The Logic of Quantum Actions: reasoning about change in quantum systems

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The older results (due to Piron, Soler, Mayet and others) on the Hilbert-complete axiomatizations of algebraic quantum logic can be improved and put into a new light by moving to a *dynamic-logical* setting, in which *physical actions* (and not only static physical properties) are logically represented.

In the paper [1], we present two *Hilbert-complete axiomatizations* for the "logic of quantum actions" (LQA): one in terms of *quantum transition systems* (QTS), and one in terms of *quantum dynamic algebras.* In this talk, I concentrate on the first setting, defining a QTS as a relational structure, subject to a number of conditions, each having a natural physical/operational interpretation. Any Hilbert space can be naturally structured as a QTS. Conversely, we have a Representation Theorem for QTS's with respect to Hilbert spaces. Next, I present a *finitary modal logic for quantum actions*, which can be seen as a *quantum version of Propositional Dynamic Logic (PDL).* The language consists of *propositional formulas* φ (expressing "static" properties) and of *programs* π (expressing "actions"):

$$\varphi ::= p \quad | \quad c \quad | \quad \neg\varphi \quad | \quad \varphi \wedge \varphi \quad | \quad [\pi]\varphi$$
$$\pi ::= \varphi? \quad | \quad U \quad | \quad U^{\dagger} \quad | \quad \pi \cup \pi \qquad \pi ; \pi$$

Formulas are interpreted using Boolean operations and "weakest preconditions" $[\pi]\varphi$ of actions (modelling *physical causality*). The program constructs are interpreted as: *successful measurements* ("tests") $\varphi?$ (i.e. *projectors* in a Hilbert space), *unitary evolutions* U (and their inverses $U^{\dagger}$), relational union $- \cup -$ (modelling *non-determinism*) and relational composition $-;-$ (modelling *temporal succession*, i.e. sequential composition of actions). The proof system is *sound and complete w.r.t. QTS's (and thus also w.r.t. Hilbert spaces).* Unlike other quantum-logical approaches, (the "static", propositional fragment of) our logic is "classical", i.e. Boolean; all the "quantum" effects are captured by the *dynamic side* of our system.

From a *philosophical* point of view, the moral is that quantum physics does not require any modification of the classical laws of Propositional Logic (governing "static" information), but only a *non-classical dynamics of information.* From a *computational* perspective, the importance of our dynamic setting is that it can be used as a bridge between traditional quantum logic and the needs of quantum computation. From an input-output point of view, computer programs are "actions"; moreover, our formalism is a quantum version of PDL, a logic that was developed by computer scientists as a tool for the formal verification of (classical) programs. Indeed, in subsequent work (to be presented at this workshop by A. Baltag in his related talk), we have shown that an extension of this formalism can be used to formally prove correctness of some quantum algorithms.

References

[1] A. Baltag and S. Smets, "Complete Axiomatizations for Quantum Actions", *International Journal of Theoretical Physics*, to appear.