# Rickart Comparability Groups and Quantum Logics

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**Abstract**

A *unital group* is a directed partially ordered abelian group $G$ with a distinguished positive element $u$, called the *unit*, such that every positive element in $G$ is a sum of a finite sequence of elements in the *unit interval* $E := \{e \in G \mid 0 \leq e \leq u\}$. Nearly every structure that has been proposed as a "quantum logic" can be realized as the unit interval in a unital group. A *compression based group* (CB-group) is a unital group together with a distinguished family $(J_p)_{p \in P}$ of compression operators. A *comparability group* is a CB-group $G$ such that each element $g \in G$ is canonically decomposed as $g = g^+ - g^-$, where $g^+ = J_p(g)$, $g^- = J_p(-g)$, and $p \in P$. A *Rickart comparability group* (RC-group) is a comparability group equipped with a mapping $' \colon G \to P$ such that, for all $p \in P$ and all $g \in G$, $J_p(g) = g \Leftrightarrow p' \leq g'$. Every element in an archimedean RC-group determines and is determined by a (rational) *spectral resolution* in $P$. An example of a RC-group is the additive subgroup $G(A)$ of an AW$^*$-algebra $A$ with 1 as the unit, $P$ as the set of projections in $A$, and $J_p(g) = pgp$ for all $g \in G(A)$. In my talk, I will argue that the most important quantum logics are realized as the unit interval in an archimedean RC-group, outline the current state of knowledge concerning the structure of CB-groups, and indicate directions in which further research could proceed.