$L_0 = \frac{1}{2}\alpha_0^2 + 2 \times \frac{1}{2}\sum_{n=1}^{\infty} \alpha_{-n} \cdot \alpha_n$$

Zero modes:

open string: $\alpha_0^\mu = (2\alpha')^{1/2} p^\mu;$

closed string: $\alpha_0^\mu = \left(\frac{\alpha'}{2}\right)^{1/2} p^\mu$

Connecting Spacetime and WorldSheet

Look:

$$L_0 = \frac{1}{2}\alpha_0^2 + 2 \times \frac{1}{2}\sum_{n=1}^{\infty} \alpha_{-n}\cdot\alpha_n$$

$$= \alpha' p^\mu p_\mu + \sum_{n=1}^{\infty} \alpha_{-n}\cdot\alpha_n$$

$$= -\alpha' M^2 + \sum_{n=1}^{\infty} \alpha_{-n}\cdot\alpha_n$$

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Connecting Spacetime and WorldSheet

Constraints become mass formula in spacetime:

$$M^2 = \frac{1}{\alpha'} \sum_{n=1}^{\infty} \alpha_{-n} \cdot \alpha_n \qquad \text{(open)}$$

$$M^2 = \frac{2}{\alpha'} \sum_{n=1}^{\infty} (\alpha_{-n} \cdot \alpha_n + \tilde{\alpha}_{-n} \cdot \tilde{\alpha}_n) \qquad \text{(closed)}$$

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Quantum Strings

Promote Poisson Brackets to Commutators....

$$[\quad , \quad]_{\mathrm{P.B.}} \to -i[\quad , \quad]$$

Fourier modes become annihilation and creation operators.

Normal order everything, etc....

There are D independent families of harmonic oscillators.
One for each boson in world-sheet theory.

Theory must be reparameterisation invariant: Virasoro constraints.
Succeeds in removing "ghosts" etc... (long story)

Quantum Strings

Revisit Virasoro:

$$
\begin{aligned}
L_0 &= \frac{1}{2}\alpha_0^2 + 2 \times \frac{1}{2}\sum_{n=1}^{\infty} \alpha_{-n}\cdot\alpha_n + \frac{D}{2}\sum_{n=1}^{\infty} n \\
&= \alpha' p^\mu p_\mu + \sum_{n=1}^{\infty} \alpha_{-n}\cdot\alpha_n + \text{const} \\
&= -\alpha' M^2 + \sum_{n=1}^{\infty} \alpha_{-n}\cdot\alpha_n + \text{const},
\end{aligned}
$$

infinite?!

No

Quantum Strings

Quantum Virasoro constraints become:

$$(L_0 - a)|\phi\rangle = 0; \qquad L_m|\phi\rangle = 0 \quad \text{for } m > 0,$$
$$(\bar{L}_0 - a)|\phi\rangle = 0; \qquad \bar{L}_m|\phi\rangle = 0 \quad \text{for } m > 0,$$

Quantum Virasoro algebra:

$$[L_m, L_n] = (m-n)L_{m+n} + \frac{D}{12}(m^3 - m)\delta_{m+n}; \qquad [\bar{L}_m, L_n] = 0$$
$$[\bar{L}_m, \bar{L}_n] = (m-n)\bar{L}_{m+n} + \frac{D}{12}(m^3 - m)\delta_{m+n}.$$

Only imposing positive modes is consistent with algebra

"Conformal anomaly"

Quantum Strings

Mass formulae?

$$M^2 = \frac{1}{\alpha'}\left(\sum_{n=1}^{\infty} \alpha_{-n}\cdot\alpha_n - a\right) \qquad \text{(open)}$$

$$M^2 = \frac{2}{\alpha'}\left(\sum_{n=1}^{\infty} (\alpha_{-n}\cdot\alpha_n + \tilde{\alpha}_{-n}\cdot\tilde{\alpha}_n) - 2a\right) \qquad \text{(closed)}$$

It turns out that:

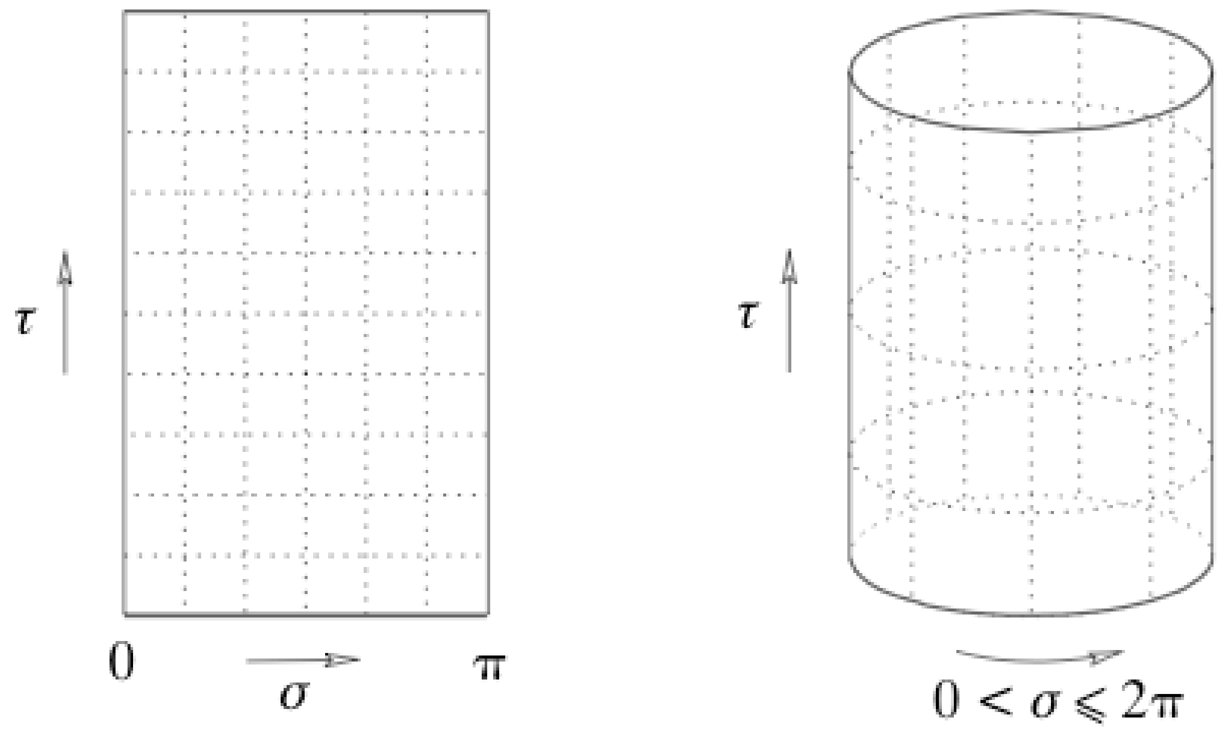
$$a = 1$$

$$D = 26$$

But see later!

Gosh: $\sum_{n=1}^{\infty} n = -\frac{1}{12}$

Think of energy as a zero point energy arising from having particle the 2d field theory in a box.



Note: Two modes (choose 0,1 directions) effectively do not contribute, due to successful application of Virasoro. This also removes "ghosts".

So have 24 bosons, with zero point energy -1/24 each. More later.

Spacetime Physics?

Example of closed string states:

$$\tilde{\alpha}^{\mu}_{-1}\alpha^{\nu}_{-1}|0>$$

oscillators add mass-energy +2

vacuum has mass-energy -2

massless states in spacetime

symmetric	$G_{\mu\nu}(x)$	“graviton”
antisymmetric	$B_{\mu\nu}(x)$	“Kalb-Ramond”
trace	$\Phi(x)$	“dilaton”

Note: vacuum gives a tachyon in spectrum. (More later)

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$$\alpha^{\mu}_{-1}|0>$$

oscillator adds
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vacuum has
mass-energy -1

massless states
in spacetime

$A_{\mu}(x)$ "photon"

Note: vacuum gives a tachyon in spectrum. (More later)

Spacetime Physics?

Consistency sometimes places a condition on D.

The condition comes about because theory has an anomaly in conformal invariance, with the following ingredients:

Counting D as number of bosons on world sheet

Each field X contributes +1: $$c = D - 1$$

There are Fadeev-Popov ghosts for fixing gauge, and they give: $$c = -26$$

The field ϕ does not in general decouple, and contributes: $$c = 1 + 3\left(\frac{26 - D}{3}\right)$$

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Comes about because ϕ has non-trivial world-sheet coupling

Spacetime Physics?

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Total vanishes, and so can have a string theory in any number of dimensions.

But one of them is different from others. Lorentz...?

Spacetime Physics?

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For Lorentz invariance, all dimensions must couple same way, resulting in:

"Critical Dimension" $$D = 26$$

Spacetime Physics?

What determines the dynamics of the spacetime fields?

$$G_{\mu\nu}(x)$$
$$B_{\mu\nu}(x)$$
$$\Phi(x)$$

Revisit worldsheet action (henceforth Euclidean):

$$S_\sigma = \frac{1}{4\pi\alpha'} \int d^2\sigma\, g^{1/2}\, \left\{ (g^{ab} G_{\mu\nu}(X) + i\epsilon^{ab} B_{\mu\nu}(X)) \partial_a X^\mu \partial_b X^\nu + \alpha' \Phi R \right\}$$

String now propagating in background of fields whose quanta it generates!

Model must still be conformally invariant...

String Perturbation Theory

Notice that the value of the dilaton couples to the Euler number of the worldsheet.

$$\chi = \frac{1}{4\pi}\int_{\mathcal{M}} d^2\sigma\, g^{1/2} R \;= 2-2h$$

# of handles

$$S_\sigma = \frac{1}{4\pi\alpha'}\int d^2\sigma\, g^{1/2}\left\{(g^{ab}G_{\mu\nu}(X) + i\epsilon^{ab}B_{\mu\nu}(X))\partial_a X^\mu \partial_b X^\nu + \alpha'\Phi R\right\}$$

So what? Well, in the path integral, amplitudes will be weighed by a topological factor: $e^{-\Phi\chi}$

$$\mathcal{Z} = \int \mathcal{D}X\mathcal{D}g\; e^{-S}$$

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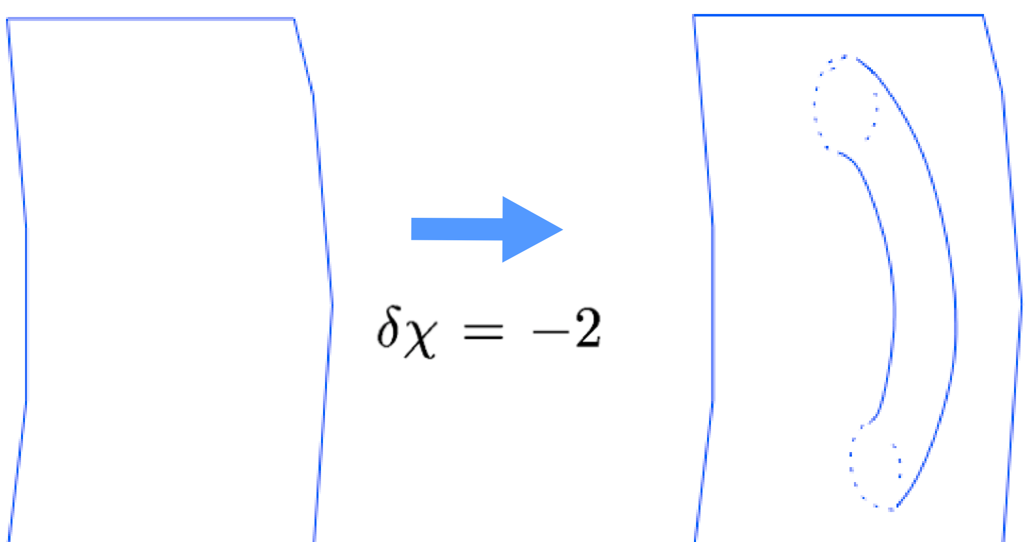
Look at different orders in perturbation theory:



Every vertex has additional factor of “closed string coupling”:

$$g_s = e^{\Phi}$$

local value of coupling set by dynamical field...

Spacetime Physics

Trace of stress tensor for the model (complicated since background fields act as highly non-linear couplings):

$G_{\mu\nu}(x)$
$B_{\mu\nu}(x)$
$\Phi(x)$

$$T^a{}_a = -\frac{1}{2\alpha'}\beta^G_{\mu\nu} g^{ab}\partial_a X^\mu \partial_b X^\nu - \frac{i}{2\alpha'}\beta^B_{\mu\nu}\epsilon^{ab}\partial_a X^\mu \partial_b X^\nu - \frac{1}{2}\beta^\Phi R$$

$$\beta^G_{\mu\nu} = \alpha'\left(R_{\mu\nu} + 2\nabla_\mu\nabla_\nu\Phi - \frac{1}{4}H_{\mu\kappa\sigma}H_\nu{}^{\kappa\sigma}\right) + O(\alpha'^2),$$

$$\beta^B_{\mu\nu} = \alpha'\left(-\frac{1}{2}\nabla^\kappa H_{\kappa\mu\nu} + \nabla^\kappa\Phi H_{\kappa\mu\nu}\right) + O(\alpha'^2),$$

$$\beta^\Phi = \alpha'\left(\frac{D-26}{6\alpha'} - \frac{1}{2}\nabla^2\Phi + \nabla_\kappa\Phi\nabla^\kappa\Phi - \frac{1}{24}H_{\kappa\mu\nu}H^{\kappa\mu\nu}\right) + O(\alpha'^2)$$

$$H_{\mu\nu\kappa} \equiv \partial_\mu B_{\nu\kappa} + \partial_\nu B_{\kappa\mu} + \partial_\kappa B_{\mu\nu}$$

Conformal invariance requires these to vanish...But this looks like a set of spacetime field equations!

Spacetime Physics

$G_{\mu\nu}(x)$
$B_{\mu\nu}(x)$
$\Phi(x)$

Those field equations can be derived from this spacetime action:

$$S = \frac{1}{2\kappa_0^2}\int d^D X(-G)^{1/2}e^{-2\Phi}\left[R + 4\nabla_\mu\Phi\nabla^\mu\Phi - \frac{1}{12}H_{\mu\nu\lambda}H^{\mu\nu\lambda} - \frac{2(D-26)}{3\alpha'} + O(\alpha')\right]$$

some dimensionful numbers...

This is the low energy effective action for the string theory, at tree level.

Spacetime Physics

Those field equations can be derived from this spacetime action:

$G_{\mu\nu}(x)$
$B_{\mu\nu}(x)$
$\Phi(x)$

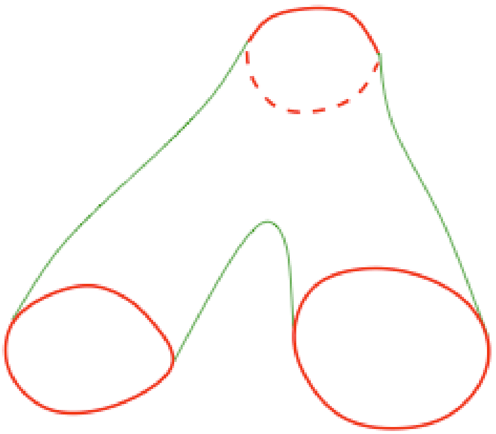
$$S = \frac{1}{2\kappa_0^2}\int d^D X(-G)^{1/2} e^{-2\Phi}\Big[R + 4\nabla_\mu\Phi\nabla^\mu\Phi - \frac{1}{12}H_{\mu\nu\lambda}H^{\mu\nu\lambda} - \frac{2(D-26)}{3\alpha'} + O(\alpha')\Big]$$

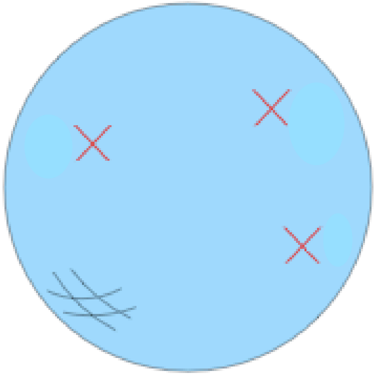
Notice the implicit power of the string coupling that appears. Everything was computed at the sphere level. Tree level in string pert. theory.

Language of Perturbation Theory

Sphere?

Can always choose conformal factor to map all external states to "vertex operators" at points....



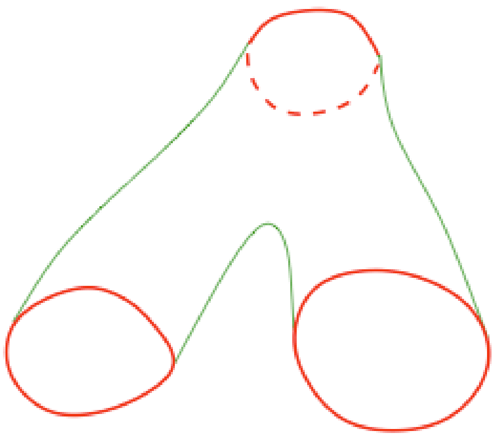


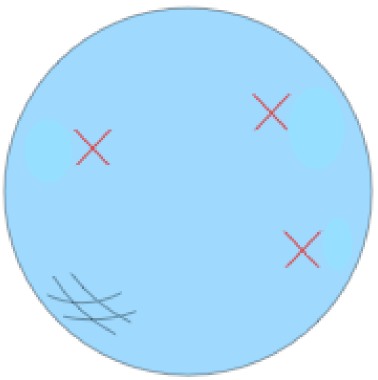
g_s^{-2}

Language of Perturbation Theory

Sphere?

Can always choose conformal factor to map all external states to “vertex operators” at points....



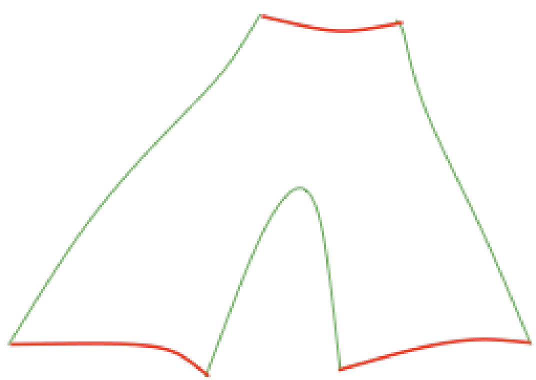


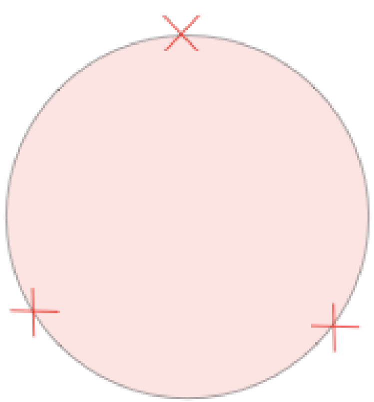
g_s^{-2}

Disc (see later)

Similarly for open strings....

...they become operators on boundary.

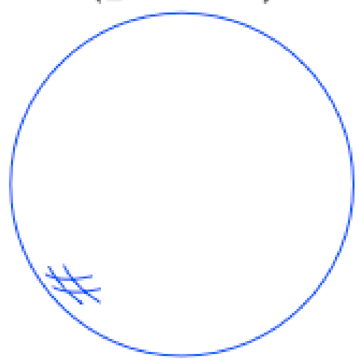
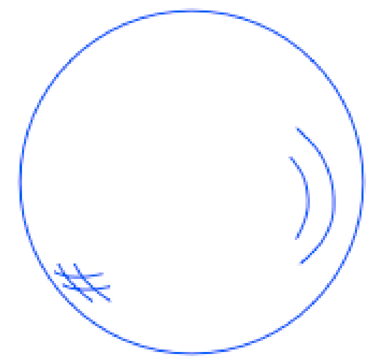
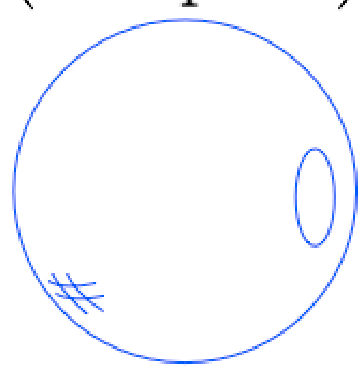
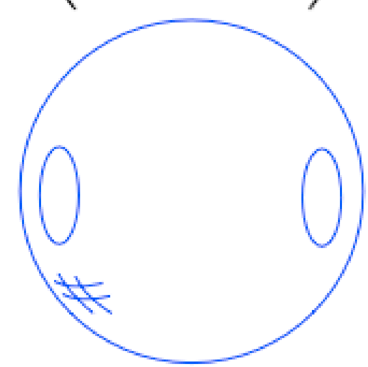




g_s^{-1}

Language of Perturbation Theory

Some more diagrams...

	g_s^{-2}	g_s^{-1}	g_s^{0}
closed oriented	sphere S^2 (plane) 	·	torus T^2 
open oriented	·	disc D_2 (half–plane) 	cylinder C_2 (annulus) 

Spacetime Physics

Can also rescale metric to present action differently:

set this to be asymptotic value of dilaton

$G_{\mu\nu}(x)$

$B_{\mu\nu}(x)$

$\Phi(x)$

$$\tilde{G}_{\mu\nu}(X) = e^{2\Omega(X)} G_{\mu\nu} = e^{4(\Phi_0 - \Phi)/(D-2)} G_{\mu\nu}$$

$$\begin{aligned} S \;=\; & \frac{1}{2\kappa^2} \int d^D X (-\tilde{G})^{1/2} \Big[R - \frac{4}{D-2} \nabla_\mu \tilde{\Phi} \nabla^\mu \tilde{\Phi} \\ & - \frac{1}{12} e^{-8\tilde{\Phi}/(D-2)} H_{\mu\nu\lambda} H^{\mu\nu\lambda} - \frac{2(D-26)}{3\alpha'} e^{4\tilde{\Phi}/(D-2)} + O(\alpha') \Big] \end{aligned}$$

Spacetime Physics

$$G_{\mu\nu}(x)$$
$$B_{\mu\nu}(x)$$
$$\Phi(x)$$

Can also rescale metric to present action differently:

$$\tilde{G}_{\mu\nu}(X) = e^{2\Omega(X)} G_{\mu\nu} = e^{4(\Phi_0 - \Phi)/(D-2)} G_{\mu\nu}$$

$$S = \frac{1}{2\kappa^2} \int d^D X (-\tilde{G})^{1/2} \left[R - \frac{4}{D-2} \nabla_\mu \tilde{\Phi} \nabla^\mu \tilde{\Phi} - \frac{1}{12} e^{-8\tilde{\Phi}/(D-2)} H_{\mu\nu\lambda} H^{\mu\nu\lambda} - \frac{2(D-26)}{3\alpha'} e^{4\tilde{\Phi}/(D-2)} + O(\alpha') \right]$$

This is the low energy effective action in "Einstein frame". (Previous was "String frame").

Recall: $g_s = e^{\Phi}$

Closed string coupling sets Newton's constant $\kappa \equiv \kappa_0 e^{\Phi_0} = (8\pi G_N)^{1/2}$

Spacetime Physics

Similar story for the open string (all Neumann):

$A_\mu(x)$

$$\int_{\partial\mathcal{M}} d\tau\, A_\mu \partial_t X^\mu$$

Coupling to boundary of the worldsheet.

$$S = -\frac{C}{4}\int d^D X\, e^{-\Phi}\,\mathrm{Tr} F_{\mu\nu}F^{\mu\nu} + O(\alpha')$$

Effective spacetime action is Maxwell, with coupling set by dilaton.

g_s^{-1} "disc order"

\# of boundaries

Pert. theory is $g_s^{-\chi}$ where now $\chi = 2 - 2h - b$

Spacetime Physics

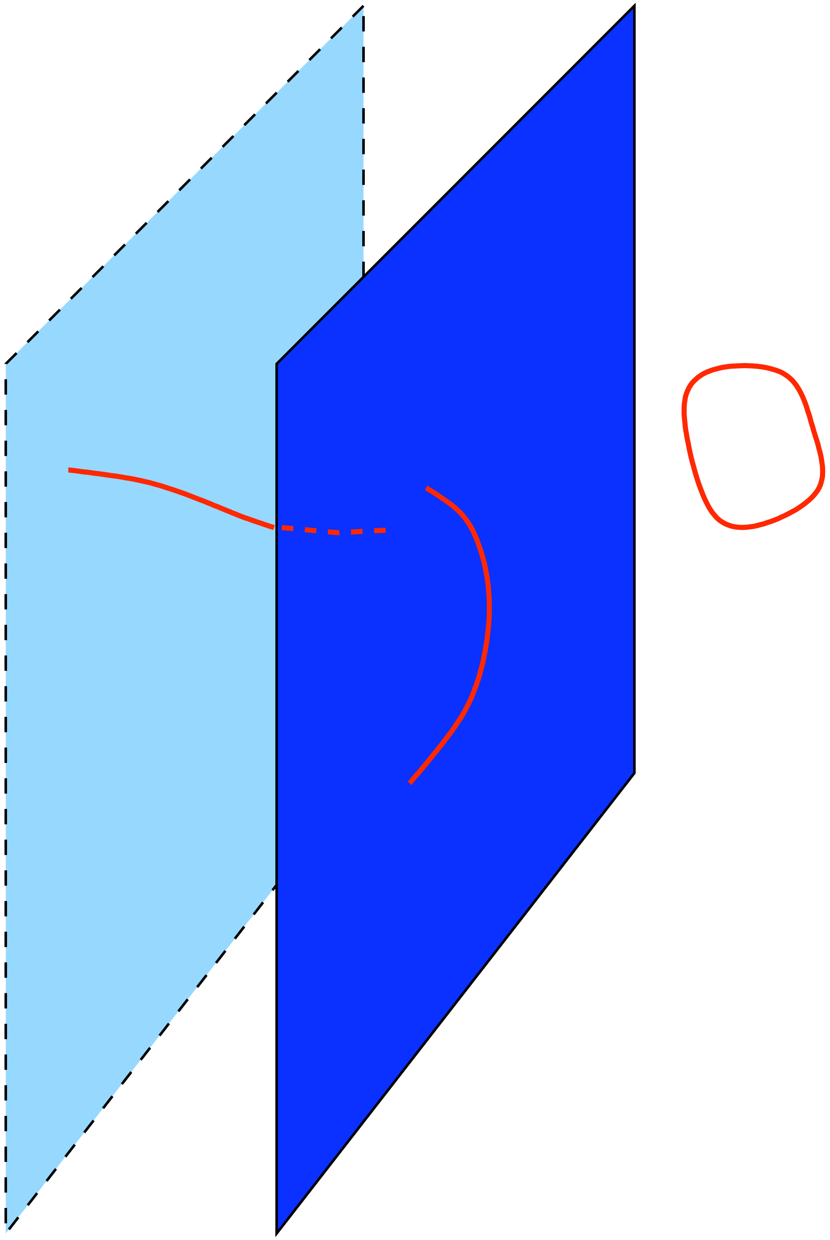
$A_\mu(x)$

When there are Dirichlet-Dirichlet and Dirichlet-Neumann directions also, there is a more geometrical language:

"D-branes"

D-Branes

Think of open string sectors as existing within a closed string theory, as hypersurfaces called "D-branes", where the endpoints lie.



p extended directions: **Dp-brane**

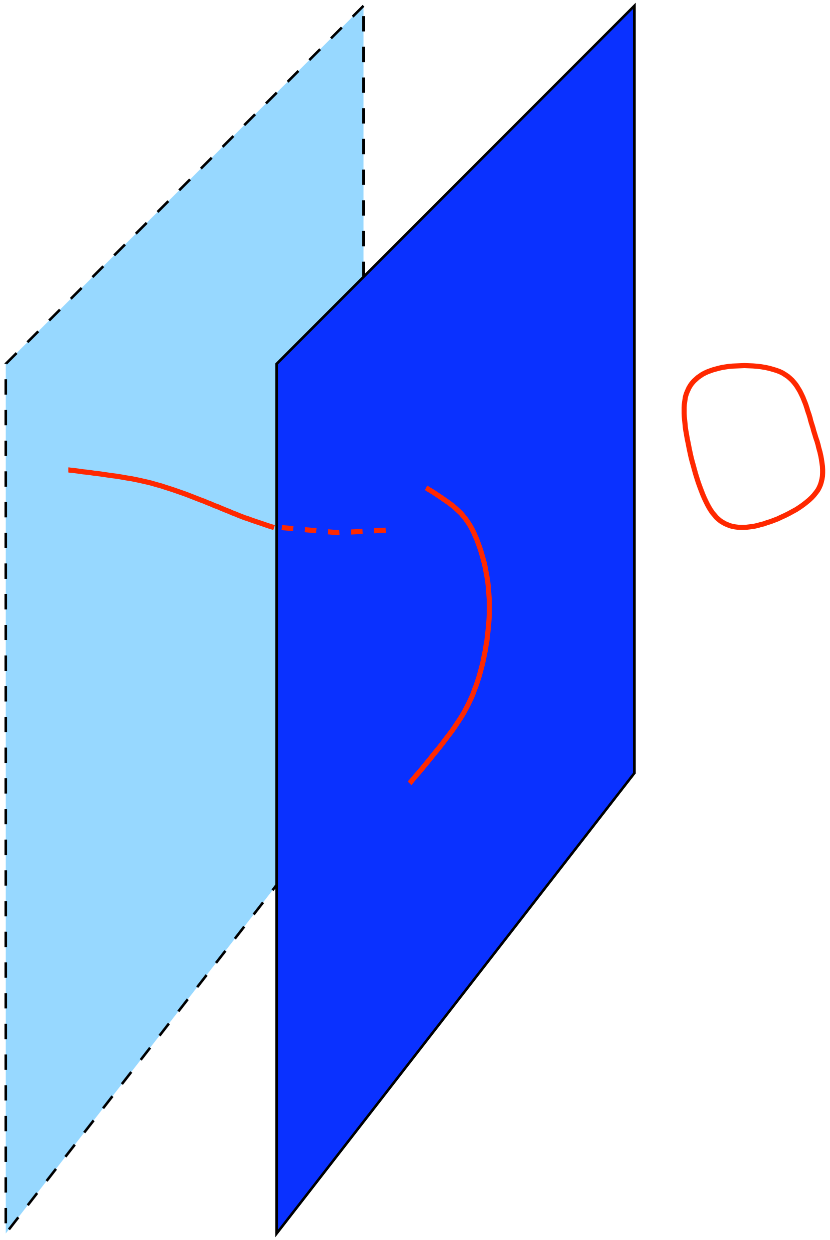
$p+1$ Neumann directions
D-p-1 Dirichlet directions

D-Branes

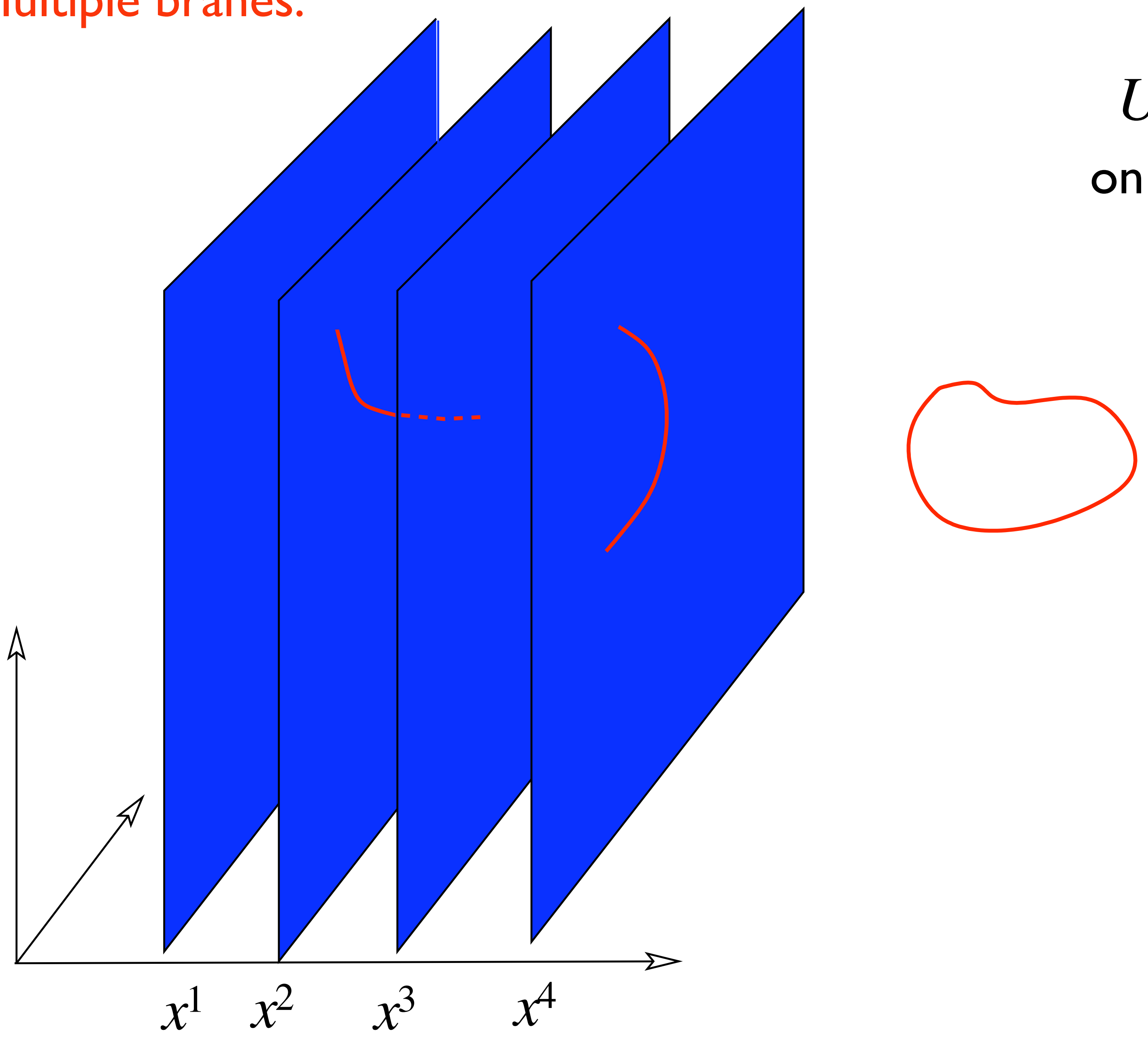
Open string degrees of freedom give a $U(1)$ gauge theory in the $(p+1)-$dimensions of its "worldvolume"

Vibrations of DN-strings and DD-strings appear as (charged) scalars in this world-volume theory.

p extended directions: **Dp-brane**



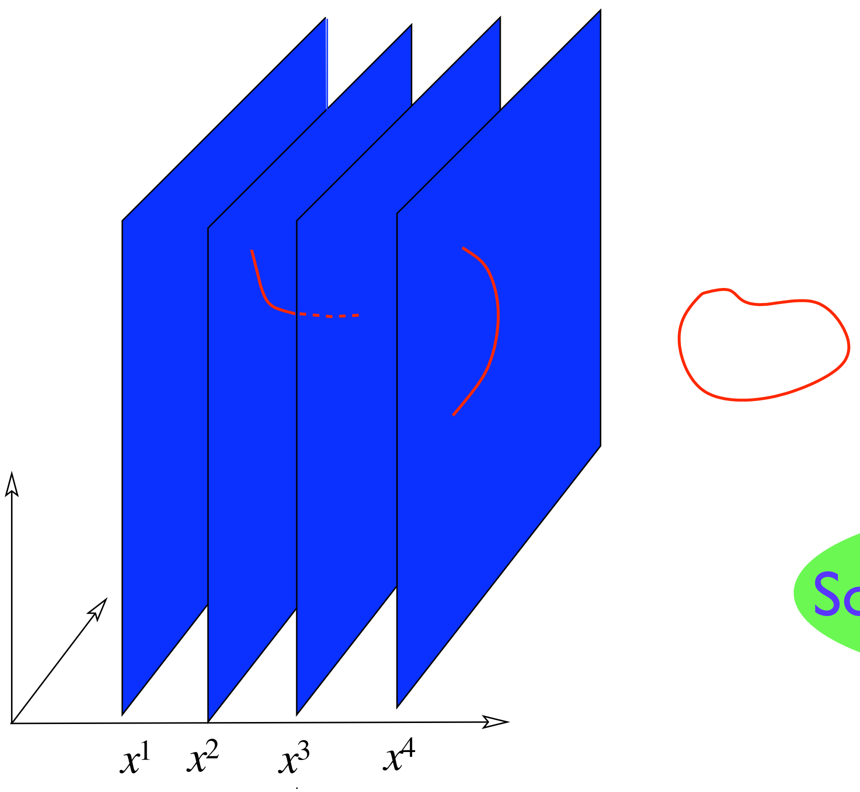
Multiple branes:



$U(N)$ gauge theory

on world-volume instead

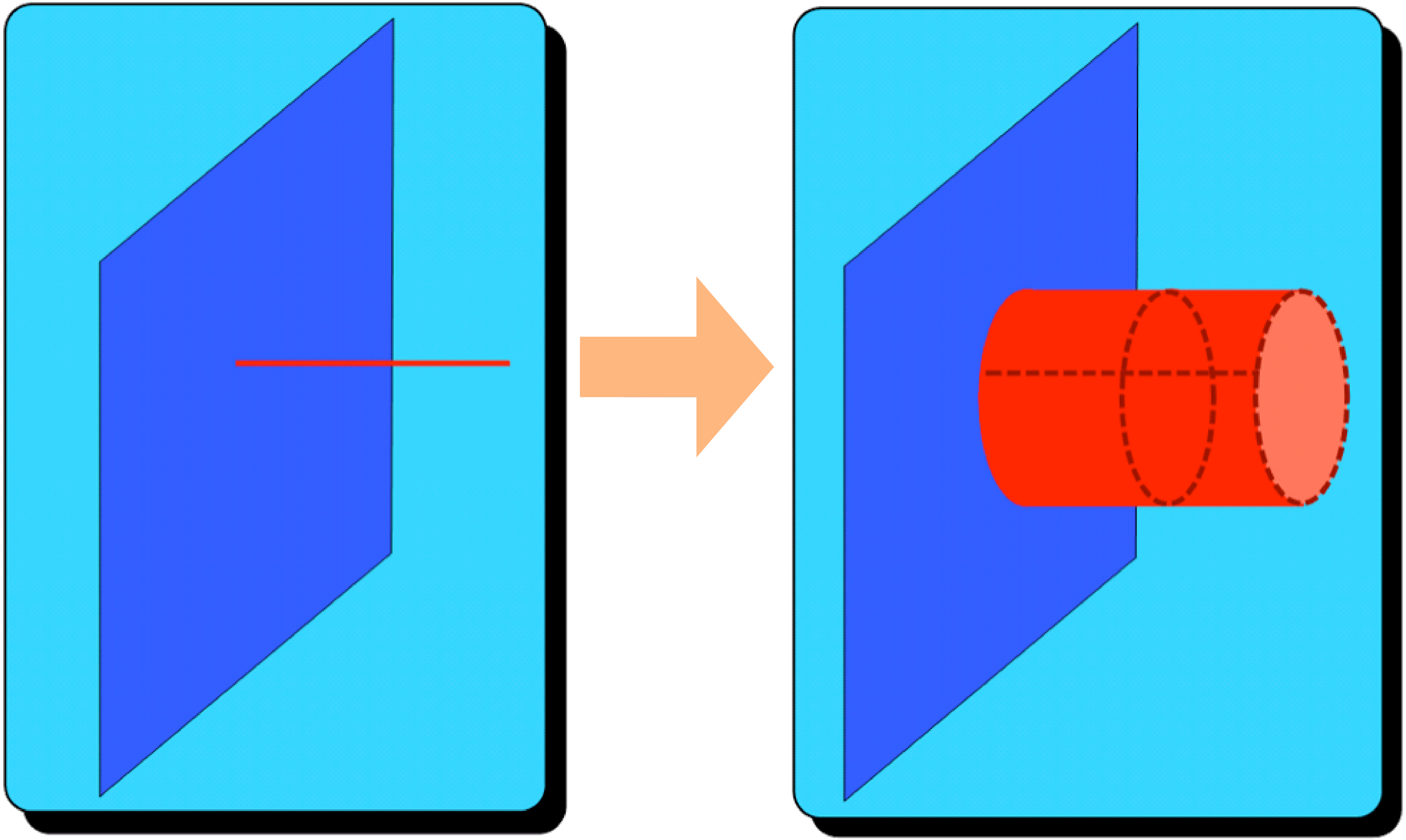
Multiple branes:



Pulling branes apart is a Higgs mechanism, from worldvolume perspective:

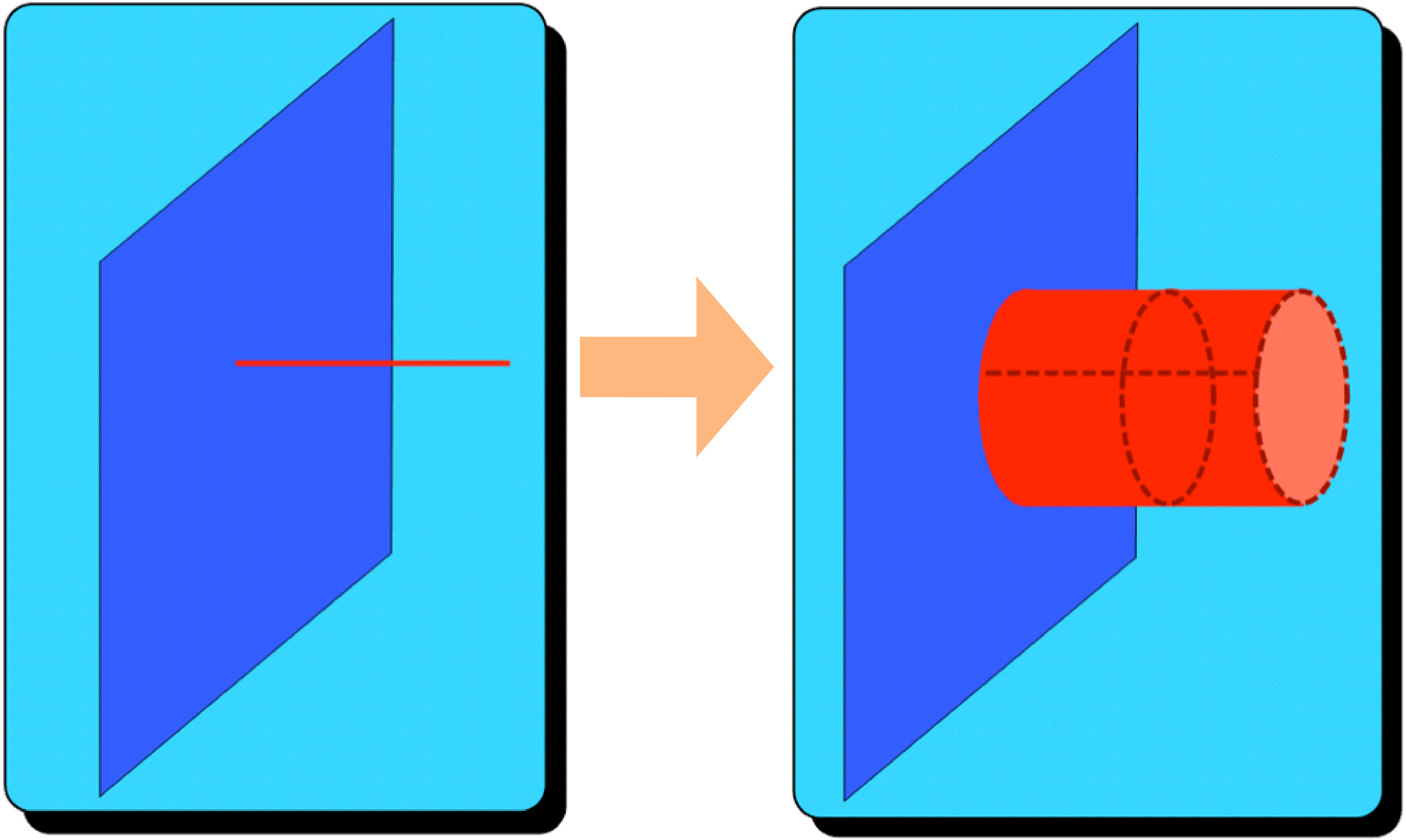
Some fields get vevs → Transverse flucts of strings with both ends on same brane

Some fields get masses → Transverse flucts of strings with ends on different branes



Within the open/closed description, one can describe the D-branes' natural sourcing of closed string fields.

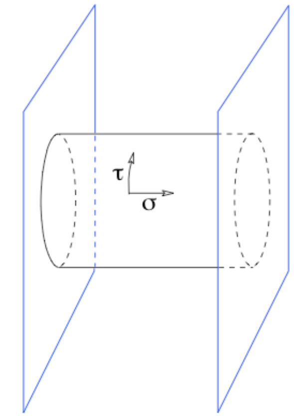
$G_{\mu\nu}, \Phi$



Sourcing gravity shows them to be genuine dynamical objects with mass per unit volume (tension).

$G_{\mu\nu}, \Phi$

Their tension can be explicitly computed using such diagrams, to fix contant C shown earlier:



$$S_p = -T_p \int d^{p+1}\xi \, e^{-\Phi} \det{}^{1/2}(G_{ab} + B_{ab} + 2\pi\alpha' F_{ab})$$

intrinsic coordinates
on D-brane

$$T_p = \frac{\sqrt{\pi}}{16\kappa_0}(4\pi^2\alpha')^{(11-p)/2}$$

"pull-back" of G, B

$$G_{ab} \equiv \frac{\partial X^\mu}{\partial \xi^a}\frac{\partial X^\nu}{\partial \xi^b} G_{\mu\nu}$$

physical tension goes inversely with string coupling:

$$\tau_p = \frac{T_p}{g_s}$$

(Get from this Dirac-Born-Infeld action to the Yang-Mills seen earlier by working at low energy.)

$$C = (2\pi\alpha')^2 T_{D-1}$$

Supersymmetric Strings

Easy to state what happens now that we have all the tools

Pretty similar story, with some extra wrinkles:

The "critical dimension" is $D=10$
The tachyon in not present
There are additional families of fields in the massless spectra

The basic closed strings are:

Type IIA
Type IIB
Heterotic
Heterotic

Can place D-branes (and other objects called "orientifolds" into these two)

Can place objects called "NS5-branes" into all of these theories

Can make the prototype open string theory:

Type I

More later!

Taking Stock

We've developed a great deal of the language of perturbative string theory.

(We'll develop a lot more of it -the supersymmetric story- on the board!)

We've seen several positive features:

The theory describes Quantum Gravity
The theory describes Gauge Theories

They fit into the same framework!

The theory generates its coupling dynamically
Only adjustable parameter seems to be one basic length scale

But there are several features of concern:

We had no means of describing how the theory generates its vacua
Had to place it into different solutions by hand
Are there non-perturbative subtleties/mechanisms which radically modify the picture?

How do we get a grip on non-perturbative features...?