Potentials for Light Moduli in Supergravity and String Theory

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Abstract

This is an acount of lectures that were given at TASI 2005 and the Shanghai Summer School in M-theory 2005 and the Perimeter Institute. I review 1) the derivation of the potential for chiral scalar fields in $\mathcal{N}$=1 supergravity 2) Relation between F and D terms for chiral scalars, Weyl anomalies and the generation of non-perturbative terms in the superpotential 3) The derivation of effective potentials for light moduli in type IIB string theory.

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I. INTRODUCTION

These lectures are aimed at giving a logically consistent acount of recent work on potentials for moduli in string theory. To this end I have tried to give a systematic presentation of the supergravity formulae that are at the basis of these discussions and to show how these potentials will arise in string theory (in particular in type IIB compactified on Calabi-Yau orientifolds with D-branes and fluxes). The lectures are organized as follows:

1. I discuss $\mathcal{N} = 1$ global supersymmetic actions for chiral scalar superfields in superspace formalism and derive the potential. I motivate the construction of the corresponding supergravity action coupled to chiral scalars and derive the F-term potential for the latter.

2. I discuss the coupling of gauge fields in global and local supersymmetry and the D-term contribution to the potential. In particular a relation between the F and D terms is derived. Finally I discuss the issue of Weyl anomalies and the derivation of the non-perturbative contribution to the superpotential from gauginoi condensation.

3. I discuss the derivation of potentials for moduli in type IIB supergravity following the work of Giddings, Kachru and Polchinski, and Kachru, Kallosh, Linde and Trivedi and how heavy moduli may be integrated out in supergravity theories to give effective theories for light moduli.

Sections 1 and 2 depend heavily on two classic text books, Wess and Bagger [1] (whose notation and conventions I use also) and Gates, Grisaru, Rocek and Siegel [2]. Section 2. also uses the work of [3][4]. Section 3 begins with a short review of [5] and[6]. I should mention here that these works are in turn based on earlier work on flux copactifications such as [7][8][9][10][11][12][13][14][15] even though I do not use these directly. Then I reveiw some of my own attempts [16][17][18, 19][20] to understand the logical structure of these derivations. It should be noted that these lectures represent my personal view on these issues rather than being a compehensive review. While they contain references to other work on these matters the list is far from being complete. The focus of these lectures is on supergravity and the derivations of moduli potentials. For comprehensive reviews of the subject of flux compactifications as such the reader should consult the recent reviews [21][22][23] [24].

II. POTENTIAL FOR CHIRAL SCALARS IN $\mathcal{N}=1$ SUPERGRAVITY

String phenomenology is based on integrating out string states and Kaluza-Klein (KK) modes to get an $\mathcal{N}=1$ supergravity (SUGRA). The standard argument in the context of superstring theory (which so far has only an on-shell S-matrix formulation)) is that one needs to compactify the extra six dimensions of string theory, on a manifold which admits one killing spinor. This ensures that the four dimensional theory has supersymmetric vacua. The fluctuations around such a vacuum must necessarily be described by four dimensional supergravity, and by requiring there is no more than one killing spinor, guarantees that there is only one four dimensional supersymmetry. Once one has derived this $\mathcal{N}=1$ SUGRA it is possible to argue that the non-supersymmetric solutions (if they exist) are also (low energy) solutions to string theory. This is the approach that is normally taken in string phenomenology and that is what we will adopt.

We will only discuss $\mathcal{N}=1$ supersymmetry in these notes so from now on this should understood. The notation and conventions are as in Wess and Bagger [1].

A. Global supersymmetry in Superspace

Global supersymmery is defined by a Weyl spinor supercharge Q_α $\alpha=1,2$ and its complex conjugae $\bar{Q}_{\dot{\alpha}}$ that transform bosons into fermions. The most convenient formulation of supersymmetry is in terms of superfields which may be viewed as fields that live in a space (superspace) with superspace coordinate $z^M=\{x^m,\theta^\mu,\theta_{\dot{\mu}}\}$with x^m, $m=0,..3$ being the usual space-time coordinates and θ^μ, $\mu=1,2$ (and its complex conjugate) being fermionic coodinates represented by Grassman numbers. The supercharge is then essentially a translation operator in the fermionic direction $Q_\alpha=\partial_\alpha-i\sigma^m_{\alpha\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}\partial_m$ (with $\partial_\alpha=\partial/\partial\theta_\alpha$, $\partial_m=\partial/\partial x^m$and $\sigma^m$are the Pauli matrices for $m=1,23$,and the unit matrix for $m=0$. The conjugate supercharge is $\bar{Q}_{\dot{\alpha}}=-\partial_{\dot{\alpha}}+i\theta^\alpha\sigma^m_{\alpha\dot{\alpha}}\partial_m$. The supercharges satisfy the algebra

$$\{Q_\alpha,\bar{Q}_{\dot{\alpha}}\} = 2i\sigma^m_{\alpha\dot{\alpha}}\partial_m \tag{1}$$

$$\{Q_\alpha,Q_\beta\}=0 \qquad \{\bar{Q}_{\dot{\alpha}},\bar{Q}_{\dot{\beta}}\}=0 \tag{2}$$

A general superfield has many terms and is a reducible representation of supersymmetry. The simplest irreducible representation is the chiral scalar superfield which in a sense depends only on the thetas and not on their complex conjugates. To make this statement in a supersymmetric way we introduce the operator $D_\alpha = \partial_\alpha + i\sigma^m_{\alpha\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}\partial_{\dot{\alpha}}$ such that it and its conjugate commute with all the super charges (and give another representaion of the superalgebra). Thus a chiral scalar superfield Φ is defined by the constraint

$$\bar{D}_{\dot{\alpha}}\Phi = 0, \tag{3}$$

whose solution is (in terms of a bosonic field $A(x)$ a fermionic field $\psi_\alpha(x)$ and an auxiliary (i.e. non-propoagating) field $F(x)$ is,

$$\Phi = A(y) + \sqrt{2}\theta^\alpha\psi_\alpha(y) + \theta^\alpha\theta_\alpha F(y)$$

where $y = x^m + i\theta\sigma^m\bar{\theta}$. The conjugate of this chiral superfield would be anti-chiral. Note that fermionic indices are raised and lowered with $\epsilon_{\alpha\beta}$: $\epsilon_{21} = \epsilon^{12} = 1$. Supersymmetry transformations take A to ψ the latter to F. The highest component of a superfield (in this case F) transforms into a total derivative. Also note that since $D^3 = \bar{D}^3 = 0$, the operator $(\bar{D}^2)\bar{D}^2$ is a (anti-) chiral projector.

The next simplest irreducible repersentation is the real superfield (V) defined by

$$V = V^\dagger,\ \text{givingcomponents}\{c, \chi^\alpha, \bar{\chi}^{\dot{\alpha}}, M, N, \lambda^\alpha, \lambda^{\dot{\alpha}}, v_m, D\}. \tag{4}$$

Note that the definition is ambiguous upto Kaehler gauge transformations

$$V \to V + \Lambda + \Lambda^\dagger \tag{5}$$

where Λ is a chiral superfield. Looking at the transformations of the components it is easy to see that the first five compnents are gauge degrees of freedom so that in a particular gauge (the so-called Wess-Zumino (WZ) gauge a real super field has a fermion (so-called gaugino) its conjugate, a real vector field (these are physical propagating fields) and a scalar auxiliary field D. Again the highest component, i.e. D, transforms into a total derivative. It is often convenient to redefine components in terms of the D_α operator. Thus for a chiral superfield the scalar the fermion and the auxiliary F field may be defined by,

$$A = \Phi|,\ \psi_\alpha = \frac{1}{\sqrt{2}}D_\alpha\Phi|,\ F = -\frac{1}{4}D^\alpha D_\alpha\Phi| \tag{6}$$

where the vertical bar is an instruction to set $\theta = \bar{\theta} = 0$ after performing the operations on the superfield. Also the D-term in a real field would now be defined by

$D = (-\frac{1}{4}\bar{D}_{\dot{\alpha}}\bar{D}^{\dot{\alpha}})(-\frac{1}{4}D_\alpha D^\alpha)\mathcal{V}|$. The product of two chiral superfields would also be chiral while the product of a chiral field and its conjugate would be a real superfield. Note also that under the space-time integral sign,

$$\int d^2\theta \to -\frac{1}{4}D_\alpha D^\alpha, \text{ and} \int d^4\theta \equiv \int d^2\theta d^2\bar{\theta} \to \frac{1}{16}\bar{D}^2 D^2 \tag{7}$$

We may now write the most general globally supersymmetric action for chiral superfields as

$$S = \int d^4x \left[\int d^4\theta K(\Phi, \bar{\Phi}) + \left(\int d^2\theta W(\Phi) + h.c.\right)\right] \tag{8}$$

In the above W the so-called superpotential is a holomorphic function of Φ and so is a chiral field whilst K the so-called Kaehler potential is a real superfield. For example W could be a polynomial in Φ while $K = \Phi\bar{\Phi}$. The integrals over the thetas are essentially instructions to pick the D or the F terms of the corresponding integrals and since these transform into total derivatives the action above is invariant under supertransformations. Clearly in the above we may replace the superfield Φ by a set of superfields $\{\Phi^i\}$.

The action also has invariance under Kaehler transformations,

$$K(\Phi, \bar{\Phi}) \to K(\Phi, \bar{\Phi}) + \Lambda(\Phi) + \bar{\Lambda}(\bar{\Phi}), \tag{9}$$

where Λ is an arbitrary chiral superfield.

Let us compute the potential for scalar fields from (8). Retaining just the scalar contributions and using (7)(6) we have

$$\int d^4\theta K \to K_{i\bar{j}}F^i F^{\bar{j}},\ \textstyle\int d^2\theta W \to F^i W_i \displaystyle\int d^2\bar{\theta}\bar{W} \to \bar{F}^{\bar{i}}\bar{W}_{\bar{i}}. \tag{10}$$

Hence we have

$$-\mathcal{V} = K_{i\bar{j}}F^i F^{\bar{j}} + F^i W_i + \bar{F}^{\bar{i}}\bar{W}_{\bar{i}}, \tag{11}$$

where $W_i = \partial W/\partial\Phi^i$ and $K_{i\bar{j}} = \partial_i\partial_{\bar{j}}K$.

The auxiliary field has an algebraic equation of motion - $\partial\mathcal{V}/\partial F^i = K_{i\bar{j}}F^{\bar{j}} + W_i = 0$. Hence we may rewrite the potential as

$$\mathcal{V} = K_{i\bar{j}}F^i\bar{F}^{\bar{j}} = K^{i\bar{j}}W_i\bar{W}_{\bar{j}} \tag{12}$$

It is useful to note that the potential can be computed either from evaluating just the F-term in the action (i.e. the $\int d^2\theta$ term) or its conjugate, or the D-term (the $\int d^4\theta$ term) with reversed sign.

Let us now compute the superfield equation of motion. To do this it is convenient to use the first replacement in (7) to write the action as

$$S = \int d^4x \int d^2\theta[-\frac{1}{4}\bar{D}^2 K(\Phi, \bar{\Phi}) + W(\Phi)] + \int d^2\bar{\theta}\bar{W}(\bar{\Phi}). \tag{13}$$

When this action is varied w.r.t. the chiral field, the variation can be pulled through the operator $\bar{D}$, so that we immediately get the field equation,

$$\frac{1}{4}\bar{D}^2 K_i = W_i \tag{14}$$

which may be rewritten as

$$\frac{1}{4}K_{i\bar{j}}\bar{D}^2\bar{\Phi}^{\bar{j}} + \frac{1}{4}K_{i\bar{j}\bar{l}}\bar{D}_{\dot{\alpha}}\bar{\Phi}^{\bar{j}}\bar{D}^{\dot{\alpha}}\bar{\Phi}^{\bar{l}} = \frac{\partial W}{\partial \Phi^i} \tag{15}$$

The lowest component of this equation gives the auxiliarly field equation $\bar{F}^{\bar{j}} = -K^{\bar{j}i}W_i + fermion\, terms$. Operating on it once with $\bar{\mathrm{D}}_{\dot{\alpha}}$and then setting the thetas to zero gives the fermion equation and doing this twice gives the bosonic equation

$$\partial^2\bar{\Phi}^{\bar{j}} = K^{\bar{j}i}\frac{D^2}{4}W_i + \ldots = -K^{\bar{j}i}W_{il}F^l + \ldots = K^{\bar{j}i}\frac{\partial \mathcal{V}}{\partial \Phi^i} + \ldots, \tag{16}$$

where the ellipses represent terms which are quadratic in the fermions and we've used the identity

$$\frac{1}{16}D^2\bar{D}^2\bar{\Phi} = \partial^2\bar{\Phi}.$$

B. Supergravity in Superspace

The global SUSY action is invariant under the constant SUSY transformations

$$z^M \to z^M - \xi^M,\, z^M \equiv \{x^m, \theta^\mu, \theta_{\dot{\mu}}\} \tag{17}$$

Local SUSY transformations naturally lead to general coordinate transformations since the anti-commutator of two supercharges generate translations, which in this case become general coordinate transformations. Thus the above transformations are replaced by the following general coordinate transformations (GCT) in superspace:

$$x^m \;\to\; -i(\theta\sigma^m\bar{\xi}(x) - \xi(x)\sigma^m\bar{\theta}) \tag{18}$$

$$\theta^\mu \to \theta^\mu - \xi^\mu(x), \qquad \bar{\theta}_{\dot{\mu}} \to \bar{\theta}_{\dot{\mu}} - \bar{\xi}_{\dot{\mu}}(x) \tag{19}$$

or $z^M \to z'^M - \xi^M(z)$.

We need to to intoduce a set of frames with Lorentz superspace indices $A = \{a, \alpha, \dot{\alpha}\}$[35] with the supervielbein one-form in suerspace being written as

$$E^A = dz^M E_M^A(z);\; E_M^A E_A^N = \delta_M^N;\; E_A^M E_M^B = \delta_A^B. \tag{20}$$

These are taken to have the following transformation properties:

$$\delta_\xi E_M^A = -\xi^N \partial_N E - (\partial_M \xi^N) E_N^A \tag{21}$$

under superspace GCT, and

$$\delta E_M^A = E_M^B L_B^A(z), \text{ where} L_B^A = \{L_b{}^a, L_\beta^\alpha, L^{\dot{\beta}}{}_{\dot{\alpha}}\}$$

(with the different L's being matrices in the Lorentz algebra in the vector and weyl (anti) spinor representations. One may also define a Lorentz connection one form

$$\phi = dz^M \phi_M \text{ with } \phi_M = \{\phi_{MB}{}^A\} \tag{22}$$

which transforms as

$$\delta\phi = \phi L - L\phi - dL \tag{23}$$

under Lorentz transformations. Also one defines a torsion superfield two-form by,

$$T^A = \nabla E^A = dE^A + E^B \phi_B^A = \frac{1}{2} dz^M dz^N T_{NM}^A = \frac{1}{2} E^C E^B T_{BC}^A. \tag{24}$$

It is important to realize that torsion on superspace does not vanish even if the metric is flat. Thus in flat space we have the super-covariant derivative with $D_A = e_A^N \partial_N$. The inverse of the matrix defines the flat space vielbein one form by $E^A = e_M^A dz^M$with $e_a^m = \delta_a^m$, $e_\alpha^m = i\sigma_{\alpha\dot{\alpha}}^m \bar{\theta}^{\dot{\alpha}} \ldots$. In flat superspace there exist transformations to coordinates in which $\phi = 0$ but torsion has non-vanishing components

$$T_{\alpha\dot{\beta}}^c = T_{\dot{\beta}\alpha}^c = 2i\sigma_{\alpha\dot{\beta}}^c. \tag{25}$$

As in ordinary differential geometry here too one can define a curvature two form by

$$\mathcal{R}_B^A = d\phi_B^A + \phi_B^C \wedge \phi_C^A \tag{26}$$

Thus one needs to solve

$$\nabla\nabla E^A = \nabla T^A = E^B \mathcal{R}_B^A \tag{27}$$

subject to torsion constraints such as (25) except that not all of the other components vanish. The result is that the minimal number of independent components of the supervielbein are

$$e_m^a,\ \psi_m^\alpha,\ \bar{\psi}_{m\dot{\alpha}}\,,M,\ b_a=\bar{b}_a \tag{28}$$

where the first is the usual vielbein, the second and third are the gravitino and its conjugate, and the last two are a complex scalar auxiliary field and a real vector auxiliarly field. In addition it turns out that all components of the torsion and the curvature superfields can be expressed in terms of three superfields

$$R,\ G_{\alpha\dot{\alpha}},\ W_{\alpha\beta\gamma} \tag{29}$$

subject to constraints such as the covariant chirality constraint

$$\bar{\nabla}_{\dot{\alpha}}R=0. \tag{30}$$

In SUGRA the chirality projector $\bar{D}^2$ of global supersymmetry is replaced by

$$\bar{\nabla}_{\dot{\alpha}}\bar{\nabla}^{\dot{\alpha}}-8R. \tag{31}$$

It turns out also that

$$R(z)|=-\frac{1}{6}M(x)\ldots,\nabla^\alpha\nabla_\alpha R(z)|=\frac{1}{3}e_a^me_b^n\mathcal{R}_{mn}^{\ \ ab}(x)+\ldots,,G_a(z)|=-\frac{1}{3}b_a(x). \tag{32}$$

C. General Matter SUGRA Action

Now we can discuss the action for supergravity coupled to chiral scalar matter. It is useful to rewrite the supervieldbein E_m^ain terms of a real axial vector field superfield H_m, and a chiral superfield φ (called the chiral compensator) which satisfies the *flat space* chirality constraint $\bar{D}_{\dot{\alpha}}\varphi=0$. It turns out that the graviton and the gravitino as well as the axial vector auxiliary field (in Wess-Zumino gauge) are contained in H_m. Also the superdeterminant[36] of the supervielbein may be written as

$$\mathbf{E}\equiv \mathrm{sdet}E_m^a=\mathcal{E}(H)\bar{\varphi}\varphi \tag{33}$$

For any superfield $\mathbf{L}$ there is a useful identity that relates different expressions for superfield actions found in the literature,

$$-\frac{1}{4}\bar{D}^2\mathbf{EL}=\varphi^3(-\frac{1}{4}\bar{\nabla}^2+2R)\mathbf{L}. \tag{34}$$

Setting $\mathbf{L} = R^{-1}$we get since $\bar{\nabla}R = 0$,

$$\varphi^3 = -\frac{1}{4}\bar{D}^2\frac{E}{2R} \tag{35}$$

consistent with the fact that the RHS is flatspace chiral. Also setting $\mathbf{L} = W(\Phi)/2R$, where $W$is a holomorphic function of the covariantly chiral scalar superfield $\Phi : \bar{\nabla}_{\dot{\alpha}}\Phi = 0$, we get

$$\varphi^3 W = -\frac{1}{4}\bar{D}^2\frac{\mathbf{E}}{2R}W(\Phi). \tag{36}$$

This identity gives us two equivalent ways of writing the SUGRA invariant term corresponding to the superpotential term in (8)

$$\int d^4xd^4\theta\frac{\mathbf{E}}{2R}W(\Phi) = \int d^4xd^2\theta\varphi^3W(\Phi). \tag{37}$$

The chiral density φ^3 is a measure for integrating (over half of superspace) any chiral term in the action.

The general SUGRA - chiral scalar action can then be written as

$$\begin{aligned} S &= -\frac{3}{\kappa^2}\int d^8z\mathbf{E}(H,\varphi)e^{-\frac{\kappa^2}{3}K(\Phi,\bar{\Phi})} \\ &\quad + \left(\int d^6z\varphi^3W(\Phi) + h.c.\right) \end{aligned} \tag{38}$$

In the above we've defined the superspace measures $d^8z \equiv d^4xd^4\theta$ and $d^6z \equiv d^4xd^2\theta$ and $\kappa^2 \equiv 8\pi G_N$. Let us motivate this: First observe that in the flat space limit $\kappa \to 0$, $\varphi \to 1$, $\mathbf{E} \to \mathbf{1}$, this reduces to the globally supersymmetric action (8). As in that case $W(\Phi)$, the superpotential, is a holomorphic function of the chiral superfields while $K(\Phi,\bar{\Phi})$ is a real superfield which is a function of the chiral fields and the anti-chiral fields. Secondly let us observe that the superfieldbein determinant contains the Einstein action.

$$-\frac{3}{\kappa^2}\int d^8z\mathbf{E} = -\frac{3}{\kappa^2}\int d^6z\varphi^3 2R = \frac{2}{\kappa^2}\int d^4xe\mathcal{R} + \ldots \tag{39}$$

In the first equality above we used the identity (34) with $\mathbf{L} = 1$, and in the second we've used $\phi^3| = e$ and $-\frac{1}{4}\nabla^2R| = -\frac{1}{12}\mathcal{R}$ where $e = \det[e_m^a]$ and $\mathcal{R}$ is the Ricci-scalar (see (32)).

D. Weyl Invariant Formalism

Before we go on to discuss the potential is useful to consider a slightly generalized formalism where the Weyl invariance becomes manifest. We introduce a chiral scalar ϕ(with

$\bar{\nabla}_{\dot{\alpha}}\phi = 0$). The action (38) (we will set $\kappa^2 = 1$ from now on) is generalized to

$$S = -3\int d^8z\mathbf{E}\phi\bar{\phi}e^{-K/3} + \left(\int d^8z\frac{\mathbf{E}}{2R}\phi^3W(\Phi) + h.c.\right)$$
$$= -3\int d^6z\varphi^3(-\frac{1}{4}\bar{\nabla}^2 + 2R)\phi\bar{\phi}e^{-K/3} + \left(\int d^6z\varphi^3\phi^3W(\Phi) + h.c.\right) \quad (40)$$

This action is invariant under Weyl transformations (with a Weyl transforamtion parameter chiral superfield $\tau : \bar{\nabla}_{\dot{\alpha}}\tau = 0$) given below.

$$\Phi \to \Phi,\ \phi \to e^{-2\tau}\phi, \quad \varphi \to e^{2\tau}\varphi,\ E^{\alpha}_{M} \to e^{(2\bar{\tau}-\tau)}(E^{\alpha}_{M} - \ldots)$$
$$E^{a}_{M} \to e^{(\tau-\bar{\tau})}E^{a}_{M},\ \mathbf{E} \to e^{2(\tau+\bar{\tau})}\tau \quad \nabla_{\alpha} \to e^{(\tau-2\bar{\tau})}(\nabla_{\alpha} - \ldots) \quad (41)$$

The omitted terms in the above are proportional to the covariant spinor derivatives of τ.

These transformations are actually an invariance of the torsion and chirality constrains of minimal supergravity. The Weyl compensator ensures that the action is also invariant.

The action is also invariant under Kaehler transformations

$$K \to K + f((\Phi,\phi) + \bar{f}(\bar{\Phi},\phi),\ W \to e^{-f(\Phi,\phi)}W,\ \phi \to e^{f(\Phi,\phi)/3}\phi \quad (42)$$

Note that unlike in the case of global SUSY here the superpotential is not invariant under Kaehler transformations. However the quantity

$$G = K + \ln|W|^2 \quad (43)$$

is invariant and in fact Kaehler invariance implies that the action is only dependent on this Kaehler invariant combination and not separately on $K$and W. It also means that only the zeroes and singularities of W have an invariant significance. Away from them it can be trnasformed to unity.

E. Calculating the Potential

To discuss the potential it is sufficient to look at conformally flat metrics $g_{mn} = \sigma^2\eta_{mn}$. This amounts to ignoring the complications coming from the fields H_m and therefore in effect also replacing $\nabla_\alpha \to D_\alpha$. In other words for the purpose of deriving the potential one may just consider the action,

$$S = -\frac{3}{\kappa^2}\int d^8z\bar{\phi}\phi e^{-\frac{\kappa}{3}K(\Phi,\bar{\Phi})} + \left(\int d^6z\phi^3W(\Phi) + h.c.\right) \quad (44)$$

Note that in the above we have chosen to fix the Weyl gauge by putting $\varphi = 1$. Also Φ represents a set of chiral scalr superfields $\{\Phi^i\}$. Alternatively we could have chosen $\phi = 1$ in which case the above would have been rewritten in terms of φ. The Weyl invariance implies that we can switch the one for the other. We can derive the equations of motion from this action following the same procedure as in the global SUSY case (see discussion around (14, 15)). We get (after setting $\kappa = 1$)

$$-\frac{1}{4}\bar{D}^2(\Phi e^{-K/3}) = \phi^2 W \tag{45}$$

$$-\frac{1}{4}\bar{D}^2(\phi\bar{\phi}e^{-K/3}K_i) = -\phi^3 W_i. \tag{46}$$

In the above the subscript i denotes differentialtion with respect to the ith chiral scalar field Φ^i. Ignoring fermionic terms ($D_\alpha\Phi$, $D_\alpha\phi$ etc) we get by taking the lowest components of the above equations (with $F_\phi = -(1/4)D^2\phi$ etc.

$$\bar{F}_{\bar{\phi}}e^{-K/3}| - \bar{\phi}\frac{1}{3}K_{\bar{i}}F^{\bar{i}}e^{-K/3}| = \phi^2 W| \tag{47}$$

$$\phi[-\frac{1}{4}\bar{D}^2(\bar{\phi}e^{-K/3})]K_i| + \phi\bar{\phi}e^{-K/3}K_{i\bar{j}}\bar{F}^{\bar{j}} = -\phi^3 W_i \tag{48}$$

Solving for the two F-terms we have,

$$\bar{F}_{\bar{\phi}} = e^{K/3}\phi^2 W + \frac{1}{3}\bar{\phi}K_{\bar{i}}\bar{F}^{\bar{i}} \tag{49}$$

$$\bar{F}^{\bar{j}} = -\frac{\phi^3}{\phi\bar{\phi}}e^{K/3}K^{i\bar{j}}D_i W \tag{50}$$

where we've defined the Kaehler covariant derivative

$$D_i W = W_i + K_i W \tag{51}$$

The potential may now be calculated from the superpotential term in the action (44) - see discussion around 12

$$-\mathcal{V} = \int d^2\theta\phi^3 W|_{boson} = -\frac{1}{4}D^2(\phi^3 W)|_{boson} \tag{52}$$

$$= 3\phi^2 F_\phi W + \phi^3 W_i F^i. \tag{53}$$

Using the expressions (49,50) we get

$$\mathcal{V} = \phi^2\bar{\phi}^2 e^{K/3}(K^{i\bar{j}}D_i W D_{\bar{j}}\bar{W} - 3|W|^2) \tag{54}$$

This potential is not quite in the canonical form since we are not in the Einstein frame. Recall that the gravitational action is hidden in the term

$$-3\int d^8z\phi\bar{\phi}\mathcal{E}(H)e^{-K/3} \tag{55}$$

To get the canonical form of the Einstein action (see 39) we need to choose

$$\phi| = \bar{\phi}| = e^{K/6}|$$

Then we finally get

$$\mathcal{V} = e^K(K^{i\bar{j}}D_iWD_{\bar{j}}\bar{W} - 3|W|^2) \tag{56}$$

This is the well-known F-term potential for chiral scalars coupled to supergravity first derived in [25].

We may rewrite this in terms of the Kaehler invariant potential G as (43)

$$\mathcal{V} = e^G(G^{i\bar{j}}G_iG_{\bar{j}} - 3) \tag{57}$$

F. Gauge Fields

Usually the Kaehler potential (metric) has isometries and if they are gauged we need to include gauge fields in the theory. A space-time dependent isometry of the Killing potential (for the moment consider just linear transformations) is represented on the chiral scalars by

$$\Phi \to e^{i\Lambda}\Phi,\ \bar{\Phi} \to \bar{\Phi}e^{-i\bar{\Lambda}}, \tag{58}$$

(where Λ is a chiral scalar superfield) is gauged by introducing a real superfield V, $V^\dagger = V$, which transforms as follows.

$$e^V \to e^{i\bar{\Lambda}}e^Ve^{-i\Lambda} \tag{59}$$

Note that in the above we've taken Φ to be a column matrix in some representation and V a square matrix in the adjoint representation $V = V^aT_a$ where T_a are the generators of the gauge group. Note that we need two copies of the gauge group $\mathcal{G} \to \mathcal{G} \times \mathcal{G}$, since the gauge transformations are reperesnted by a chiral superfield. As in the Abelian case one can go to the Wess-Zumino gauge in which

$$V = \{A_m, \lambda_\alpha, D\}, V^3 = 0. \tag{60}$$

Here $A_m$is the usual gauge field, λ is its fermionic partner the gaugino and D is an auxiliary field. Observe that under gauge transformations

$$\bar{\Phi}e^V \to \bar{\Phi}e^V e^{-i\Lambda}, \tag{61}$$

so that invariants are constructed out of Φ and $\bar{\Phi}e^V$. Thus for instance the global gauge invariant Kaehler potential $\bar{\Phi}\Phi$ is to be replaced by $\bar{\Phi}e^V\Phi$. In general we replace

$$K(\Phi, \bar{\Phi}) \to K(\Phi, \bar{\Phi}e^V) \tag{62}$$

,The gauge field strength is given by a chiral superfield

$$\mathcal{W}_\alpha = -(\frac{1}{4}\bar{\nabla}^2 - 2R)e^{-V}\nabla_\alpha e^V = \mathcal{W}^a_\alpha T_a \tag{63}$$

(note that the first factor is the chiral projector) whose components in the WZ gauge are $\mathcal{W}_\alpha = \{\lambda_\alpha, F_{mn}, D\}$. The kinetic terms for the gauge fields may then be written as

$$\int d^4x d^4\theta \frac{\mathbf{E}}{2R}(\frac{1}{4}f_{ab}\mathcal{W}^{a\alpha}\mathcal{W}^b_\alpha + h.c.), \tag{64}$$

where in general the gauge coupling function f_{ab}, an invariant tensor of the gauge group, may be a function of the chiral fields which are neutral under the group. The bosonic part of this action contains the usual gauge field kinetic term as well as the axion coupling term,

$$-\frac{1}{4}\int d^4x\sqrt{g}(\Re f_{ab}F^a_{mn}F^{bmn} - \frac{1}{2}\Im f_{ab}\epsilon^{mnpq}F^a_{mn}F^b_{pq})$$

Note that with the Weyl transformation rule $V \to V$ which implies $\mathcal{W} \to e^{-3\tau}W$, the above action is Weyl invariant without any Weyl compensator factor.

To derive the equations of motion for the gauge field from (64) we use the following trick (for simplicity we'll just consider the global SUSY case).

Define $\Delta V \equiv e^{-V}\delta e^V = \delta V + ... = \Delta V^a T^a$. Then

$$\begin{aligned} \delta_V \frac{1}{2}\int d^6z f_{ab}\mathcal{W}^{\alpha a}\mathcal{W}^b_\alpha &= \int d^6z f_{ab}\delta_V tr(T^a e^{-V}D^\alpha e^V)\mathcal{W}^b_\alpha \\ &= \int d^6z f_{ab} tr(T^a[e^{-V}D^\alpha e^V, \Delta V]\mathcal{W}^b_\alpha \\ &= \int d^6z f_{ab}(\nabla^\alpha \Delta V)^a \mathcal{W}^b_\alpha = -\int d^6z \Delta V^a \nabla^\alpha(f_{ab}\mathcal{W}^{\lfloor}_\alpha) \end{aligned}$$

thus giving the contribution $\nabla^\alpha(f_{ab}\mathcal{W}^{\lfloor}_\alpha)$ to the gauge field eom. Also note that under an infinitesimal gauge transformation, $\Delta_\Lambda V = -i\Lambda + ie^{-V}\bar{\Lambda}e^V \equiv -i\Lambda + i\tilde{\Lambda}$.

Now we are in a position to evaluate the D-term contribution to the potential for chiral scalars. As before in evaluating the potential contributions we ignore curvature terms and Lorentz connection terms. We make the Kahler potential term gauge invariant by writing $K(\Phi,\bar{\Phi}) \to K(\Phi,\bar{\Phi}e^V) + \xi trV)$, where we've also added a Fayet-Illiopoulos (FI) term when the guage group contains a U((1) generator. . So the equations of motion for the auxiliary fields and hence the potential can be derived from

$$S = -3\int d^8z\bar{\phi}\phi e^{-\frac{1}{3}(K(\Phi,\bar{\Phi}e^V)+\xi trV)} + \{\int d^6z(\phi^3W + f_{ab}\mathcal{W}^a\mathcal{W}^\lfloor) + h.c.\} \tag{65}$$

The invariance of Kgives,

$$(i\Lambda - i\tilde{\Lambda})\frac{\delta K}{\delta V} + \Lambda^a k_a^i K_i + \tilde{\Lambda}^a k_a^{\bar{i}} K_{\bar{i}} = 0$$

where k_a is the Killing vector corresponding to the gauge generator with label a. From this we have the relation

$$\frac{\delta K}{\delta V^a} = ik_a^i K_i \tag{66}$$

Under the gauge transformation the FI term transforms $\xi trV \to \xi trV + \xi tr(i\Lambda - i\bar{\Lambda})$. To make the action invariant we need to have the chiral compensator field also transform under U(1) gauge transformations as $\phi \to e^{itr\Lambda\frac{\xi}{3}}\phi$. Then in order to keep the superpotential term invariant we need

$$\delta(\phi^3W) = 3i\Lambda^a trT^a\frac{\xi}{3}\phi^3W + \phi^3\Lambda^a k_a^i\partial_i W = 0$$

giving us the useful relation

$$k_a^i\partial_i W = -i\xi trT^aW \tag{67}$$

It is convenient to redefine $\phi \to \phi/W^{1/3}$and eliminate the superpotential from the action (65) which is now rewritten with $K \to G = K + \ln|W|^2$and $W \to 1$. Then we have the following equations of motion.

$$\begin{aligned}
-\frac{1}{4}\bar{D}^2(\bar{\phi}e^{-\tilde{G}/3}) &= \phi^2 \\
-\phi\bar{\phi}e^{-\tilde{G}/3}\frac{1}{4}\bar{\nabla}^2 G_i &= -\phi^3 G_i - \frac{1}{4}f_{ab,i}(\Phi)\mathcal{W}^{\alpha a}\mathcal{W}^b_\alpha \\
-\phi\bar{\phi}e^{-\tilde{G}/3}ik_a^iG_i + \frac{1}{2}\nabla^\alpha(f_{ab}\mathcal{W}^b_\alpha) &+ \frac{1}{2}\bar{\nabla}^{\dot{\alpha}}(\bar{f}_{ab}\mathcal{W}^b_{\dot{\alpha}} = 0
\end{aligned}$$

In the above we have ignored fermionic terms and used (66) with $K \to G$. Also in getting the the second equation we used the first. The third equation gives the D-term ($D^a = \frac{1}{2}\nabla^\alpha \mathcal{W}_\alpha|$)

$$2\Re f_{ab} D^b = \phi\bar{\phi} e^{-\tilde{G}/3} |ik^{ai} G_i|$$

Choosing the Einstein frame (see discussion after ...) $\phi| = \bar{\phi}| = e^{\tilde{G}/6}| = e^{G/6}|$ (the last is valid in WZ gauge) we finally have,

$$2\Re f_{ab} D^b = ik^{ai} G_i| = ik^{ai} \frac{D_i W}{W}| = ik^{ai} K_i + \xi tr T^a \tag{68}$$

where in the last equality we've used the relation (67). The last form of the D-term is the more familiar one and may be used whether or not W is zero. However the penultimate form shows that at minima where the F-terms $D_i W$ are zero with ($W \neq 0$) i.e. at a generic supersymmeric AdS minimum, the D-term is also zero. In other words such a minimum cannot be lifted by D-terms even with an FI term.

Now we are in a position to canculate the addtional contribution to the potential from the gauge theory D-terms. These are

$$-\int d^2\theta \frac{1}{4} f_{ab} \mathcal{W}^{a\alpha} \mathcal{W}^b_\alpha|_{scalar} + h.c. = \frac{1}{2}\Re f_{ab}(\Phi)| D^a D^b$$

Thus the complete potential for chiral scalars coupled to gauge fields and supergravity is

$$\mathcal{V} = e^K (K^{i\bar{j}} D_i W D_{\bar{j}} \bar{W} - 3|W|^2) + \frac{1}{2}\Re f_{ab}(\Phi)| D^a D^b \tag{69}$$

with the D-term given by (68). This is the most important formula in string (and SUGRA) phenomenolgy.

G. Quantum effects and the Veneziano-Yankielowicz superpotential

At the quantum level the Weyl transformations discussed in subsection (II D) are anomalous. The origin of the anomaly is the same one-loop effect that causes chiral gauge transformations to be anomalous. In fact the Weyl transformations include an R-transformation. For putting $\tau(x, \theta) = i\nu$ we see from (41) that the chiral fermions transform as

$$\psi_\alpha \to e^{3i\nu} \psi_\alpha,\ \lambda_\alpha \to e^{-3i\nu} \lambda_\alpha,$$

where ψ is the fermion in the chiral scalar superfield and λ is the gaugino. The anomaly can be thought of as arising from the non-invariance of the functional integral measure (following Fujikawa's argument) and by supersymmetrizing the usual (ABJ) expression we have (note that by abuse of notation we are now labelling the gauge groups by a rather than the generators of a given group - the trace over each gauge group is being suppressed)

$$[d\Phi][d\bar{\Phi}][dV] \to e^{i\nu\frac{3c_a}{16\pi^2}\int d^8z\frac{E}{2R}(\mathcal{W}^{\alpha a}\mathcal{W}^a_\alpha+h.c.)}[d\Phi][d\bar{\Phi}][dV]. \tag{70}$$

The anomaly coefficient is given by

$$c_a = T(G_a) - \sum_r n_r T_a(r) \tag{71}$$

where $T_a(r) = tr_r(T_a^2)$ for a representation r of the gauge group G_a, and $r = G_a$means that the corresponding trace is evaluated in the adjoint representation. Note that by the standard Adler-Bardeen argument this anomaly is exact.

This anomaly needs to be cancelled since it is actually an anomaly in a local (super) Weyl symmetry which furthermore is necessary to obtain Einstein gravity. This can be done by modifying the gauge coupling function in the following manner [3]:

$$f_a(\Phi) \to f_a(\Phi,\phi) = f_a(\Phi) - \frac{3c_a}{8\pi^2}\ln\phi. \tag{72}$$

The cancellation of the anomalous transformation of the measure then follows from the Weyl transformation $\phi \to e^{-2\tau}\phi$ of the Weyl compensator.

If one of the gauge groups that contributes to this anomaly is confining (exhibits gaugino condensation and develops a mass gap as in QCD) then this quantum anomaly actually results in a non-perturbative addition to the classical superpotential [26] (the present argument is a simplified version of one given in [4]). Non-renormalization theorems imply that the classical superpotential is not corrected in perturbation theory but there is no such restriction non-perturbatively.

Suppose the non-perturbatively generated scale of the confining gauge group is Λ. Then one expects the gauginos to condense with $<\lambda\bar{\lambda}>\sim\Lambda^3\sim M^3e^{-1/g^2_{YM}(M)}$ and all fields that are charged under the gauge group to acquire masses of $O(\Lambda)$. In this case for energies $E \ll \Lambda$ we may integrate out this gauge theory giving an effective contribution to the low energy action of the form,

$$e^{-\Gamma(\Phi,\phi)} = \int [dV]\exp\left\{-\frac{1}{4}\int [f_a(\Phi) - \frac{3c_a}{8\pi^2}\ln\phi]\mathcal{W}^{a\alpha}\mathcal{W}_{a\alpha} + h.c.\right\} \tag{73}$$

(Note that here the chiral fields $\Phi$are neutral under the group - fields which are charged are integrated over). However (super) Weyl invariance tells us that the superpotetial term in action must come multiplied by a factor of ϕ^3. The above then tells us that the superpotential term in Γ must be of the form

$$\phi^3 W_{np} = w_a \phi^3 e^{-\frac{8\pi^2}{c_a} f_a(\Phi)}.$$

If there are several gauge groups that develop a mass gap then (below the lowest such scale) there will be a sum of such terms - one for each gauge group.

Also since the Kaehler potential in the action occures in combination with the Weyl compensator in the combination $\phi\bar{\phi}e^{-K/3}$ the anomaly cancelling term also generates a correction to the Kaehler potential so that the total Kaehler potential is given by the equation

$$e^{-K/3} = e^{-K_p/3} + e^{-K_{np}/3}$$

where $\exp(-K_{np}/3) = k_a \exp\{-\frac{8\pi^2}{3c_a}(f_a(\Phi)+\bar{f}_a(\bar{\Phi})\}$ and K_p is the perturbatively corrected classical Kaehler potential. Of course since unlike the superpotential there is no reason for the Kaehler potential not to be corrected in perturbation theory, these NP terms are less important.

III. POTENTIAL FOR MODULI IN IIB STRING THEORY

A. Classical equations and the flux potential

We will now discuss the derivation of the effective potential for the dilaton and the complex structure moduli in type IIB string theory compactified on Calabi-Yau (CY) orientifolds. We start with the (bosonic part of the) ten dimensional IIB action (in other words we are in the supergravity approximation i.e. at energy scales which are much less than the string scale ($E \ll M_s = 1/2\pi\alpha'$). The action is (putting $2\kappa_{10}^2 = 1$)

$$\begin{aligned} S &= \int d^{10}x\sqrt{g}\{R - \frac{1}{2\tau_I^2}\partial_M\tau\partial^M\bar{\tau} - \frac{1}{2.3!}\frac{1}{\tau_I}G_{MNP}\bar{G}^{MNP} \\ &- \frac{1}{4.5!}\tilde{F}_{MNPQR}\tilde{F}^{MNPQR}\} - \frac{1}{4i}\int\frac{1}{\tau_I}C_4\wedge G_3\wedge\bar{G}_3 \end{aligned} \quad (74)$$

Here we have put

$$\tau = C_0 + ie^{-\phi},\ G_3 = F_3 - \tau H_3,\ F_3 = dC_2,\ H_3 = dB_2 \quad (75)$$

$$\tilde{F}_5 = F_5 - \frac{1}{2}C_2\wedge H_3 \quad +\tfrac{1}{2}B_2\wedge F_3, \quad F_5 = dC_4 \quad (76)$$

In the above the numerical index denotes the rank of the corresponding form, $\phi$is the dilaton, C_i is a RR potential of rank i and B_2 is the NSNS potential.

Now we need to compactify this theory to four dimensions. For phenomenological reasons it is desirable that the four dimensional theory is $\mathcal{N}=1$ SUGRA. Since IIB has 32 supersymmetries compactification on a Calabi-Yau gives eight supersymmetries (i.e. $\mathcal{N}=2$) in four dimensions but if we orientifold, we can reduce this to $\mathcal{N}=1$. However it will turn out that tadpole cancellations (i.e. satisfaction of Gauss' law constraints) will require the addtion of D-branes and or fluxes in the internal directions. We will not get into the details here but it turns out that with a particular orientifold projection we will end up with orientifold three-planes and D3 and D7 branes (which actually is a limit of an F-theory construction). For the moment let us focus on just the three brane action whose leading terms take the form

$$S_{loc} = \sum_i (-T_3 \int_i d^4x \sqrt{g^{(4)}} + \mu_3 \int_i C_4) \tag{77}$$

This is an action that is localized at a set points i on the CY orientifold X where the D-branes (orientifolds) are located. The equations of motion/Bianchi identities coming from the total action $S + S_{loc}$ are,

$$\begin{aligned} R_{MN} - \frac{1}{2} g_{MN} R &= \frac{1}{2} T_{MN} \\ d\frac{*(G_3 + \bar{G}_3)}{2\tau_I} + H_3 \wedge \tilde{F}_5 &= 0 \\ d\frac{*(\tau\bar{G}_3 + \bar{\tau}G_3)}{2\tau_I} + F_3 \wedge \tilde{F}_5 &= 0 \\ d\tilde{F}_5 = H_3 \wedge F_3 &- \sum_i \mu_3 \delta_6^i \\ *F_5 &= F_5 \end{aligned}$$

In the above δ_6^i is a delta function on X localized at the point i where a brane/orientifold plane is located and the last is the self-duality condition for the five form which is imposed by hand as usual.

Now we need to do a Kaluza-Klein reduction of these equations to derive the effective four dimensional equations from these ten dimensional equations, when as we discussed above six of the dimensions are compactified on a Calabi-Yau orientifold X. This is unfortunately not as straightforward as one might think. The problem is that a simple truncation leads to inconsistencies.

One may try to derive the four dimensional effective action by introducing the metric ansatz

$$ds^2 = e^{2\omega(y)-6u(x)}\tilde{g}_{\mu\nu}(x)dx^\mu dx^\nu + e^{-2\omega(y)+2u(x)}\tilde{g}_{mn}(x,y)dy^m dy^n \tag{78}$$

with $\partial_\mu \det \tilde{g}_{mn} = 0$. The coordinates x^μ are taken over the four dimensional space so that $\tilde{g}_{\mu\nu}$ is the metric measured in our observed world, while y^m are the coordinates on the compact space X. For simplicity let us assume that the number of Kaehler structure moduli in X is just one ($h_{11} = 1$) but we shall keep the number of complex structure moduli h_{21} arbitrary. The Kaehler deformation (which is thus just the volume modulus corresponding to changes in the overall size of X) is given by the factor e^{2u}. In fact we may normalize the internal metric by putting $\int_X e^{-2\omega}\sqrt{\det \tilde{g}_{mn}} = 1$ so that the physical volume of X is e^{6u}. The factor e^{-6u} in the first term is then introduced in order to ensure that $\tilde{g}_{\mu\nu}$ is the four-dimensional Einstein metric.

The effective potential was derived in [5] by reducing the ten D action using the static version of this ansatz (i.e with $u = 0$ and $\partial_\mu g_{mn} = 0$) and ignoring the warp factor $e^{2\omega(y)}$, and the expression (the tilde denotes the use of the metric $\tilde{g}$ in the inner product)

$$V = \int d^6y\sqrt{\tilde{g}^{(6)}}\frac{e^{4\omega-12u}}{24\tau_I}|iG_3 \widetilde{- *_6}G_3|^2 \tag{79}$$

was obtained. However, except at the minimum of the potential the static ansatz cannot really be used and immedately leads to the no-go theorem forbidding positive potentials [27][28][29] and the resolution, as pointed out in [16], is to include time dependence of the volume modulus $u(x)$. Furthermore in the presence of fluxes and D-branes/orientifolds, the warp factor is necessarily non-trivial as can be seen from the internal components of the Einstein equation. An attempt was made to include the moduli and a non-trivial warp factor in [30] but it was shown in [16] that a consistent derivation was not possible without including all the Kaluza-Klein (KK) modes, *(including non-diagonal terms in 78.* In fact it was argued that the full ten-dimensional equations with time dependent moduli (and except at the minimum of the potential the moduli are necessarily time-dependent) and non-trivial warp factor, imply that the metric ansatz (78) is invalid. In other words the effective potential (79) would have additional terms involving KK modes and the derivatives of the warp factor.

If we ignore these issues (for progress towards resolving them see [31]) then the above potential can be written in the $\mathcal{N}=1$ SUGRA form. The superpotential is of the form proposed in [13] (see also [12])

$$W = W_{flux} \equiv \int \Omega \wedge G_3$$

Here Ω is the holomorphic three form on X. W_{flux} depends implicitly on the complex structure moduli z_i (through Ω) and the dilaton $S \equiv i\tau$. However it is important to note that it is independent of the Kahler moduli T (note $\Re T = e^{4u}$. Non-renormalization theorems for the superpotential then imply that this remains true to all orders in perturbation theory.

$$K = -\ln(S+\bar{S}) - 3\ln(T+\bar{T}) + k(z^i, \bar{z}^{\bar{j}}). \tag{80}$$

This leads to a no scale potential for S, z_i [5]

$$V = e^K (D_S W D_{\bar{S}} \bar{W} K^{S\bar{S}} + D_z W D_{\bar{z}} \bar{W} K^{z\bar{z}}) \geq 0.$$

This has a minimum at

$$D_S W = \partial_S W + K_S W = 0,\ D_z W = \partial_z W + K_z W = 0 \tag{81}$$

However generically$F_T = D_T W_0 = -3\frac{W_0}{T+\bar{T}} \neq 0$. So SUSY is broken in general but the potential at the minimum $V_0 = 0$. However T (and so the overall size of the internal manifold) is unfixed - hence the name no-scale. This is of course unacceptable since we need to get rid of all Brans-Dicke scalars.

B. Non-perurbative terms and the KKLT potential

A proposal to fix T was given by Kachru et al [6] KKLT. Let us discuss this.

The no-scale feature arises from the fact that the superpotential is independent of the Kaehler modulus T. By standard non-renormalization theorems the superpotential does not get corrected in perturbation theory. However there can be non-perturbaive terms. There are two sources for such terms in the present context.

a) Confining Gauge theory on a stack of$D7$ branes wrapping wrapping a 4-cycle in X.

b) String instanton effects.

In case a) one can see from examining the DBI action for a stack of D7-branes that the gauge theory has a gauge coupling function f (see subsection (II F) that is proportional to $\Re T$. The latter comes from the volume of the 4-cycle which is simply given in terms of $\Re T$ since we only have one Kaehler modulus. It should also be observed that in the Einstein frame there is no addional modulus dependence of f at the classical level. By the arguments of subsection (II G) we then get an additional term in the superpotential so that the total superpotential becomes

$$W = W_{flux} + Ce^{-aT}, \tag{82}$$

where C, a constants that depend on the particular D-brane configuration. A similar term arises from mechanism b). In either case a non-vanishing pre-factor C requires that the configuration generating it have two fermionic zero modes.

KKLT proceed to discuss the physics of the potential that is generated from this by resorting to a two stage argument. First ignore the NP term and solve for S, z by imposing (81). This results in a constant superpotential W_0 which is simply W_{flux} evaluated at the solution to (81). Now add the NP term. Then we have a potential for the single (complex) modulus T with the Kahler potential given by the second term of (80) (upto a constant) and the superpotential by (82) with $W_{flux} \rightarrow W_0$. The resulting potential just has one AdS SUSY min. with $F_T = \partial_T W - 3\frac{W}{T+\bar{T}} = 0$ and $V_0 = -3|W|^2/(T+\bar{T})^3$. The self-consistency of the argument requires that the value at which the stabilization occurs is such that $\Re T >> 1$ $a\Re T >> 1$. The first of these comes from the fact that since our starting point is the ten dimensional supergravity we are effectively assuming that the Kaluza-Klein mass scale (proportinal to $T^{-1/4}$) is small compared to the string scale. The second comes from the fact that the NP term is just the leading term in an expansion (an instanton sum) which is only valid only if this is satisfied. Then the stabilization condition requires that $W_0 << 1$. Generically of course W_0 is of order unity, but given a sufficient number of fluxes it is possible that there would be configurations where this is satisfied.

To get a SUSY breaking dS minimum KKLT add a $\bar{D}_3$ term to the SUGRA potential to get finally a potential

$$V = V_{sugra} + \frac{d}{(ReT)^2}.$$

This procedure is rather ad hoc! The Dbar branes break susy at string scale (there would be tower of string states associated with it which break the opposite half of supersymmetry to the D-brane/orientifold system). Thus it is unclear how one can use the four dimensional $N = 1$ SUGRA formalism at all. A rigorous derivation would require that we try to repeat the arguments of GKP in the presence of the Dbar brnaes. Given that we already had problems of decoupling KK modes even in the case with more controle it seems unlikely that one could find a rigorous derivation of such a potential with SUSY broken already at the ten-dimensional level.

Can the Dbar term be interpreted as a D term in N=1 SUGRA? This is unclear but even if this is true, there is no uplift of a F-term supersymmeric AdS vacuum. As we discussed in subsection (II F) the F and D terms are related (when the superpotenital is non-vanishing) by

$$f_{ab}D^b = \frac{ik_a^i D_i W}{W}$$

So for an F-term supersymmetric minimum which is AdS i.e. $D_i W = 0$, $W \neq 0$, the D-term also vanishes so that it cannot be used to uplift the potential[37]. One might ask however whether by including more NP terms in the superpotential one can get a dS minimum by F-terms alone. Within the context of the two stage procdure this is impossible though it can be done if at least two light moduli are included[32].

However one should ask whether this two step procedure is justified - even assuming large masses for S,z? The full superpotential is given by (82) and it is clear that even if one is justified in assuming that for some range of values of T the masses of S and z are heavy it is clear that the equations for integrating out these fields will not set them to be constant but to be functions of T. Thus the correct procedure would yield a potential that is much more complicated than what one obtains in the two stage calculation where the “heavy” fields have been set to constants. There is in fact no approximation scheme in which the two stage procedure is justified. However we will find that assuming there are flux configurations such that $M_S, M_z \sim M_{KK}$ so that we can integrate out S, z, it is possible to avoid the ad hoc uplifiting procedure.

C. Integrating out in SUSY and SUGRA

Before we discuss the KKLT potential however it is worthwhile discussing the general procedure of (classically) integrating out fields in field theory. There does not seem to be much discussion of these issues in the case of supersymmetric (especially supergraivty) theories so it is perhaps worthwhile going through this exercise in some detail.

Suppose the potential of a scalar field theory with a heavy (Φ) and a light (ϕ) field is

$$V(\Phi, \phi) = \frac{1}{2}M^2\Phi^2 + \tilde{V}(\Phi, \phi)$$

Solving the equation of motion (EOM) for Φ we get

$$\Phi = \frac{1}{\Box - M^2}\frac{\partial\tilde{V}}{\partial\Phi} = -\frac{1}{M^2}\frac{\partial\tilde{V}}{\partial\Phi} + \frac{1}{M^2}O(\frac{\Box}{M^2}\frac{\partial\tilde{V}}{\partial\Phi}).$$

So upto terms $O(E/M)$ Φ solves

$$\frac{\partial V}{\partial\Phi} = 0$$

The effective potential for light fields is then

$$V(\Phi(\phi), \phi).$$

Now let us look at a SUSY (global) theory:

$$S = \int d^4xd^4\theta\bar{\Phi}^i\Phi^i + (\int d^4xd^2\theta W(\Phi^i) + c.c.).$$

with superfield EOM

$$-\frac{1}{4}\bar{D}^2\bar{\Phi}^i + \frac{\partial W}{\partial \Phi^i} = 0.$$

Let $i = H$ for heavy and $i = l, l'$ for light fields. The standard argument for integrating out Φ^H is that one should use

$$\frac{\partial W}{\partial \Phi^H} = 0. \tag{83}$$

However this condition is valid only under certain restrictions. Consider the following superpotential

$$W(\Phi^H, \Phi^l) = \frac{1}{2}M\Phi^{H2} + \tilde{W}(\Phi^H, \Phi^l).$$

Solving the E of M for heavy field:

$$\Phi^H = \frac{1}{\Box - M^2}(M\frac{\partial \tilde{W}}{\partial \Phi^H} + \frac{\bar{D}^2}{4}\frac{\partial \bar{\tilde{W}}}{\partial \bar{\Phi}^H})$$

Expand in powers of $\Box/M^2$ as before to get,

$$\frac{\partial W}{\partial \Phi^H} = -\frac{\bar{D}^2}{4M}\frac{\partial \bar{\tilde{W}}}{\partial \bar{\Phi}^H} + O(\frac{\Box}{M^2}). \tag{84}$$

So we see that when $\tilde{W} \neq 0$, to get the usual condition (83) we need in addiont to $\frac{E^2}{M^2} << 1$, also $|\Phi_l| << M$.

Let us illustrate this withh a simple example having one heavy H and one light field L.

$$\int d^4\theta(\bar{H}H + \bar{L}L) + \left[\int d^2\theta\frac{1}{2}(MH^2 + HL^2) + c.c.\right]$$

The potential is easily worked out giving,

$$\begin{aligned} -V &= \bar{F}F + \bar{f}f + \frac{1}{2}(Fa^2 + \bar{F}\bar{a}^2) + (Aaf + \bar{A}\bar{a}\bar{f}) \\ &+ M(AF + \bar{A}\bar{F}) \end{aligned}$$

Ignoring fermions let us put $H = (A, F)$, $l = (a, f)$.

The heavy eqns. are

$$\begin{aligned} -\Box\bar{A} &= MF + af \\ \bar{F} &= -\frac{a^2}{2} - MA \end{aligned}$$

Plug these solution for F into V ignoring $O(\Box/M^2)$.to get

$$-V = \bar{f}f(1 + \frac{\bar{a}a}{M^2}) - \frac{1}{2M}(fa^3 + \bar{f}\bar{a}^3).$$

Eliminating the light auxiliary field using its equation of motion,

$$V = \frac{|a|^6}{4M^2}(1 + \frac{|a|^2}{M^2})^{-1}$$

If $\partial W/\partial H = 0$ had been used the term $|a|^2/M^2$ would not be obtained.

So the usual condition is valid for

$$\frac{|a|^2}{M^2} << 1.$$

What is the corresponding condition in SUGRA?

We expect (83) to be replaced by

$$D_H W = \partial_H W + \frac{1}{M_p^2} W \partial_H K = 0 \tag{85}$$

Using the flat space chiral compensator formalism of GGRS [2] we have (with $M_p^2 = 1$)

$$S = -3\int d^4xd^4\theta\bar{\phi}\phi e^{-K/3} + (\int d^4xd^2\theta\phi^3 W + h.c.) \tag{86}$$

Take

$$K = \overline{H}H + K^l(L, \overline{L}), \tag{87}$$

with a superopotential

$$W = \frac{1}{2}MH^2 + \tilde{W}(H, L), \tag{88}$$

We need to solve heavy field equation to $O(E^2/M^2)$. Effectively this amounts to putting $\Box H = 0$. So we get

$$
\begin{aligned}
D^2(e^{K/3M_p^2}\phi^2)D_H W + \ & 4Me^{2K/M_p^2}\phi^2\bar{\phi}^2(1+\tfrac{\bar{H}H}{M_p^2})D_{\bar{H}}\bar{W} \\
& = -e^{K/3M_p^2}\phi^2 D^2 D_H \tilde{W} \\
& \quad + O(D_\alpha H)
\end{aligned}
\tag{89}
$$

Note that the global limit $M_p \to \infty$, $\phi \to 1$ gives the previous result (84). As in that case here too we can ignore the first term in the RHS for $|L| << M$. But what about the fermionic terms? It turns out that $D_H W = 0$ is a sufficient condition. Spinor derivativation of this condition gives $W\overline{D}_{\dot\alpha}\overline{H}$=0 and so at generic points in field space the fermion terms vanish. However the condition $D_H W = 0$ is not a necessary one - infact it is too strong and we cannot ignore the fermion terms (even for $L << M$). To see this take

$$K = K^h(H, \overline{H}) + K^l(L, \overline{L})$$

The condition (85) is

$$\partial_H W + \partial_H K^h W = 0 \tag{90}$$

This is not a chiral eqn. Take the anti-chiral derivative to get

$$W K^h_{H\bar{H}} \overline{\mathcal{D}}_{\dot\alpha}\bar{H} = 0 \tag{91}$$

Suppose (90) is solved by $H = H(L, \bar{L})$. From chirality of H, L and (91),

$$\bar{\mathcal{D}}_{\dot\alpha} H = \frac{\partial H}{\partial L}\bar{\mathcal{D}}_{\dot\alpha} L + \frac{\partial H}{\partial \bar{L}}\bar{\mathcal{D}}_{\dot\alpha}\bar{L} = \frac{\partial H}{\partial \bar{L}}\bar{\mathcal{D}}_{\dot\alpha}\bar{L} = 0.$$

$$\mathcal{D}_\alpha H = \frac{\partial H}{\partial L}\mathcal{D}_\alpha L + \frac{\partial H}{\partial \bar{L}}\mathcal{D}_\alpha \bar{L} = \frac{\partial H}{\partial L}\mathcal{D}_\alpha L = 0$$

This implies that there are no fermions in the light field theory! The lesson is that $D_H W = 0$ can be imposed for computing scalar potential (for $L << M$) but should not be used as a superfield relation, since in that case we would need to keep the fermion squared terms in (89).

Note that even with $L << M < M_p$, non-trivial SUGRA terms will be present since

$$W(L.\bar{L}) = W_0 + ...$$

with $W_0 \sim O(M_p)$ in typical situations so SUGRA corrections to global SUSY $K_L W/M_p^2$, $|W|^2/M_p^2$ are not necessarily neglible!

D. String theory potentials for light moduli

Let us now get back to string theory effective potentials. First consider a simple model where the compact dimensions are taken to be on a rigid CY manifold. This means we just have the Kaehler modulus that corresponds to changing the overall size of the manifold. This then gives a model with just S and T.

The classical Kaehler potential is

$$K = -\ln(S + \bar{S}) - 3\ln(T + \bar{T})$$

The superpotential coming from fluxes and a NP term is

$$W = A + SB + Ce^{-aT}$$

Assume that the fluxes are such that S is heavy so we may integrate it out using

$$D_S W = B - \frac{A + SB + Ce^{-aT}}{S + \bar{S}} = 0$$

This is solved by

$$\bar{S} = (A + Ce^{-aT})/B$$

This equation is not holomorphic! This is not necessarily a problem since the SUGRA action is determined by the Kaehler invariant combination

$$G = K(\Phi, \bar{\Phi}) + \ln W(\Phi) + \ln \bar{W}(\bar{\Phi}).$$

After using the solution for S we get an effective potential for T that is determined by

$$\begin{aligned} G &= -\ln(\frac{(A + Ce^{-aT})}{B} + \frac{(\bar{A} + \bar{C}e^{-a\bar{T}})}{\bar{B}}) - 3\ln(T + \bar{T}) \\ &\quad + \ln(A + B\frac{(\bar{A} + \bar{C}e^{-a\bar{T}})}{\bar{B}} + Ce^{-aT}) + c.c.). \end{aligned}$$

On the other hand if the KKLT two stage process is used, $S = A/B$, so that

$$G = -3\ln(T + \bar{T}) + (\ln(A + B\frac{\bar{A}}{\bar{B}} + Ce^{-aT}) + c.c.)$$

The problem is that the missing NP terms for example the term

$$\frac{B}{\bar{B}}\bar{C}e^{-a\bar{T}}$$

is of the same magnitude as the ones that are being kept! Note that this calculation would still be valid when there are more than one Kaehler modulus (but keeping $h_{21} = 0$) with the replacement $Ce^{-aT} \to \sum_i C_i e^{-a_i T_i}$.

This model actually has no minima in S, T space as shown in [33]. The SUSY extremum at $D_S W = D_T W = 0$ is a saddle point. Of course since we have AdS supersymmetry that's OK! But obviously it cannot even be uplifted by $\bar{D}$ terms to get a phenomenologically viablemodel.

Let us now consider models with complex structure moduli z^i but with just one Kaehler modulus:

$$K = -\ln(S+\bar{S}) - 3\ln(T+\bar{T}) + k(z^i, \bar{z}^{\bar{j}})$$

$$W = A(z^i) + SB(z^i) + Ce^{-aT}$$

Where, as before, the first two terms give the flux superpotential and the third term is coming from NP effects. The Kaehler derivatives are

$$\begin{aligned} D_T W &= -aCe^{-aT} - \frac{3}{T+\bar{T}}W, \\ D_S W &= B - \frac{W}{S+\bar{S}}, \\ D_i W &= \partial_i A + S\partial_i B + \partial_i k W. \end{aligned}$$

Let us assume that the z^i are heavy and integrate them out using $D_i W = 0$. Now unlike in the previous case we cannot solve this explicitly but it is easy to show that there is no holomorphic solution contrary to claims in the literature.

Let us assume that there is a holomorphic solution. So

$$W = W_{eff} + Ce^{-aT},$$

with

$$W_{eff} = A(z^i(S,T)) + B(z^i(S,T))S,$$

and

$$K = -\ln(S+\bar{S}) - 3\ln(T+\bar{T}) + k(z(S,T), \bar{z}(\bar{S},\bar{T})).$$

The eqn to be solved is

$$D_i W = \partial_i A(z^i) + S\partial_i B(z^i) + W(S,T,z^i)k_i = 0. \tag{92}$$

The assumption is that this is solved by $z^i = z^i(S,T)$. Differentiate (92) w.r.t. $\bar{S}$;

$$W(S,T,z^i(S,T))k_{i,\bar{j}}\frac{\partial \bar{z}^{\bar{j}}}{\partial \bar{S}} = 0.$$

At generic points $W \neq 0$ and $k_{i\bar{j}}$ is non-degenerate.

So

$$\frac{\partial z^i}{\partial S} = 0$$

Similarly we get $\frac{\partial z^i}{\partial T} = 0$. But this obviously cannot be the case so we conclude that the z^i cannot be holomorphic functions of S,T. So we expect a solution of the form $z^i = z^i(S,T,\bar{S},\bar{T})$.

As in the simple case (with no z) that we solved explicitly, we need to compute V from

$$\begin{aligned} G = K &= -\ln(S+\bar{S}) - 3\ln(T+\bar{T}) \\ &+ k(z(S,T,\bar{S},\bar{T}),\bar{z}(\bar{S},\bar{T},S,T)) \\ &+ \ln|W(S,T,z(S,T,\bar{S},\bar{T}))|^2 \end{aligned}$$

Since it is hard to deal with two complex variables let us assume that S is heavy as well for some choice of fluxes, i.e put $D_S W = 0$ so that

$$z^i = z^i(T,\bar{T}) \text{ and } S = S(T,\bar{T})$$

Let us assume that there is a power series solution, valid for $a\Re T >> 1$.

$$\begin{aligned} S &= \alpha + \beta C e^{-aT} + \gamma \bar{C} e^{-a\bar{T}} + ... \\ z^i &= \alpha^i + \beta^i C e^{-aT} + \gamma^i \bar{C} e^{-a\bar{T}} + ... \end{aligned}$$

where $\alpha, ...\gamma^i$ are functions of flux integers. Then

$$
\begin{aligned}
G &= \ln(v + bCe^{-aT} + \bar{b}\bar{C}e^{-a\bar{T}} + cC^2e^{-2aT} + \bar{c}\bar{C}^2e^{-2a\bar{T}} \\
&\quad + d|C|^2e^{-a(T+\bar{T})} + \ldots) - 3\ln(T + \bar{T}),
\end{aligned}
$$

and the potential becomes

$$
\begin{aligned}
\mathcal{V} &= \frac{1}{(T+\bar{T})^2}[a(bCe^{-aT} + 2cC^2e^{-2aT} + c.c.) \\
&+ \ a|C|^2((4\frac{a|b|^2}{v} - 3ad)\frac{T+\bar{T}}{3} + 2d)e^{-a(T+\bar{T})}]
\end{aligned}
$$

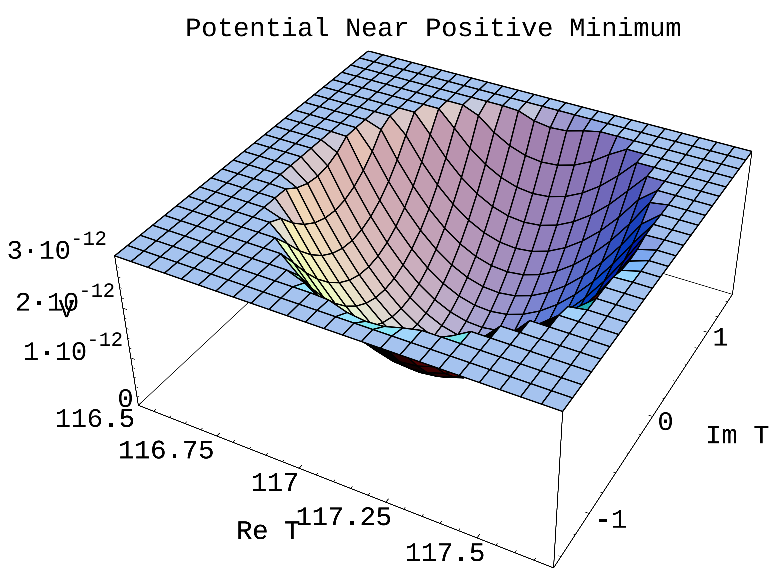
This differs from a two stage calculation by terms such as

$$
2cC^2e^{-2aT} + c.c.
$$

which are obviously of the same magnitude as terms which are kept. Note that even in the real direction this potential has more parameters than in the two stage process. The reason is that in the latter all the flux parameters are subsumed into W_0.

With the two stage version we can show that there are no dS minima even with multiple condensate terms.[32]. The above on the other hand can have positive minima!

An explicit example is the following



We choose the following parameters:

$$a = \frac{2\pi}{320} \quad v = 0.22941751641574312 \quad b = 1$$
$$c = -1.4097828718993035$$
$$d = 15.786002156414208$$

This then has a local minimum at $\Re T_{min} = 117.138$, $\Im T = 0$, $V_{min} = 10^{-15}$. The example is of course somewhat artificial and certainly does not give a viable phenomenology. What it does demonstrate is that the uplift terms of KKLT are not really essential. Finding a viable model without them however might be a challange! Of course in any case it is not really necessary to get dS minima at the classical level - quantum contributions from integrating out high frequency modes from the string scale down to cosmological scales, together with contributions from the standard model and QCD phase transitions could well lift a ADS vacuum whose CC is not too large (i.e. of the order of the standard model scale).

E. Conclusions

In this section we argued that in SUSY theories imposing $\partial_H W = 0$ to integrate out a heavy field H is valid if the light field space is restricted to $|L| << M$. In SUGRA the corresponding equation is $D_H W = 0$, but it has to be used with caution - in particular it is not valid as an equation for superfields, but it is valid subject to the same restrictions as in the SUSY case for the purpose of calculating the potential.

In applying this to potentials arising from flux compactifications corrected by non-perturbative terms, in order to derive an effetive potential for a light Kaehler modulus we found that the correct procedure leads to a more complicated potential than what one gets if one followed the two stage procedure of KKLT. The additional terms are of the same magnitude as the terms which kept in the two stage procedure. It was argued that these may lead to phenomenologically viable models for inflation without the need for ad hoc uplifting terms.

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Appendix

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[35] These frame indices are often called flat indices in the supergravity literature whilst the index set M are called curved indices)

[36] For a definінition see for example section 3.7 of [2]. Here we will only need to know that they are superdensities which enable one to define superspace integrals which are invariant under local SUSY transformations.

[37] Burgess et al give a proposal for getting explicit FI D terms[34]. The above argument shows that they cannot be used to lift AdS SUSY minima as in the KKLT case but some such mechanism might work if the F term potential gave SUSY breaking AdS minimum.