Quantizing and Dequantizing Reference Frames

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Outline

The coherence as fact vs. coherence as fiction controversy

A resolution: Classical reference frames and quantum reference frames as alternative paradigms of description

The lessons I wish to draw from this:
- Quantum states are about relations between systems
- Many, if not all, superselection rules can be circumvented in principle

Coherence: Fact or fiction?

There are many contexts in which the debate arises:

Superconductors – for superpositions of charge eigenstates
BECs – for superpositions of atom number eigenstates
Lasers – for superpositions of photon number eigenstates

We discuss the optical case, although the discussion would be similar for the others.

Optical coherence: a convenient myth?

K. Molmer, Phys. Rev. A. **55**, 3195 (1997)

Standard assumption:

$$|\alpha\rangle = \sum_{n=0}^{\infty} \frac{e^{-|\alpha|^2/2}\alpha^n}{\sqrt{n!}}|n\rangle$$

But this is seen to be false if we quantize the atoms in the gain medium:

•assuming the gain medium is in an energy eigenstate,

•applying the RWA (energy conservation)

! atoms and field evolve to an entangled state

$$\rho = \sum_{n=0}^{\infty} p_n |n\rangle\langle n|$$

$$p_n = \frac{e^{-|\alpha|^2}|\alpha|^{2n}}{n!}$$

Thus, coherence is a fiction!

A possible dialogue

C: Your assumption was that the source has no coherence, but this is false

NC: Even if the source had a phase, we don't *know* it, therefore it is described by a mixture over all phases. Assuming a real coherent state is to commit the preferred ensemble fallacy

$$\rho = \int_0^{2\pi} \frac{d\phi}{2\pi} |\alpha\rangle\langle\alpha| = \sum_{n=0}^{\infty} p_n |n\rangle\langle n|$$

C: This is a proper mixture, the PEF only applies to improper mixtures

NC: You are forced to assume the existence of coherences in the initial condition

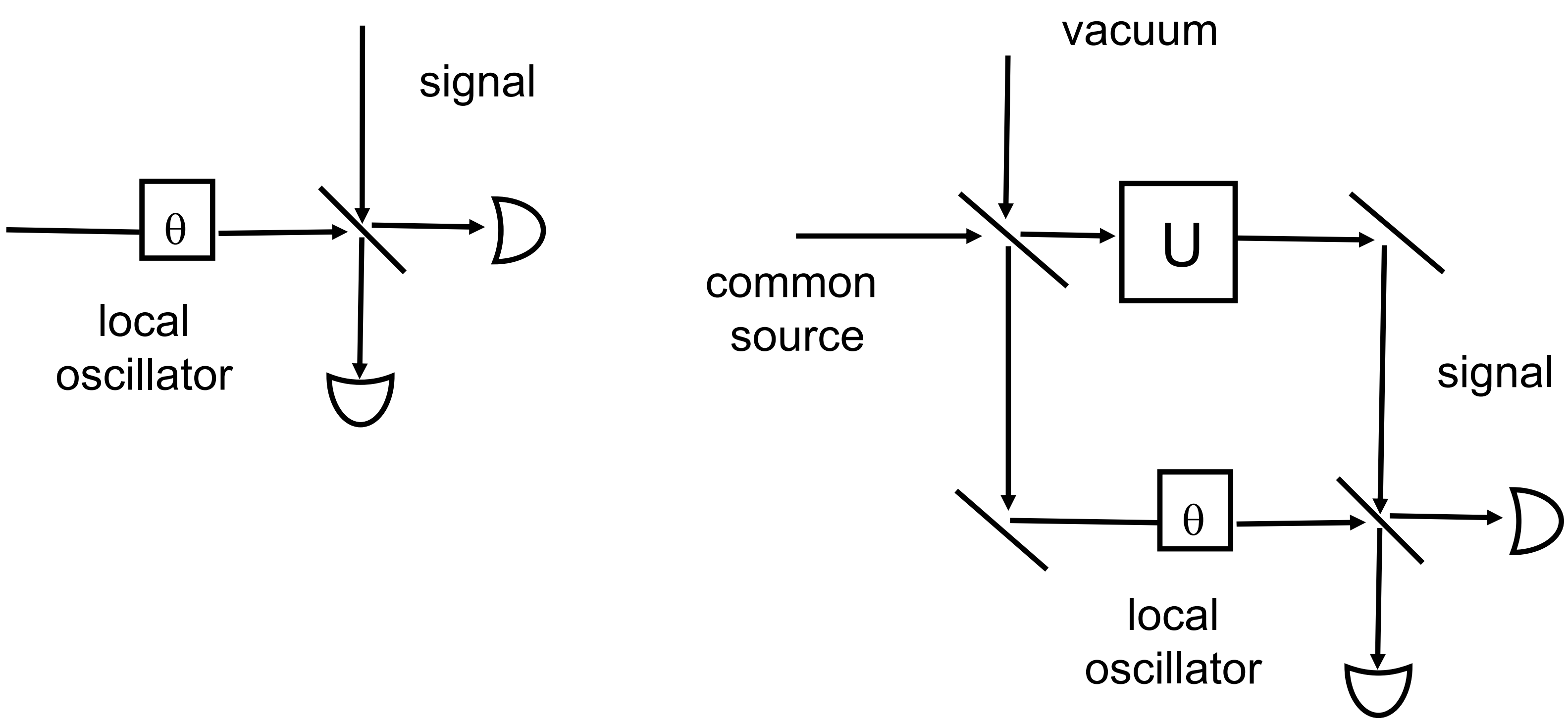
C: So what?

NC: I don't buy it.

C: Experiments have shown that lasers have a well-defined phase

NC: Your argument is circular

Example: Homodyne detection



C: Your reasoning applies equally well to coherent states of radio waves, and to coherent superpositions of linear and angular momentum, so you'll have to deny that radio waves have a well-defined phase and that particles are localized!

NC: Hmm. I guess you're right. But weird things happen when you start quantizing everything. We have to learn to live with it.

C: How are you going to understand the classical limit?

NC: I'm working on it…

C: How do you explain the fact that my calculations are in agreement with all experiments?

NC: Lasers are not *really* in coherent states, but mathematically, there is no harm in assuming that they are.

The debate is about whether or not coherence describes the intrinsic properties of the system

Why not say that "really" there are only relations, and the coherence describes these?

Relational view of quantum states

The quantum state describes the relation between a system and a reference frame (RF)

Non-eigenstate of	Classical RF
linear momentum	spatial frame (e.g. GPS satellites)
angular momentum	orientation frame (e.g. gyroscopes)
photon number	clock
atom number	BEC phase
charge	Superconducting phase

Coherence paradigm = classical RF paradigm

No coherence paradigm = quantum RF paradigm

Equally valid descriptions

Aharonov and Susskind, Phys. Rev. **155**, 1428 (1967).

Is there a framework that works for all these cases?

Quantizing RFs

Suppose the system state w.r.t the classical RF is:
$|\psi$i 2 H_S

Quantize all physical objects that can serve as a RF.
Introduce a Hilbert space H_R

<u>Naïve approach</u>: assign $|\chi$i - $|\psi$i 2 H_R - H_S

E.g. For optical case, one could take $|\chi$i to be a coherent state $|\alpha$i

<u>Better approach</u>: Assign ρ on H_R - H_S

$$\rho = \frac{1}{2\pi}\int_0^{2\pi} d\phi \, |\phi\rangle \langle\phi| \otimes T(\phi)|\psi\rangle\langle\psi|T^\dagger(\phi)$$

G = group of transformations for the relevant d.o.f.

No classical RF for G ⇒ Operations and observables must be invariant under collective action of G (Superselection rule)

Suppose T:G ! GL(H) is a collective representation of G

A G-invariant CP map $\mathcal{O}$ satisfies

$$\mathcal{O}[T(g)\rho T^{\dagger}(g)] = T(g)\mathcal{O}[\rho]T^{\dagger}(g) \qquad \forall\ g \in G$$

A G-invariant POVM $\{E_k\}$ satisfies

$$T(g)E_k T^{\dagger}(g) = E_k \qquad \forall\ g \in G$$

Equivalence classes of states:

$\rho \equiv \rho'$ if $\mathsf{Tr}[A\rho] = \mathsf{Tr}[A\rho']$ for all G-invariant A

or

$$\mathcal{G}(\rho) = \mathcal{G}(\rho')$$

where

$$\mathcal{G}[\rho] \equiv \begin{cases} (\dim\ G)^{-1} \sum_{g \in G} T(g) \rho T^\dagger(g)\,, & \text{finite groups} \\ \int_G \mathrm{d}v(g)\, T(g) \rho T^\dagger(g)\,, & \text{Lie groups} \end{cases}$$

Convention: represent each equivalence class by the G-invariant state

$$\rho = \mathcal{G}(\rho)$$

Problem with naïve approach to quantization:
There is no observational difference among states

$$U(g)|\chi\rangle \otimes U(g)|\psi\rangle$$

for different $g \in G$

There is no real difference associated with this distinction

The only real degree of freedom is in the relative orientation

We must find a set of G-invariant states in H_R - H_S
that encode the possible relations
Can these simulate the states in H_S? Yes.

See: Kitaev, Mayers, Preskill, quant-ph/0310088

Most general state:

Density operator ρ

$$\rho > 0$$

$$\mathrm{Tr}(\rho) = 1$$

Most general measurement:

POVM $\{E_k\}$

$$E_k > 0$$

$$\sum_k E_k = I$$

Most general transformation:

CP map $\mathcal{O}$

$$\mathcal{O}(\rho) = \sum_\mu A_\mu \rho A_\mu^\dagger$$

$$\sum_\mu A_\mu^\dagger A_\mu = I$$

Generalized Born rule:

$$\mathrm{Prob}(k) = \mathrm{Tr}[\mathcal{O}(\rho) E_k]$$

Classical RF paradigm

States	ρ	defined on $\mathcal{H}_S$
Measurements	$\{E_k\}$	
Transformations	$\mathcal{O}$	

Quantum RF paradigm

States	$\tilde{\rho}$	defined on $\mathcal{H}_R \otimes \mathcal{H}_S$ and G-invariant
Measurements	$\{\tilde{E}_k\}$	
Transformations	$\tilde{\mathcal{O}}$	

Find a mapping

$$\rho \longrightarrow \tilde{\rho}$$
$$E_k \longrightarrow \tilde{E}_k$$
$$\mathcal{O} \longrightarrow \tilde{\mathcal{O}}$$

such that

$$\mathrm{Tr}_S[\mathcal{O}(\rho)E_k] = \mathrm{Tr}_{RS}[\tilde{\mathcal{O}}(\tilde{\rho})\tilde{E}_k]$$

Define

$$\tilde{\rho} = d_R^{-1} \$(\rho)$$

$$\tilde{E}_k = \$(E_k)$$

$$\tilde{A}_\mu = \$(A_\mu)$$

where

$$\$: A \mapsto \sum_{g \in G} |g\rangle \langle g| \otimes T(g) A T^\dagger(g)$$

with

$$\langle g|g'\rangle = \delta_{g,g'}$$

$$T(g') |g\rangle = |g' \circ g\rangle, \text{ for all } g, g' \in G$$

Property 1: $\$(A)$ is G-invariant

Proof: $(T(g') \otimes T(g'))\$(A)(T^\dagger(g') \otimes T^\dagger(g'))$

$$= \sum_{g \in G} T(g') |g\rangle \langle g| T^\dagger(g') \otimes T(g')T(g)AT^\dagger(g)T^\dagger(g')$$

$$= \sum_{g \in G} \left|g' \circ g\right\rangle \left\langle g' \circ g\right| \otimes T(g' \circ g)AT^\dagger(g' \circ g)$$

$$= \$(A).$$

Property 2: $\$(A+B) = \$(A) + \$(B)$

and

$\$(AB) = \$(A)\$(B)$

Proof: $\sum_{g \in G} |g\rangle \langle g| \otimes T(g)AT^\dagger(g) \sum_{g' \in G} |g'\rangle \langle g'| \otimes T(g')BT^\dagger(g')$

$$= \sum_{g \in G} |g\rangle \langle g| \otimes T(g)AT^\dagger(g)T(g)BT^\dagger(g)$$

$$= \sum_{g \in G} |g\rangle \langle g| \otimes T(g)ABT^\dagger(g)$$

Property 3: $\text{Tr}_{RS}(\$(A)) = d_R \text{Tr}_S(A)$

Property 4: if $A > 0$ then $\$(A) > 0$

Property 5: $\$(I_S) = I_{RS}$

3,4 $\rightarrow$ if ρ is a density operator, so is $\tilde{\rho}$

2,4,5 $\rightarrow$ if $\{E_k\}$ is a POVM, so is $\{\tilde{E}_k\}$

2,5 $\rightarrow$ if $\mathcal{O}$ is a CP map, so is $\tilde{\mathcal{O}}$

$$\begin{aligned}\text{Tr}_{RS}[\tilde{\mathcal{O}}(\tilde{\rho})\tilde{E}_k] &= \text{Tr}_{RS}[\textstyle\sum_k \$(A_\mu) d_R^{-1} \$(\rho) \$(A_\mu^\dagger) \$(E_k)] \\ &= d_R^{-1} \text{Tr}_{RS}[\$(\textstyle\sum_k A_\mu \rho A_\mu^\dagger E_k)] \\ &= \text{Tr}_S[\mathcal{O}(\rho) E_k]\end{aligned}$$

Example: Superpositions of charge eigenstates

Consider a coherent superposition of charge eigenstates on H_S

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

This is simulated by the U(1)-invariant state

$$\rho = \frac{1}{2\pi}\int_0^{2\theta} d\theta\, |\theta\rangle\, \langle\theta| \otimes T(\theta)|\psi\rangle\langle\psi|T^\dagger(\theta)$$

$$|\theta\rangle = \frac{1}{\sqrt{2\pi}} \sum_{q=-\infty}^{\infty} e^{-iq\theta}|q\rangle$$

$$T(\theta) = e^{-i\theta\hat{Q}}$$

which may be written as

$$\rho = \sum_{q=-\infty}^{\infty} |\psi_q\rangle\, \langle\psi_q|$$

where $|\psi_q\rangle = \alpha|q+1\rangle|0\rangle + \beta|q\rangle|1\rangle$

Dequantizing RFs

Wrong approach: Trace over H_R

(Think of the Hydrogen atom).

Right approach: Project into an irrep and trace over the factor space that supports the collective representation of G (keep only the decoherence-free subsystem)

Shared RFs

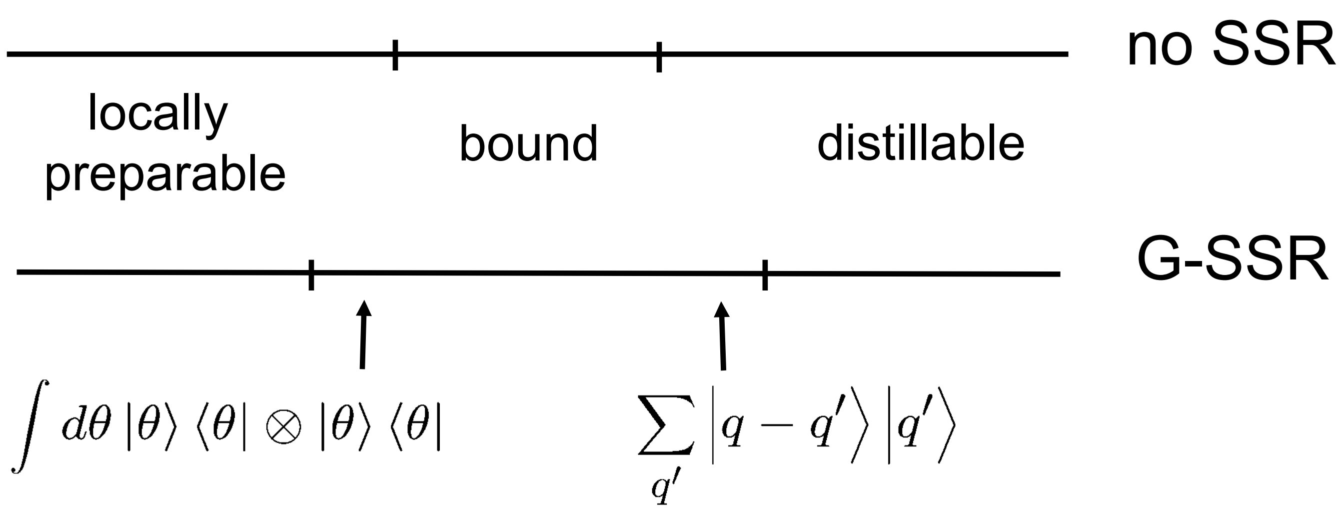
$$\rho = \sum_{g \in G} |g\rangle\langle g| \otimes |g\rangle\langle g|$$

Due to the SSR, this is not locally preparable
It is operationally non-separable

See: Verstraete and Cirac, PRL 91, 010404 (2003)

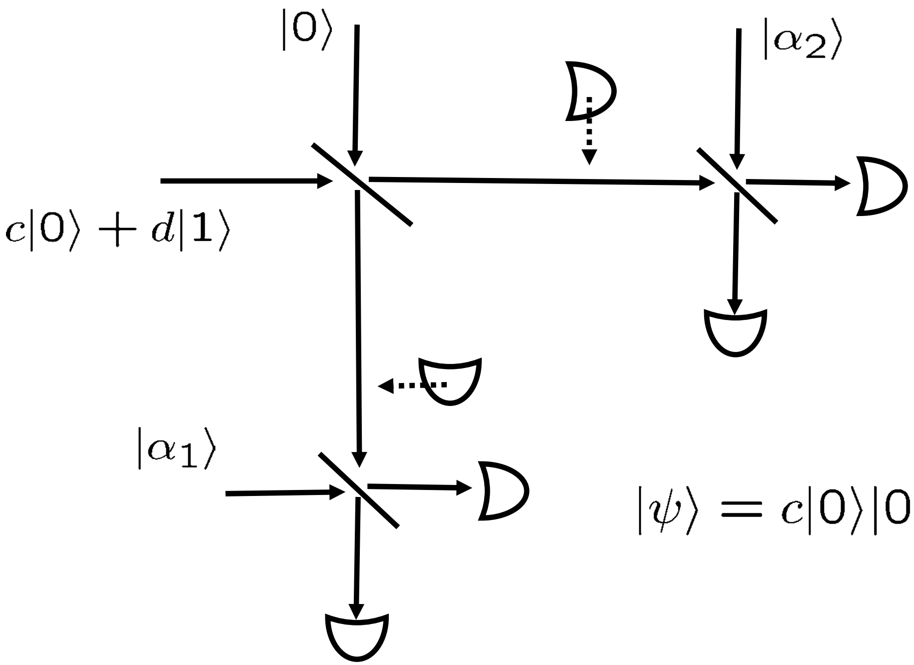
However, there is clearly also no distillable entanglement

Therefore: bound entangled



Application: Nonlocality of a single photon

Hardy, PRL 73, 2279 (1994)



$$|\psi\rangle = c|0\rangle|0\rangle + \frac{id}{\sqrt{2}}|1\rangle|0\rangle + \frac{d}{\sqrt{2}}|0\rangle|1\rangle$$

Individually, the G-invariant versions of $|\psi$i and $|\alpha_1$i $|\alpha_2$i are bound entangled, but together, they have distillable entanglement

$$|1,0\rangle|0,1\rangle + c|0,1\rangle|1,0\rangle$$

Entanglement, like coherence does not describe intrinsic properties but rather relations

Conclusions

- Quantum states describe relations
- Coherence and entanglement describe features of relations
- One can break superselection rules given appropriate resources

Future research

Investigate:

- Possibility of condensates for novel degrees of freedom
- Dequantizing of finite RFs
- Degradation of finite RFs
- Broken symmetry in particle physics
- Relevance to quantum gravity?