Quantum communication in non-inertial frames.

G. J. Milburn

The University of Queensland, QLD 4072 Australia.

Paul Alsing, UNM, Albuquerque.

Quantum information and relativity.

1. Is entanglement preserved when the local parties are in uniform relative motion ?
2. Is entanglement preserved when one or both local parties are accelerated ?

Q1: Lorentz invariance of entanglement.

Q2: Entanglement in non inertial frames, entanglement in curved spacetime.

Decoherence $\equiv$ degradation of entanglement.

Use teleportation fidelity as a measure of decoherence.

Quantum entanglement and superposition.

Must distinguish correlations in the entangled state

$$|s=0\rangle = \frac{1}{\sqrt{2}}(|\uparrow_z\rangle_1|\downarrow_z\rangle_2 - |\downarrow_z\rangle_1|\uparrow_z\rangle_2)$$

from correlations in the mixed state

$$\rho = \frac{1}{2}\left(|\uparrow_z\rangle_1\langle\uparrow_z| \otimes |\downarrow_z\rangle_2\langle\downarrow_z| + |\downarrow_z\rangle_1\langle\downarrow_z| \otimes |\uparrow_z\rangle_2\langle\uparrow_z|\right)$$

For both:

Prob(z-spin is opposite)=1

Lorentz invariance of entanglement.

Peres, Scudo & Terno, Phys. Rev. Lett., **88**, *230402 (2002).*

Alsing & GJM, Quant. Inf. Comp. **2**, *487, (2002).*

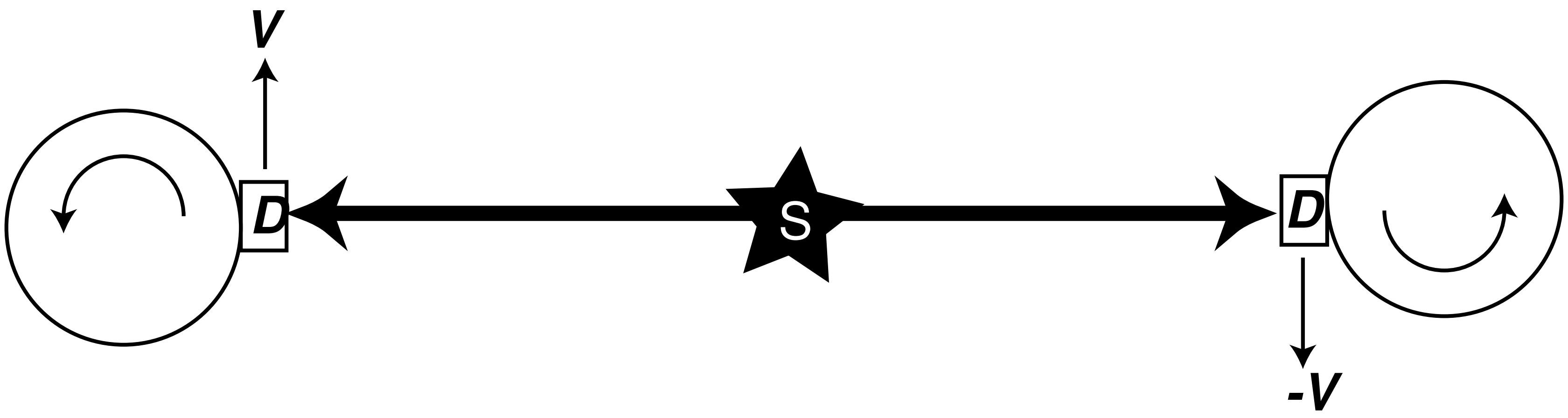
Gingrich & Adami, Phys. Rev. Lett., **89**, *270402, (2002).*

Yes, but which local degrees of freedom are entangled may change.

Example: spin-momentum of entangled wave packets.

This may appear as decoherence, if local detectors only measure one degree of freedom.

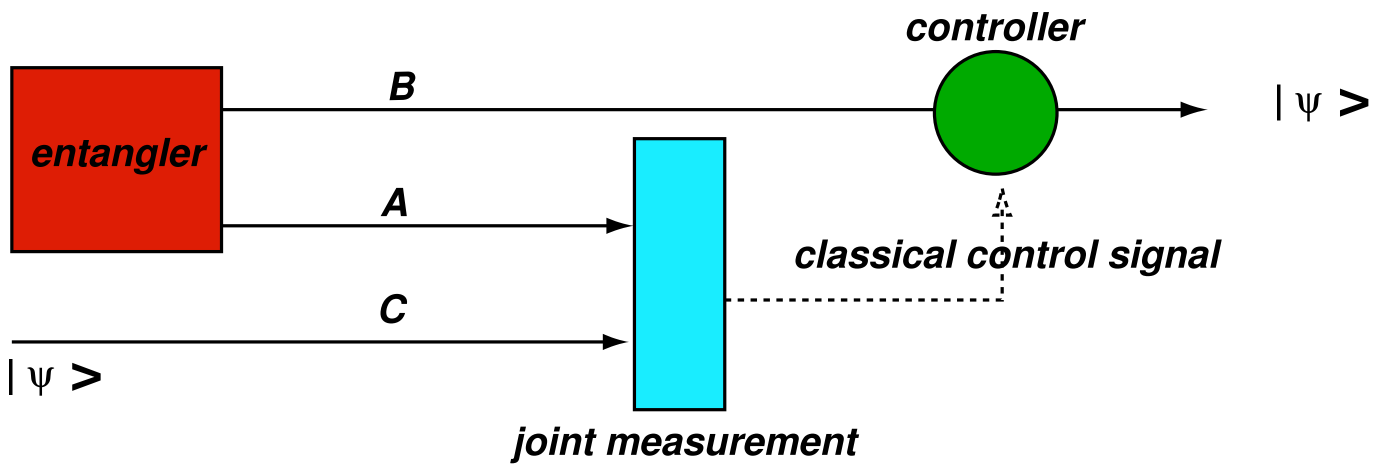
Bell measurements with moving detectors.



D polarisation detectors.

S source of entangled pairs.

Standard teleportation: quantum control.



Communicate state of client (C) from transmitter (A) to receiver (B), without A or B learning about the state.

Teleportation of qubits.

Generic two state system with basis, $\{|0\rangle, |1\rangle\}$, a *qubit*.

Take three qubits (A, B, C) with A, B entangled.

Joint state:

$$|\Psi\rangle = |\psi\rangle_C \otimes (|00\rangle_{AB} + |11\rangle_{AB})$$

where $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$.

Notation: $|xy\rangle \equiv |x\rangle_A \otimes |y\rangle_B$.

Teleportation of qubits.

Re arrange joint state:

$$
\begin{aligned}
|\Psi\rangle &= (\alpha|0\rangle_C + \beta|1\rangle_C) \otimes (|00\rangle_{AB} + |11\rangle_{AB}) \\
&= (\alpha|0\rangle + \beta|1\rangle) \otimes (|00\rangle + |11\rangle) \\
\\
&= (|00\rangle + |11\rangle)(\alpha|0\rangle + \beta|1\rangle) + (|01\rangle + |10\rangle)(\alpha|1\rangle + \beta|0\rangle) \\
&\quad + (|00\rangle - |11\rangle)(\alpha|0\rangle - \beta|1\rangle) + (|01\rangle - |10\rangle)(\alpha|1\rangle - \beta|0\rangle)
\end{aligned}
$$

Joint measurements on AB: *Bell operator* $\hat{\mathcal{B}}$

eigenstates $\{(00\rangle \pm |11\rangle), (|01\rangle \pm |10\rangle)\}$

Note: all four results are equally likely...no information on $|\psi\rangle$.

Teleportation of qubits.

Example: measure $\hat{\mathcal{B}}$, get result corresponding to $(|00\rangle - |11\rangle)$.

Conditional state for mode B: $\alpha|0\rangle - \beta|1\rangle$.

Send result to B, with instructions to apply the unitary operator $Z = |0\rangle\langle 0| - |1\rangle\langle 1|$

Final conditional state after this unitary is $\alpha|0\rangle_B + |1\rangle_B$...the same as the state of the client.

But can you be sure? *Errors, noise and decoherence*

Errors, noise and decoherence.

Bit flip errors on physical qubits.

$$X|0\rangle = |1\rangle, \qquad X|1\rangle = |0\rangle$$

X is a Pauli operator, σ_x.

Phase flip errors on physical errors.

$$Z(|0\rangle + |1\rangle) = |0\rangle - |1\rangle$$

Z is a Pauli operator, σ_z

Errors, noise and decoherence.

Decoherence ?

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

Phase error with probability p.

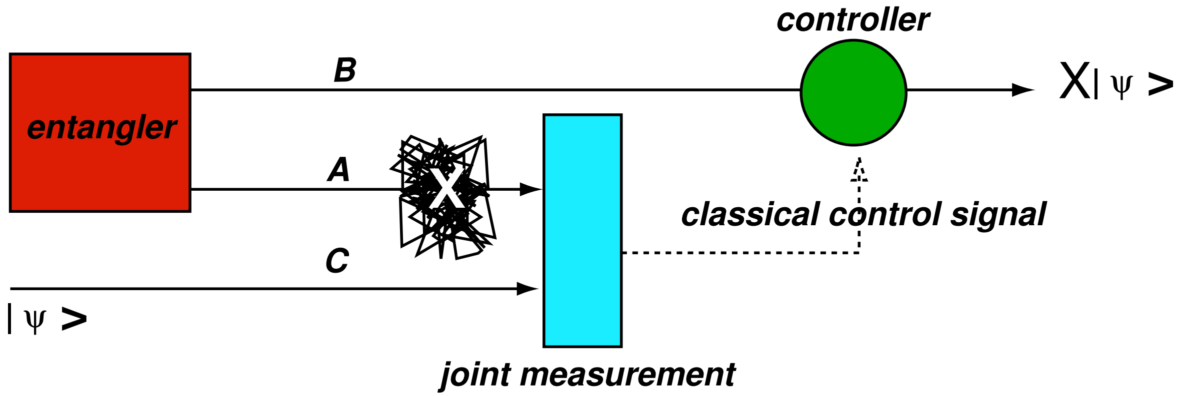
Average over possible events:

$$\begin{aligned} |\psi\rangle\langle\psi| &\rightarrow (1-p)|\psi\rangle\langle\psi| + pZ|\psi\rangle\langle\psi|Z \\ &= |\alpha|^2|0\rangle\langle 0| + |\beta|^2|1\rangle\langle 1| \\ &\quad +(1-2p)\left(\alpha\beta^*|0\rangle\langle 1| + \alpha^*\beta|1\rangle\langle 0|\right) \end{aligned}$$

Complete decoherence when $p = 1/2$.

Errors in teleportation.

Bit flip error in resource state, prob p.



Output state is mixed:

$$\rho_B = (1-p)|\psi\rangle\langle\psi| + pX|\psi\rangle\langle\psi|X$$

Errors in teleportation.

Quality of teleportation: *Fidelity.*

$$F = \langle\psi|\rho|\psi\rangle = (1-p) + p|\alpha\beta^* + \alpha^*\beta|^2$$

If $\alpha = \beta = 1/\sqrt{2}$, input state is an X eigenstate, $F = 1$.

Average over all input states $\overline{F} = 1 - p$.

How well can you do without entanglement?

A classical level.

Assume classical correlation, but no entanglement,

$$\rho_{AB} = \frac{1}{2}\left(|00\rangle\langle 00| + |11\rangle\langle 11|\right)$$

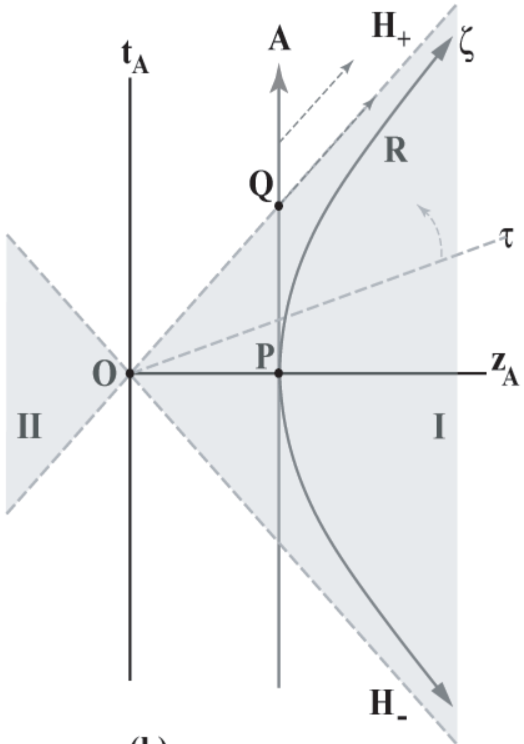
$$\rho_B = |\alpha|^2 |0\rangle\langle 0| + |\beta|^2 |1\rangle\langle 1|$$

Average over all input states: $\overline{F} = \frac{2}{3}$.

Can tolerate X errors, so long as the error probability $p < 1/3$.

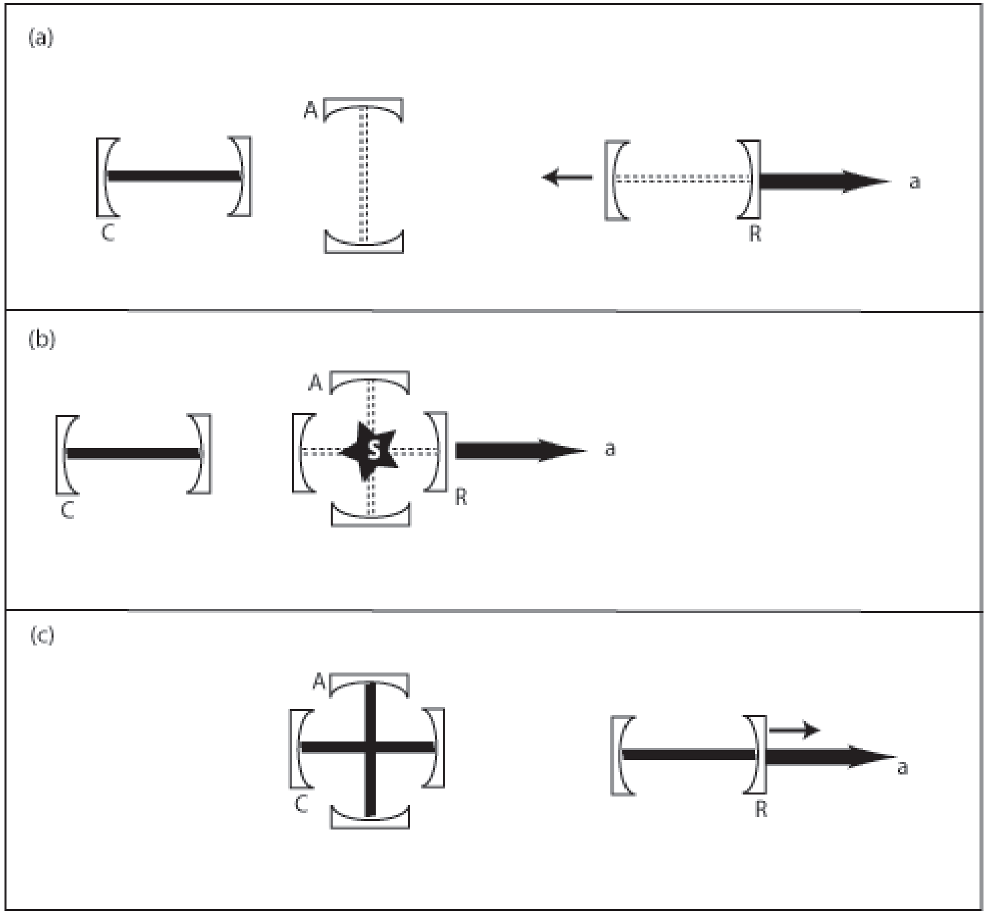
Teleportation with an accelerated partner.

Alice is an inertial observer. Rob is an accelerated observer.



(b)

Teleportation with an accelerated partner.



Two orthogonal spatial modes with the same frequency in each cavity.

Entanglement resource

Dual rail code: Encode qubit as single photon excitations of *Minkowski* vacuum in each spatial mode

$$|\mathbf{0}\rangle_M = |1\rangle_{A_1}|0\rangle_{A_2}, \qquad |\mathbf{1}\rangle_M = |0\rangle_{A_1}|1\rangle_{A_2},$$

Entanglement resource: two photon excitation of *Minkowski* vacuum modes for A and R.

$$|\mathbf{0}\rangle_M|\mathbf{0}\rangle_M + |\mathbf{1}\rangle_M|\mathbf{1}\rangle_M$$

Client state is unknown:

$$|\psi\rangle_M = \alpha|\mathbf{0}\rangle_M + \beta|\mathbf{1}\rangle_M$$

Entanglement resource for Rob.

Minkowski modes are *global*.

Rob's detectors only respond to his *local Rindler* modes.

$$
\begin{aligned}
|0\rangle_M &= \frac{1}{\cosh r} \sum_{n=0}^{\infty} \tanh^n r \, |n\rangle_I \otimes |n\rangle_{II} \\
|1\rangle_M &= \frac{1}{\cosh^2 r} \sum_{n=0}^{\infty} \tanh^n r \, \sqrt{n+1} \, |n+1\rangle_I \otimes |n\rangle_{II}
\end{aligned}
$$

$$
\tanh r = e^{-2\pi c \omega_0 / a}
$$

ω_0 is the common resonance frequency of the two cavities.

Entanglement resource for Rob.

Rob can only detect photons for states in the Rindler wedge labelled I.

Must trace resource state over region II

$$\rho^{(I)} = Tr_{II}(\rho^{(M)}).$$

Teleportation.

Joint readout (Bell basis) of qubits in cavities A and C.

Example: result is $(0,0)$

Conditional states for Rob

$$\begin{aligned}\rho_{00}^{(I)} &= \frac{1}{\cosh^6 r}\sum_{n=0}^{\infty}\sum_{m=0}^{n}(\tanh^2 r)^{n-1}\left[(n-m)|\alpha|^2 + m|\beta|^2\right] \\ &\quad \times |m, n-m\rangle_I\langle m, n-m| \\ &\quad +(\,\alpha\,\beta^*\tanh^{2n} r\,\sqrt{(m+1)(n-m+1)} \\ &\quad \times |m, n-m+1\rangle_I\langle m+1, n-m| + \text{h.c.}).\end{aligned}$$

Teleportation fidelity.

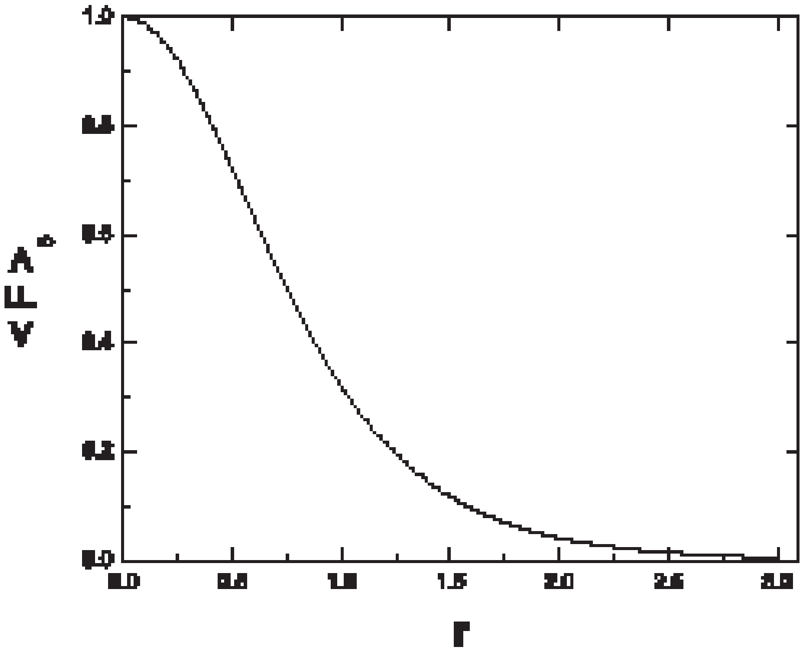
The state Rob would like to get is

$$|\boldsymbol{\psi}\rangle_I = \frac{1}{\sqrt{2}}\,(\,\alpha|\mathbf{0}\rangle_I + \beta|\mathbf{1}\rangle_I\,)\,.$$

The fidelity of the teleportation is then

$$F^{(I)} \equiv Tr_I(|\boldsymbol{\psi}\rangle_I\langle\boldsymbol{\psi}|\,\rho^{(I)}) = 1/\cosh^6 r.$$

Teleportation fidelity.



Operational meaning of this fidelity.

Repeat protocol so that Rob has many copies of the (imperfectly) teleported state.

Rob does state tomography and sends results back to the client.

The client can then infer the fidelity of the protocol.

Experimental 'test'.

Replace field with collective vibrational modes of N trapped ions.

$$\text{photons} \rightarrow \text{phonons}$$

Use internal states of ions as local detectors of phonons

Distant analogy, Sonic black holes. See *Unruh, Phys. Rev. Lett. 46, 1351 (1981).*

Physical picture of Unruh radiation.

Milloni and Alsing, quant-ph/0401170.

Plane-wave, (ω_K, K), parallel to the direction along which the observer is accelerated.

In the instantaneous rest frame of the observer the frequency ω' is

$$\omega'_K(\tau) = \omega_K e^{-a\tau/c}$$

Observer sees a field with a time dependent phase,

$$\phi(\tau) = \frac{\omega_K c}{a} e^{-a\tau/c}$$

Power spectrum:

$$\left| \int_{-\infty}^{\infty} e^{i\Omega\tau} e^{i(\omega_K c/a) e^{a\tau/c}} \right|^2 = \frac{2\pi c}{\omega a} \frac{1}{e^{2\pi\Omega c/a} - 1}$$

Planck distribution, temperature: $T = \hbar a/2\pi kc$.

Quantum case.

$$\hat{\phi} = \sum_K \left(\frac{2\pi c^2}{\omega_K V}\right)^{1/2} \left[a_K e^{-i\omega_K t} + a_K^\dagger e^{i\omega_K t}\right]$$

Define,

$$\hat{g}(\Omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} dt \hat{\phi} e^{i\Omega t} = \sum_K \left(\frac{2\pi c^2}{\omega_K V}\right)^{1/2} a_K \delta(\omega_K - \Omega), \qquad \Omega > 0$$

Power spectrum:

$$\langle \hat{g}^\dagger(\Omega)\hat{g}(\Omega')\rangle = \sum_K \left(\frac{2\pi c^2}{\omega_K V}\right)^{1/2} \langle a_K^\dagger a_K\rangle \delta(\Omega - \Omega')\delta(\omega_K - \Omega)$$

Quantum case.

Minkowski observer in thermal state:

$$\langle \hat{g}^\dagger(\Omega)\hat{g}(\Omega')\rangle = \frac{2\hbar c/\Omega}{e^{2\pi\Omega c/a} - 1}\delta(\Omega - \Omega')$$

Quantum case.

Rindler observer in Minkowski vacuum:

$$\langle \hat{g}^\dagger(\Omega)\hat{g}(\Omega')\rangle = \frac{2\hbar c/\Omega}{e^{2\pi\Omega c/a}-1}\delta(\Omega-\Omega')$$

Planck distribution, temperature, $T=\hbar a/2\pi kc$.

Ion traps.

Ion trap technology has its roots in high precision spectroscopy and frequency standards.

Relies on laser cooling techniques.

In 1995 Cirac and Zoller: this technology could be used to create qubit gate networks for qubits realised as two level atoms.

Ion trap QC is the technology with the best chance to reach the US government's Quantum Computer Technology Roadmap goals.

What is an ion trap?

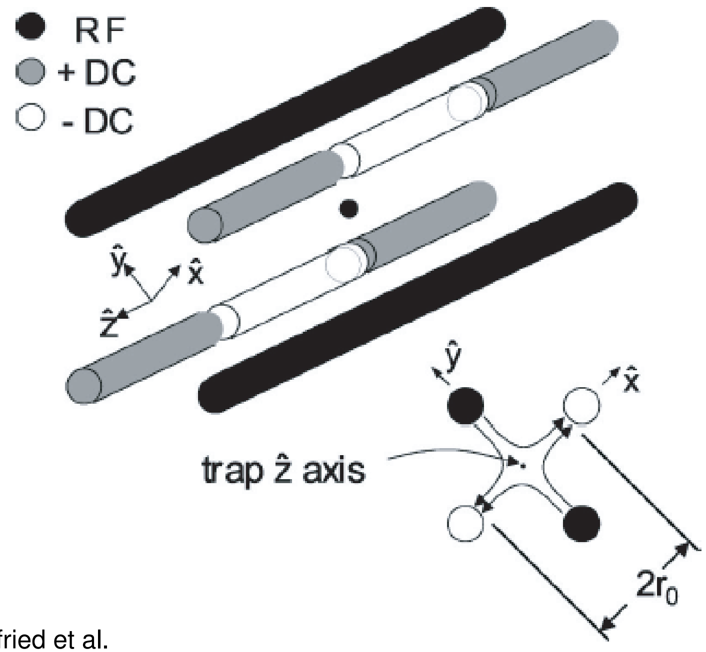
Objective: to harmonically trap a single ion in an electromagnetic trap, and cool to the vibrational ground state.

Problem: Laplace's equation $\rightarrow$ cannot make a stable electrostatic trap.

Solution: Time dependent potentials.

Examples.

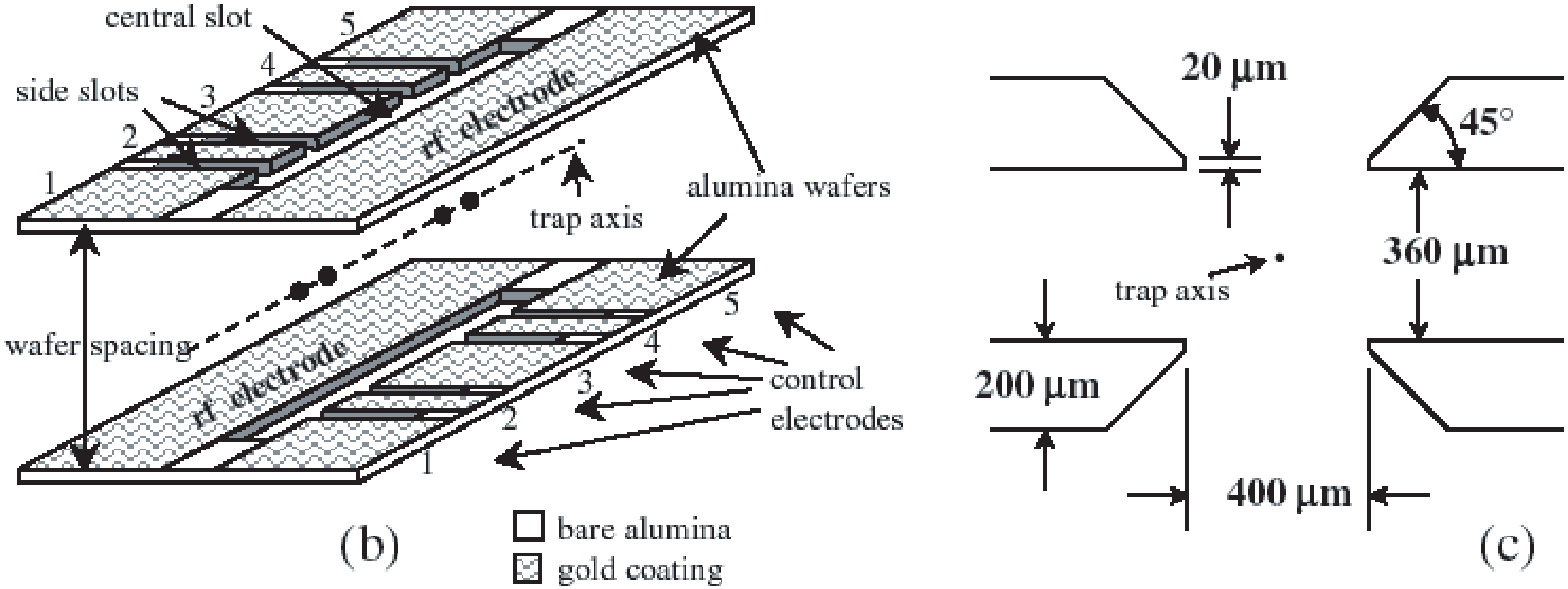
Linear RF trap.



Leibfried et al.
Rev. Mod. Phys. vol 75, 281 (2003)

Examples.

Wafer stack trap.



Liebfried et al. J. Phys. B **36**, 5999, (2003)

Single ion theory.

Let $\{\hat{q}_i(t), i = 1, \ldots, N\}$ be small oscillations around equilbrium for the i'th ion.

$$H_I(t) = -i\frac{\hbar\Omega}{2}\sigma_x(t)\left[e^{ik\hat{q}_1(t)-i\omega_L t} - hc\right]$$

$\sigma_x(t) = \left(|g\rangle\langle e|e^{-i\omega_A t} + |e\rangle\langle g|e^{i\omega_A t}\right)$ $\qquad$ ω_L: laser frequency.

. $\qquad$ ω_A : atomic transition.

In the Lamb-Dicke regime:

$$\eta = k_L \Delta q_{rms} << 1$$

expand $e^{ik\hat{q}_1(t)} = 1 + ik\hat{q}_1(t) + \ldots$

Ions as phonon detectors.

$$H_I(t) = \frac{\hbar\eta\Omega}{2}\sigma_x(t)\hat{q}_1(t)$$

Equivalent to the atom-photon interaction for EM detector.

Use as a *phonon counter*.

$$|\Psi\rangle_{in} = |g\rangle_{el}|\psi\rangle_{ph}$$

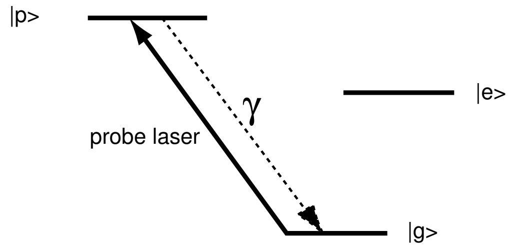
Prob. to find atom excited is

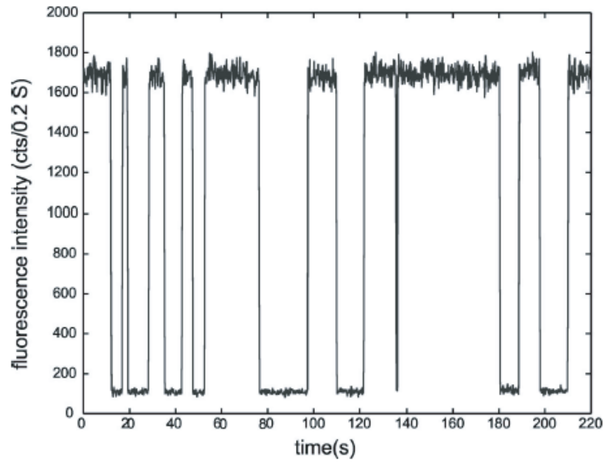
$$p_e \propto \langle \hat{q}_1^\dagger(\omega_A)\hat{q}_1(\omega_A)\rangle$$

$$\hat{q}_1(\omega) = \int_{-\infty}^{\infty} dt e^{i\omega t}\hat{q}_1(t)$$

Measurement of atomic states.

Fluorescent shelving technique.





Liebfried et al. 2003

total detection efficiency of 10^{-3} , $\rightarrow$ 50 000 counts/s with $\gamma = 10^8 \mathrm{s}^{-1}$.

Projective measurement of atomic state: efficiency $> \ 99\%$

Multiple ions.

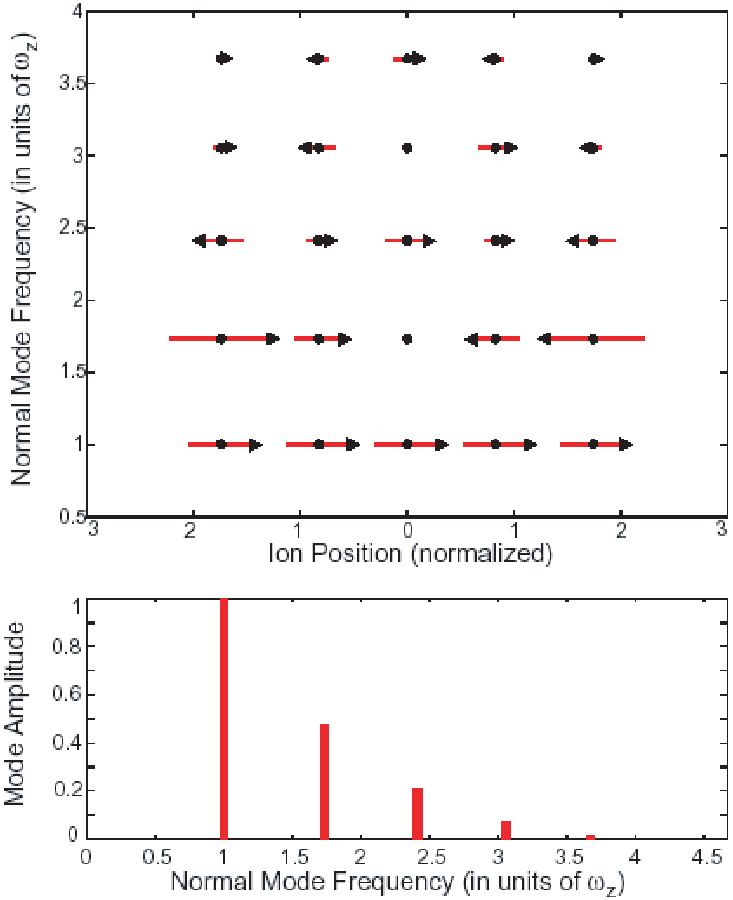


Figure 1. Normal modes: Frequencies and amplitudes for 5 ions. Note that in the upper part the amplitudes for the center of mass mode have been reduced to 30% for clarity.

Blatt group, Innsbruck.

Normal modes.

James, Appl. Phys. B 66, 181190 (1998)

Let $\{\hat{q}_i(t), i = 1, \ldots, N\}$ be small oscillations around equilbrium for the i'th ion.

Normal mode at frequency $\nu_p = \sqrt{\mu}_p \nu$, is

$$\hat{Q}_p(t) = \sum_{n=1}^{N} b_n^{(p)} \hat{q}_n(t)$$

Normal modes are *global modes*.

Laser cooling techniques prepare all modes in the ground state: $|0\rangle$

Normal modes.

Define creation and annihilation operators for normal modes:

$$\hat{Q}_p(t) = i\sqrt{\frac{\hbar}{2m\nu_p}}(a_p e^{-i\nu_p t} - a_p^\dagger e^{i\nu_p t})$$

$$\nu_p = \sqrt{\mu_p}\nu$$

where ν is the trap frequency, μ_p normal mode eigenvalue.

eg, $\mu_1 = 1, \quad \mu_1 = \sqrt{3}$

Local modes in terms of normal modes.

$$\hat{q}_m(t) = i\sqrt{\frac{\hbar}{2m\nu N}} \sum_{p=1}^{N} s_m^{(p)} (a_p e^{-i\nu_p t} - a^\dagger e^{i\nu_p t})$$

$$s_m^{(p)} = \frac{\sqrt{N} b_m^{(p)}}{\mu_p^{1/4}}$$

Modulate trap parameters.

$$\nu \longrightarrow \nu(t) = \nu e^{-\gamma t}$$

Solution for normal mode annhilation operators:

$$a_p(t) = a_p(0) e^{i\nu/\gamma} \exp\left[-i\frac{\nu}{\gamma}e^{-\gamma t}\right]$$

Thus local mode operator seen by k'th ion-detector:

$$\hat{q}_m(t) = i\sqrt{\frac{\hbar}{2m\nu N}} \sum_{p=1}^{N} s_m^{(p)} (a_p e^{i\nu/\gamma} \exp\left[-i\frac{\nu}{\gamma}e^{-\gamma t}\right] - hc)$$

Local detector sees thermal phonons.

Detector only responds to local mode $\hat{q}_1(t)$.

Prob. to find atom excited is

$$p_e \propto \langle \hat{q}_1^\dagger(\omega_A)\hat{q}_1(\omega_A)\rangle$$

$$\langle \hat{q}_1^\dagger(\omega_A)\hat{q}_1(\omega_A)\rangle = \frac{\hbar}{2m\omega_A}\left(\frac{1}{e^{\hbar\omega_A/kT}-1}\right)$$

$$T = \frac{\hbar\gamma}{k_B}$$

Conclusion.

- When one partner of a teleportation protocol is accelerated, the fidelity is reduced.
- Teleportation fidelity is a measure of entanglement.
- Connection between horizons and decoherence of quantum entanglement.
- Possible to simulate Unruh-Davies radiation in an ion trap...what else ?