Distributed measurement-based quantum computation

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Measurement-based models provide an exciting new framework for thinking about quantum computation. While quantum circuits are still widely considered as a convenient tool, and of course many experimental implementations research physical models based on its concepts, using measurements to steer quantum computation definitely came to be accepted as a serious alternative in recent times. Due to their inherent probabilistic nature, measurements were long thought to be a disturbance to quantum computations – unavoidable though they are when wanting to read out the final output of a computation. That measurements can be an active component of a computation has been known for quite some time through the teleportation protocol. Only much later it was realized that also in fault-tolerant constructions, measurements can be quite useful. Soon thereafter, with the advent of models such as the one-way quantum computer [RBB03] and the teleportation model [Nie03, Leu04], it was established that measurements could not only be a recurring component of a computation, but the actual driving force behind it. Moreover, the measurement paradigm throws a whole new light on the basic requirements of actual physical implementations of a quantum computer, and how these can be provided.

However, measurements are not the only crucial ingredient of these models: they are also inherently *distributed*. Indeed, it is the realization that an variation on the teleportation protocol not only transports but also *transforms* quantum information, which is the basis of the teleportation model. Likewise, the one-way quantum computer is all about transforming via measurement and transportation, this time by way of a generic entangled state, the *graph state*. One-qubit measurements on this state transform the logical qubits, i.e. the inputs, whilst transporting them via a path of graph state qubits. Again, non-local correlations provided by particular entanglement properties together with measurements, steer the computation. Of course quantum measurements remain intrinsically probabilistic, but this can be solved by applying corrections dependent on previous measurement outcomes, rendering computations effectively deterministic. All of this is very nicely captured by the *measurement calculus* [DKP04], a formal framework for one-way computations. Measurement patterns are defined essentially by sequences of commands allowed in the one-way model. From this one can define an (equivalent) operational and denotational semantics, and show that notions of composition are well-defined. More importantly, there is an associated *rewrite system* which allows one to put any pattern into a standard form. The measurement calculus, which can be seen as an *assembly language*, proves to be a valuable tool for formal investigations into all measurement-based models; f.e. one can easily show how the teleportation model reduces to the one-way model, via a conversion between the associated calculi [DKP05].

Because of the inherent distributive aspect, measurement-based models for quantum computation are well-suited as a starting point for a formal model for distributed quantum computations. By this we mean *macroscopically* distributed, i.e. we are talking about coordinated actions between different parties. Of these there are many examples in quantum computation [NC00]: teleportation,

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of course, but also entanglement swapping, logic gate teleportation, cryptographic protocols, and also quantum versions of classical distributed applications such as leader election [DP04]. However, a formal language for distributed quantum computation is lacking. While some endeavors have been made into using techniques from classical process calculi [LJ04, GN04], these have remained rather descriptive and are, to our opinion, not very well-suited to really get a grip on the low-level quantum aspects. In this work, we define the *distributed measurement calculus*, an assembly language for distributed applications, directly built on the most basic distributed model of all: the one-way quantum computer. We first develop a *global* view on distributed computations. A distributed patterns is described in rounds, where during each round measurement patterns are carried out in parallel, after which classical and/or quantum communication occurs. A formal semantics is defined much as it was for the ordinary measurement calculus. Next, we switch to a *local* view and describe the system as a set of *agents* communicating asynchronously and operating on a globally entangled quantum state, Again, we define a formal semantics, and furthermore define a *bisimilarity relation.* We use this to prove the bisimilarity of several exemplary *quantum channels.* Most importantly, we prove that transporting a qubit physically between agents is bisimilar to teleporting it between them.

References

[DKP04] Vincent Danos, Elham Kashefi, and Prakash Panangaden. Robust and parsimonious realisations of unitaries in the one-way model. 2004.

[DKP05] Vincent Danos, Elham Kashefi, and Prakash Panangaden. 1-qubit versus 2-qubit measurement-based quantum computation. 2005.

[DP04] Ellie D'Hondt and Prakash Panangaden. Quantum weakest preconditions. In Peter Selinger, editor, *Proceedings of the 2nd Workshop on Quantum Programming Languages (QPL04)*, Turku, Finland, 2004. Turku Centre for Computer Science, TUCS General Publication No 33.

[GN04] Simon J. Gay and Rajagopal Nagarajan. Communicating quantum processes. In Peter Selinger, editor, *Proceedings of the 2nd Workshop on Quantum Programming Languages (QPL04)*, Turku, Finland, 2004. Turku Centre for Computer Science, TUCS General Publication No 33.

[Leu04] Debbie W. Leung. Quantum computation by measurements. *Int. J. of Quant. Comp.*, 2(1):33–43, 2004.

[LJ04] Marie Lalire and Philippe Jorrand. A process algebraic approach to concurrent and distributed quantum computation: operational semantics. In Peter Selinger, editor, *Proceedings of the 2nd Workshop on Quantum Programming Languages (QPL04)*, Turku, Finland, 2004. Turku Centre for Computer Science, TUCS General Publication No 33.

[NC00] Michael A. Nielsen and Isaac Chuang. *Quantum computation and quantum information.* Cambridge university press, 2000.

[Nie03] Michael A. Nielsen. Universal quantum computation using only projective measurement, quantum memory, and preparation of the 0 state. *Phys. Lett. A*, 2-3(308):96–100, 2003.

[RBB03] Robert Raussendorf, Daniel E. Browne, and Hans J. Briegel. Measurement-based quantum computation on cluster states. *Phys. Rev. A*, 68(2):022312, 2003.