Quantum Measure Theory and its Interpretation [†]

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Abstract

We propose a realistic, spacetime interpretation of quantum theory in which reality constitutes a *single* history obeying a "law of motion" that makes definite, but incomplete, predictions about its behavior. We associate a "quantum measure" $|S|$ to sets S of histories, and point out that $|S|$ fulfills a sum rule generalizing that of classical probability theory. We interpret $|S|$ as a "propensity", making this precise by stating a criterion for $|S| = 0$ to imply "preclusion" (meaning that the true history will not lie in S). The criterion involves triads of correlated events, and in application to electron-electron scattering, for example, it yields definite predictions about the electron trajectories themselves, independently of any measuring devices which might or might not be present. (In this way, we can give an objective account of measurements.) Two unfinished aspects of the interpretation involve *conditonal* preclusion (which apparently requires a notion of coarse-graining for its formulation) and the need to "locate spacetime regions in advance" without the aid of a fixed background metric (which can be achieved in the context of conditional preclusion via a construction which makes sense both in continuum gravity and in the discrete setting of causal set theory).

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Three Principles

Let me begin by listing three principles and asking whether or not they are compatible with quantum mechanical practice (as opposed to one or another interpretation of a particular mathematical version of the quantum formalism). There are many reasons for raising such a question, but to my mind the most important is the need to construct an interpretive framework for quantum gravity, which I believe we will attain only by holding onto these principles (cf. [1]). In the present talk, I will try to convince you that this belief is viable by sketching an interpretative framework for quantum mechanics in general, which honors the principles, but which will still allow us to continue using quantum theory in the manner to which we have grown accustomed.

The three principles in question are those of *Realism*, of the *Spacetime Character* of reality, and of the *Single World*.

By realism/objectivity I mean for example that in electron-electron scattering, the electrons exist and have definite trajectories, and that consequently a statement of the form "the electrons never scatter at 90 degrees" is meaningful in itself, without needing to be reinterpreted as shorthand for "If we set up detectors at 90 degrees they will never register an event".

By the spacetime- (or better the "history-") character of reality, I mean for example that it is the 4-metric $^4g_{ab}$ which is real and not the "wave-function of the universe" ψ, or for that matter, some purely spatial positive-definite metric $^3g_{jk}$.

By the principle of the single world, I mean for example that in an electron diffraction experiment, the electron traverses a single definite slit, and not somehow two slits simultaneously, or one slit in one "world" and another slit in some other world. But "real" and "single" does not entail "deterministic", and I do not mean to deny that (on current evidence) the world is fundamentally stochastic—so that, for example, the electron's past trajectory does not determine fully what its future trajectory will be.

The motivation for these principles comes partly from familiar philosophical worries and partly from the projected needs of quantum gravity. The so-called Copenhagen Interpretation has no answer to the question "Who shall observe the observer?", no way to give a rational account of "wave function collapse", and more generally no escape from the vicious circle that we ourselves are made out of atoms and therefore cannot be more real than they are (cf. [2]). In the early universe—one of the main anticipated fields of application of quantum gravity—these questions assume a much greater practical importance, because then there were no observers at

all, and having to refer to that era through the indirect medium of present-day observations would complicate unbearably the already difficult questions of quantum cosmology. For quantum gravity more generally, its fundamental diffeomorphism-invariance means that only global properties of the metric have physical meaning, and it is hard to see how such properties could be reduced to statements about objects tied to spacelike hypersurfaces, like the wave-function and spatial metric of canonical quantum gravity. Hence the need for a spacetime character. As for the "singleness" of the world, it is hard for me to imagine what the contrary hypothesis might mean, or how physics can have any predictive content at all if everything conceivable actually occurs. *

But do the facts of quantum mechanics actually allow us to hold on to these three principles? Perhaps the strongest argument that they must, is merely an appeal to the obvious truth that we (or at least most of us) *experience* a single world of really existing objects extended in spacetime. If this is in fact the nature of our experience, then logically it ought not to be possible to force us to some other viewpoint—especially if the character of that other viewpoint is precisely to elevate our subjective experience above what we naively take to be objective reality. However convincing such reasoning might be, though, it does not yet suggest concretely how a realistic, spacetime account of quantum mechanics would go. For that, we must look to the theory itself.

The Sum-over-histories

Of the existing formulations of quantum mechanics, the only one which provides a starting point for constructing a spacetime framework is of course the sum-over-histories formulation, which in fact is explicitly called a "spacetime approach" in one of the founding papers on the subject [3]. In that formulation, the central dynamical quantity is what I will call the "quantum measure" † $|S|$ of a subset S of the space $\mathcal{H}$ of all possible histories γ [3][4][5]. Here "history" is a synonym for "possible reality", the concrete meaning of which depends on what one takes

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* Beyond this general remark, I find it hard to comment explicitly on the "many worlds" interpretation, because I have never achieved a clear understanding of what it is intended to mean. If I had to put my finger on what seems most obscure to me, I probably would choose to emphasize the so-called "pointer basis problem".

† Not to be confused with the so-called measure-factor in the path integral, which corresponds to only part of what is here meant by the measure.

to be the basic form of matter one is dealing with: a collection of particle world-lines or spacetime fields; a 4-dimensional Lorentzian manifold; a causal set [6]; or whatever. * I will maintain that the task of understanding quantum mechanics in a manner compatible with the three principles enunciated above will be accomplished if we succeed in understanding the meaning of the quantum measure in a way which frees it from any reference to the customary apparatus of expectation-values and wave-function collapse.

In non-relativistic quantum mechanics or quantum field theory, the measure $|S|$ of a particular subset S of the space of all histories $\mathcal{H}$ is a nonnegative real number computed by a rather peculiar looking rule [4] [7] [8], which involves not just individual "paths" or "histories", but *pairs* of them, corresponding to the familiar fact that probabilities turn out to be quadratic rather than linear in the basic amplitudes. Specifically the rule is

$$|S| = \sum_{\gamma_1, \gamma_2 \in S_T} \operatorname{Re} D(\gamma_1, \gamma_2), \qquad (1)$$

where initial conditions are assumed to be given at some early time (before any time to which the properties defining S refer), and one has introduced a *truncation time* (or hypersurface) T which is late enough to be after any time to which the properties defining S refer. Then γ_1 and γ_2 are truncated histories which emerge from the initial spacetime region with some (joint) amplitude α and propagate to T; and $D(\gamma_1, \gamma_2)$ is either zero or the product of α with the amplitude of γ_2 times the complex conjugate of the amplitude of γ_1, according as the paths γ_1 and γ_2 either do not or do come together at T (in the field theory case "coming together" means

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* Notice that this usage is in one way much more restrictive than that of reference [7], where arbitrary sequences of projection operators qualify as "histories". Notice as well, that a change in how one identifies the basic "substance" will in general change the meaning of the principle that one and only one history is realized. Thus, when in the introduction I illustrated this principle with the statement that the electron traverses only a single slit, I was assuming that electrons and spacetime itself are, if not fundamental, then emergent in such a way that the concept of electron trajectory continues to make sense as a property of a single history. By way of contrast, the analogous assumption would seem considerably less warranted in the case of photon trajectories, for example, which have to be reinterpreted in terms of field configurations if one takes the electromagnetic field to be the fundamental reality.

having equal restriction to the hypersurface T). Pictorially this expression appears as in figure 1. The corresponding formula in operator language is $|S| = \operatorname{tr} C^* \rho_i C$ where ρ_i is an initial density operator and C is the operator which propagates ρ forward when the domain of integration in the path-integral is restricted to just those γ belonging to S. (Notice that the real-part operator Re can be omitted without changing the value of the sum (1).)
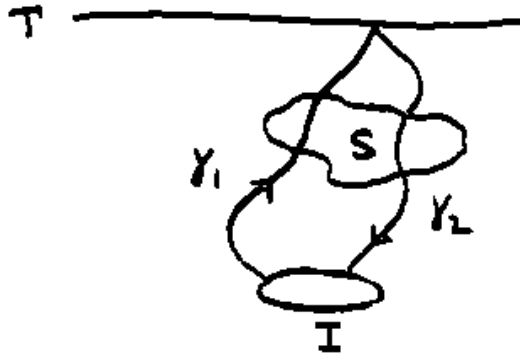

Figure 1 Definition of the quantum measure in non-relativistic quantum mechanics

In the special case that the set S is defined by requiring the history to belong to specified regions of configuration space at specified moments of time, $|S|$ reduces to the usual quantum mechanical probability that the corresponding position-measurements will all yield affirmative results, but this special case can exhaust the physical meaning of the measure only to the extent that we retreat from a fully spacetime picture to something like a pre-relativistic one, thereby running head-on into what is often called the "problem of time in quantum gravity." To avoid such an eventuality, we must find an interpretation of $|S|$ which frankly adopts a space-time standpoint rather than appealing to some notion of position-measurements at specified times.

To grasp the meaning of quantum mechanics from such a standpoint means first of all to understand why the measure assumes the peculiar "quadratic" form that it does, and second of all — and more importantly — to specify the physical meaning of the measure $|\cdot|$ (in effect its predictive content) without appealing to the notion of observation or measurement as an undefined primitive of the theory. Let us take up these two questions in turn.

Quantum Measure Theory

Seen in the way I am advocating, quantum mechanics is a direct descendent of the classical theory of stochastic processes [9], and differs from it only in that a different "probability calculus" is involved from that of classical measure theory (a point stressed early on in [10]). Classical measure theory also attaches a number $|S| \geq 0$ to each (measurable) set of histories, but there $|S|$ has an immediate interpretation as a probability, and accordingly obeys the classical sum rule (with '$\sqcup$' denoting disjoint union),

$$I_2(A, B) := |A \sqcup B| - |A| - |B| = 0, \tag{2}$$

which allows the measure to be given a frequency interpretation.

In the quantum generalization, $|S|$ is still ≥ 0 but (2) gets weakened to the following pair of axioms, any solution of which may be called a *quantum measure.*

$$|N| = 0 \ \Rightarrow \ |A \sqcup N| = |A| \tag{3}$$

$$I_3(A, B, C) := |A \sqcup B \sqcup C| - |A \sqcup B| - |A \sqcup C| - |B \sqcup C| + |A| + |B| + |C| = 0 \tag{4}$$

Of these two axioms, only the first seems clearly essential for the type of interpretation to be given below. The second on the other hand, makes the quantum measure what it is, and can be thought of concretely in terms of the "3-slit experiment", or more generally in connection with any process in which three mutually exclusive alternatives interfere [11]. Indeed, the 3-slit-experiment can be said to epitomize quantum mechanics by illustrating the possibility of interference on one hand (i.e. the fact that I_2 does not always vanish), and on the other hand the fact that no *new* type of interference arises when one passes from two alternatives to three or more.

In this sense, quantum mechanics is a rather mild generalization of classical measure theory, and (4) is only the first of an infinite hierarchy of successively more general sum-rules,

$$I_2 = 0 \ \Rightarrow \ I_3 = 0 \ \Rightarrow \ I_4 = 0 \ \Rightarrow \ \cdots,$$

each formed using the same pattern of even minus odd, and each of which might serve as the basis for a further generalization of the quantum formalism. [11] An experimental search for 3-alternative interference, therefore, would provide a "null

test" of one of the hallmarks of quantum mechanics, and—were the test to fail—might suggest the need for generalizing (4) in the direction of one of the higher sum-rules. [†]

Here, however, I want to emphasize not the possibility of further generalization, but the extent to which the sum-rule (4) accounts for the "quadratic character" of the quantum measure as expressed in (1). In fact one can prove that, given any set-function $|\cdot|$ for which $I_3(A, B, C)$ vanishes identically, the definition

$$I(A, B) := |A \cup B| + |A \cap B| - |A\backslash B| - |B\backslash A|$$

provides a function of pairs of sets which: (i) extends the function I_2 of eq. (2) to overlapping arguments; (ii) is additive separately in each argument:

$$I(A \sqcup B, C) = I(A, C) + I(B, C);$$

and (iii) allows $|\cdot|$ itself to be recovered via the equality

$$|A| = \frac{1}{2}I(A, A).$$

[Thus $I(A, B)$ corresponds to (twice) the real part of the "decoherence functional" $D(A, B)$.]

In this sense, the fact that the quantum measure can be derived from a "decoherence function" is explained by the sum-rule (4). However (4) by itself does not explain the appearance in quantum theory of complex numbers—together with the associated unitarity—nor does it entail the "Markov" character of the measure (meaning the fact that amplitudes evolve locally in time via the Schrödinger equation). On the other hand realistic quantum systems are in practice always open, and therefore their evolution is strictly speaking neither unitary nor Markovian (cf. [13], [14] and the first reference in [5]). Thus, the extra generality afforded by the "quantum measure theory" framework is, in practice, already needed for the correct

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[†] These sum-rules appear to have a theoretical relevance entirely independent of whether there exist physical processes which directly embody them. As pointed out by D. Meyer [12], such processes, if they existed, could be used to solve in polynomial time certain classes of problems which would take exponentially long using classical or quantum computations. Indeed, there seems to be an entire hierarchy of new computational complexity classes, corresponding to the sum rules $I_n = 0$ for $n = 4, 5, 6, \ldots$.

description of everyday systems, whatever may be its ultimate fate in connection with quantum gravity.

The Meaning of the Measure

We saw above that the quantum measure reduces in special cases to a quantity which one could consistently interpret as the probability of an affirmative outcome of a sequence of position measurements. More generally one can presumably think of $|S|$ as a kind of "propensity of realization" of the set $S \subseteq \mathcal{H}$. We might expect, for example, that the ratio $|S| / |\mathcal{H} \backslash S|$ indicates how much "more likely" the actual history is to be found in S than in its complement. But since $| \cdot |$ does not in general obey the classical sum rule (2), it is not a probability in the ordinary sense, and it is not obvious how to make such a notion of "more likely" quantitative in a meaningful and consistent manner. In pondering this task, it seems appropriate to enquire more closely into what is meant by probability in the classical case.

Even classically, probability is hard to understand because it seems to govern how a history *will* develop, but in retrospect is nowhere visible in how the history actually *has* developed. (The chance of rain as of yesterday was only 20%, but today it is raining.) In some hard to define sense, probability refers to that which doesn't exist as well as to that which does, a characteristic which is reflected in the fact that we often speak of it in terms of imaginary ensembles of universes.

One way to extract a positive meaning out of probabilistic assertions without invoking an infinite ensemble of universes is to appeal to the notion of *preclusion**, which is meant to express the idea that certain events are so unlikely as to be "essentially impossible". We may say, for example, that in a trial of ten thousand tosses of a fair coin, it is "precluded" that heads will come up ten thousand times. More generally, we can try to substitute the concept of preclusion for that of probability, and seek the dynamical content of a theory in its statements of preclusion. In this way, the predictions we can make become "definite" but incomplete.[†] That

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* This term is borrowed from [15] and belongs to the interpretation of probability which seems to go by the name of "Cournot's Principle".

† Unfortunately, they also become imprecise to the extent that the criterion for preclusion is itself imprecise, which it necessarily is classically, and quite possibly also must be quantum mechanically. More generally, it may be that one loses something by reducing probability to preclusion, but at least the resulting concept is relatively precise and objective.

is, the statement that a certain subset $S \subseteq \mathcal{H}$ is precluded, means that the *actual* history γ will not belong to S; it is thus a *definite* assertion about γ, but not one that determines γ in all its details, as a prediction in celestial mechanics would, for example.

Having decided to interpret probability in terms of preclusion, one still has the further task of incorporating the fact that what is precluded is not fixed once and for all, but rather "changes with time"; or in other words the fact that the future is *conditioned on* the past. (Thus, it may be that in 1858 the American Civil War was still not inevitable, but that peace became precluded when John Brown was hanged—at least this was an implication of his gallows address). To do this one needs to be able to make *conditional* statements of the form, "If the past has such and such properties then such and such a future possibility is precluded". With a classical probability measure, the criterion for conditional preclusion can be derived from the criterion for absolute preclusion just by relativizing the measure to the appropriate subspace of $\mathcal{H}$. With a quantum measure, things will be more complicated.

So far in this section we have mainly been reviewing how one can interpret *classical* probability-measures in terms of preclusion. The challenge now is to find a similar scheme for interpreting the quantum measure $| \cdot |$, a scheme powerful enough to allow us to make the predictions on whose success our belief in quantum mechanics is based. In the remainder of this paper, I will propose a candidate scheme of this kind. It is a bit complicated, but in its own way natural, and perhaps even consistent. I would be surprised if the further working out of this interpretation did not change many of its details (for example the shapes of the spacetime regions which enter), but I also feel that it is broadly on the right track.

Criteria for Preclusion (unconditional case)

Let us assume, for the moment, a non-dynamical spacetime M with a fixed causal structure, and consider an event E *defined in* a region $R \subseteq M$. Here "an event" means just a set of histories ($E \subseteq \mathcal{H}$), and describing it as "defined in R" will mean that the properties which determine whether a given history γ belongs to E or not pertain only to the portion of γ within R (specifically to γ's *intersection* with R if it is a set of world lines, its *restriction* to R if it is a field, etc). We now ask, "When is such an event E precluded?"

An answer given along the lines of [16] would be something like, "E is precluded when $|E| = 0$ (or $< \epsilon$) and we *can* operationally distinguish E from its complement,
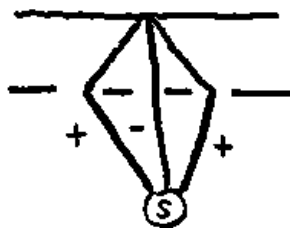
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but we *cannot* operationally distinguish subsets of E from each other". (For example, in a two-slit experiment, the arrival of the electron at a dark band on the screen would be precluded because it has measure zero, and we *can* tell whether the electron has landed there rather than elsewhere, but we *cannot* tell which of the two slits it traversed.)

The trouble with this criterion, of course, is that it appeals to our possibility of knowledge, which is at once subjective and vague. * On the other hand, it won't do just to drop all qualification and say that E is precluded whenever its measure is sufficiently small. That criterion would be objective and well-defined, but it would in general lead to the absurd result that *everything* would be precluded.

A striking illustration of this difficulty is furnished by the three-slit experiment referred to earlier, which we may idealize for present purposes as a source emitting spinless particles which impinge on a diffraction grating with three slits. (See figure 2.) Let P be a spacetime region—idealized as a point—which is aligned with the central slit and consequently sits within a "bright band" of the diffraction pattern. For such a point, the rule (1) yields a nonzero value for the measure $|\{1, 2, 3\}|$ of the set of all world lines that arrive at P via any one of the slits, and we know, in fact, that if we look for the particle at P by placing a detector there, we will often find it. On the other hand, we can choose the separation between the slits so that, when taken in pairs $\{1, 2\}$ or $\{2, 3\}$, the amplitudes cancel, and correspondingly, the measures of $E = \{1, 2\}$ and of $F = \{2, 3\}$ will vanish: $|E| = |F| = 0$. An unrestricted preclusion rule would then entail that the actual history could belong neither to E nor to F, whence it could not arrive at P at all—a false prediction. More generally, one can typically embed any given history γ in a subset $S \subseteq \mathcal{H}$ of zero measure, whence every possibility without exception would be ruled out by an unrestricted preclusion rule. [†]

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* which is not to say that the criterion could not still be perfectly satisfactory in many situations.

[†] In a certain sense the same problem is present even classically (where for example, each single Brownian path taken individually is of measure zero), but the contradiction in the quantum case is made worse by the effects of interference. If, despite this, one could somehow "learn to live with the contradiction", as one does in the case of classical probability theory, then the remainder of this paper would be superfluous.

Figure 2 Possible histories in three-slit diffraction

To avoid such nonsensical conclusions, we are obliged to place further limits on the application of the preclusion concept. In seeking inspiration for such a modified criterion, it is natural to refer mentally to measurement situations; for they are one arena where contradictions of the above sort are avoided. We may therefore suspect that there is something special about measurement situations which, if correctly identified, would provide a prototype on which a more general, objective interpretation of the quantum measure could be modeled. But what is this special feature? If your answer turns on "amplification" and the decohering of macroscopically distinct alternatives, you will be led in the direction of a "consistent histories" interpretation of quantum mechanics. If, on the contrary, you fasten on the correlation which a measurement sets up between observer and observed, you will be led to the sort of scheme I am about to propose. *

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* This may be a good place to comment on the relation of my interpretation to the consistent histories scheme, where by the latter I mean especially the version described in [7]. The two approaches agree on the spacetime character of reality; both take the measure $|\cdot|$ to be the fundamental quantity governing the dynamics; and both attempt to honor the "principle of realism-objectivity" by working with "beables" (properties of histories) rather than "observables". As presented in [7] and elsewhere, the realistic aspect is not so much stressed, but this seems more a matter of philosophical predisposition than of a genuine difference in the schemes. The real difference lies in the interpretation of the measure. The "consistent historians" would in effect reduce the quantum sum-rule $I_3 = 0$ back to the classical one $I_2 = 0$ by limiting the domain of $|\cdot|$ to a suitable family of subsets ("coarse-grained histories"). In this way "consistency" is achieved, in the sense that one has returned

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This scheme, specifically, will be founded on the idea that the quantum measure acquires predictive content only when variables belonging to two kinematically independent subsystems become correlated, or more accurately, since neither "kinematic independence" nor "subsystem" has any definite meaning in general, when there occur *correlations between events which* (in the technical sense introduced above) *are defined in spacelike separated regions of M*.

This is the basic idea, but its most straightforward implementation does not entirely eliminate the problem of "overlapping preclusions", of which the 3-slit diffraction described above is only the simplest instance. More complicated instances remain, where the problems arise from specious correlations between non-interacting systems containing "null events" (events of measure 0), or from correlations of the sort that occur in the Kochen-Specker version of the "EPRB"-experiment. There is no space here to explain these problems in detail (a fuller account will appear elsewhere), but the attempt to frame a preclusion criterion which can exclude both types of difficulty leads one to introduce a third spacetime region, whose effect is not only to remove the unwanted preclusions, but also (as an unanticipated benefit) to broaden the range of inferences one can draw about the past in situations where the new criterion *is* satisfied. (Incidentally, most of the problem situations just alluded to have implications for consistent histories as well, where they typically provide new types of examples of mutually inconsistent coarse-grainings. For instance, in the 3-slit situation above, $\{1, 2\} \sqcup \{3\}$ and $\{1\} \sqcup \{2, 3\}$ are both "consistent" coarse-grainings in themselves, but for the first, only alternative $\{3\}$ can happen, whereas for the second, only alternative $\{1\}$ can happen.)

The specific criterion or *preclusion rule* which emerges may be stated formally as follows (see figure 3). Let I, II and III be a triple or "triad" of spacetime

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to classical probability theory, but direct contact with the micro-world tends to be lost (because decoherence tends to require macroscopic objects), and the principle of the single world is sacrificed to the extent that the choice of coarse-graining is non-unique, leading either to conflicting probability predictions or the admission of several distinct "worlds", each with its own dynamical laws [17]. In contrast, my approach embraces the non-classical behavior of the measure as that which makes quantum mechanics what it is, and tries to interpret $|\cdot|$ directly via a preclusion rule which is strong enough to justify standard applications of the quantum formalism, but weak enough to avoid the kind of contradiction exposed above. Despite these differences there are many parallels and connections between the schemes, some of which should be apparent in the following pages.

regions, and for each $R = I, II, III$ let E_i^R ($i = 1 \ldots n$) be a system of n events defined in R. Let regions I and II be spacelike to each other, with III equal to the common future of $I \cup II$; or in formulas:

$$I \natural II \qquad \text{and} \qquad III = \text{future}(I) \cap \text{future}(II),$$

where '$R \natural S$' means that every element of R is spacelike to every element of S, and 'future (R)' denotes the set of all points which are to the future of *every* element of R. For each $R = I, II, III$, let the E_i^R partition $\mathcal{H}$ (meaning that $\mathcal{H} = \sqcup_i E_i^R$ is their disjoint union; such systems of events are often called "exclusive and exhaustive"). Further, let $\widetilde{E}_i^R$ denote an arbitrary "sub-event" (i.e. subset) of E_i^R, also defined in R. Call a triple i, j, k "diagonal" when $i = j = k$, and "off-diagonal" otherwise. Then, the criterion comprises two assertions. First, if

$$|\widetilde{E}_i^I \cap \widetilde{E}_j^{II} \cap \widetilde{E}_k^{III}| = 0 \tag{5}$$

in all off-diagonal cases and for all choices of sub-events $\widetilde{E}_i^R$, *then every off-diagonal event $E_i^I \cap E_j^{II} \cap E_k^{III}$ is precluded.* Second, if in addition to (5) we have

$$|E_i^I \cap E_i^{II} \cap E_i^{III}| = 0 \tag{6}$$

for some particular i, *then that E_i itself is precluded.* (Notice that in the presence of (5), (6) entails $|E_i^I| = |E_i^{II}| = |E_i^{III}| = 0$ by the axiom (3).) Moreover, these implications will, if necessary [†], be taken to hold when "$= 0$" is replaced by "$< \epsilon$, for sufficiently small ϵ".
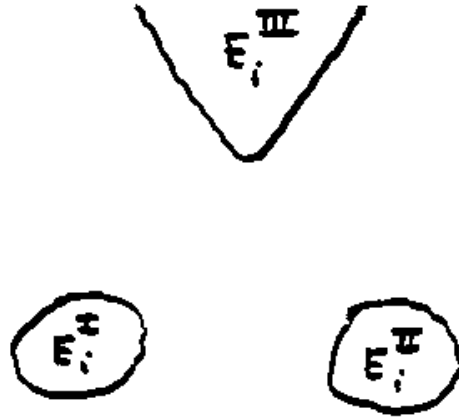

Figure 3 A triad of spacetime regions

[†] That such a "fuzzing" might conceivably be dispensed with, is due to the fact that the measure of a nontrivial quantum alternative can vanish exactly, unlike in classical probability theory.

The effect of this rule is to delineate a class of situations in which one can uphold the idea that sets of zero measure are precluded, even if (as we have seen) this idea cannot consistently be maintained in general. Specifically, the criterion tells us that certain three-way correlations which are "present in the measure" will in fact be present in the history itself*, and that when such a correlation is in force, not only are all the "off diagonal" possibilities precluded, but also any of the "diagonal" ones which are themselves of zero measure.
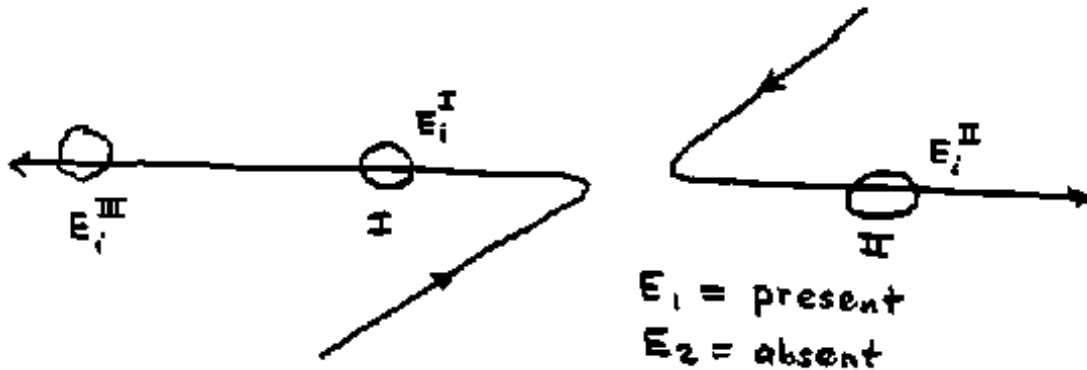

Figure 4 a Correlations in electron-electron scattering

We can illustrate the meaning of our preclusion criterion with an idealized account of the scattering of low-energy polarized electrons in their center of mass frame (see Figure 4). In that situation the amplitude for the electrons to emerge other than in approximately opposite directions is zero, as is the amplitude for them to scatter at 90 degrees (due to the fermionic cancelation between "exchanged" trajectories). Then, let us choose our regions I and II to be situated simultaneously in time, and spatially opposite each other with respect to the scattering center; and let their extension be large enough so that the corresponding "uncertainty principle disturbances" of the electrons' energy and momentum are negligible. Let us take event E_1^I to be the *presence* of an electron in I, and E_2^I to be the complementary possibility (absence of any electron there), with the E_i^{II} defined analogously. For the corresponding pair of events in region III, we can take the presence or absence

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* The indirect wording of the criterion, which expresses a correlation as the negation of its negation, could be avoided if the opposite of preclusion could be defined directly, but this seems to be impossible because there is no analog of being of probability 1 for the quantum measure (which is unbounded above, due to the possibility of *constructive* interference).

of an electron at some convenient location within *III* to which the continuation of a scattering trajectory through region *I* (say) would lead.
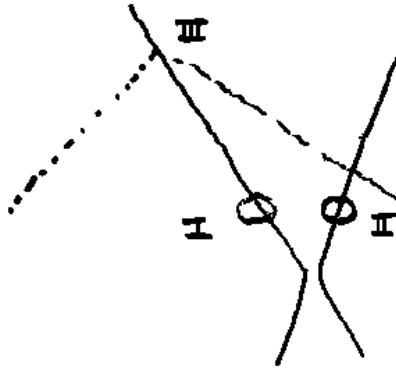

Figure 4 b The same scattering as a spacetime diagram

Then the first clause of the preclusion criterion will be satisfied, whence we can assert that either there will be electrons in both regions *I* and *II* or in neither (and the former happens if and only if the electron traversing region *I* also is found at the "confirming" location within region *III*). By appealing to a sufficient number of such triads, we can then claim that the electrons do in reality emerge opposite each other (to an accuracy governed by the region size). Recall that this assertion concerns the trajectories themselves, and is to be regarded as meaningful whether or not the regions are provided with electron detectors.[†]

For an application of the second clause of the preclusion criterion, we can now choose regions *I* and *II* to be at 90° and 270° respectively, and then we predict that no electron will appear in either region. Once again, this is a prediction about the electrons themselves, independently of any apparatus which we might install to help confirm the prediction experimentally.

(The illustration of the preclusion criterion we have just given deals directly with the behavior of microscopic objects. For macroscopic objects such as measuring instruments, the possible triads are much more numerous and harder to analyze

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[†] A slightly more elaborate argument along similar lines implies that, in a diffraction experiment, the electron actually avoids the "dark" regions, whether or not a detecting screen is present. Such a conclusion might seem even more striking than the one just discussed for electron scattering. However it also turns out to be less "stable" against small changes to the formulation of the preclusion criterion.

completely, but it appears plausible that every macroscopic event is enmeshed in a wealth of triads in such a manner that one can conclude that, in an actual ensemble of repeated measurements, it is precluded to obtain a set of results very far from those predicted by application of the usual probability rules. Here one would in effect be using the concept of preclusion in its classical form, relying on the independence of the separate trials and the decoherence of the individual outcomes to make the law of large numbers work as it does classically.)

Although there is no room for a thorough motivation of the above preclusion rule (or "criterion"), a few comments and observations seem in order.

First, the criterion is time-asymmetric due to the placement of region III to the future, rather than the past, of the other two regions. Perhaps this asymmetry corresponds in some manner to the T-asymmetry of "wave-function collapse" in traditional interpretations, or perhaps it is more fundamental, since the collapse-asymmetry is well known to disappear when one expresses the probabilities directly in terms of products of the corresponding projections operators [18].

Second, the correlations which figure in our criterion are obvious generalizations of correlations which routinely occur in the course of ordinary measurements, where E_i^I (say) would be a particular property of the "micro-object", E_i^{II} the corresponding response of the "instrument", and E_i^{III}, for example, a record kept of the result. The only aspect of our rule that could not be directly motivated by this paradigm is the requirement that regions I and II be spacelike separated. (Notice however that the asymmetry in ordinary measurements between observer and observed is *not* mirrored by any asymmetry between regions I and II in our preclusion rule.)

But why did we need region III at all? As mentioned above, the motivating example is a Gedankenexperiment in which 117 Stern-Gerlach analyzers for each of two correlated spin-1 particles are set in place with each analyzer followed by a "recombiner" which undoes its effect. With appropriate settings for the analyzers one predicts more overlapping correlations than the particles can consistently satisfy. *

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* The logical contradiction involved here is the same one which shows that local hidden variable theories can never reproduce the pattern of perfect correlations predicted by quantum mechanics for a pair of suitably "entangled" spin-1 particles. This is a stronger contradiction than provided by Bell's inequality, because it is not merely probabilistic, but operates instead at the level of logic. That the analysis of [19] can be used to strengthen Bell's result in this way was pointed out in [20], and made the basis of a Gedankenexperiment utilizing Stern-Gerlach spin-analyzers and

The effect of requiring correlation with region III as well, is to force a given $I - II$ correlation to "persist" long enough that some of the others with which it would conflict can no longer have *their* regions I and II mutually spacelike.

Region III is also helpful when we want to draw conclusions about the past in the manner of geology. Since none of the exploits of the dinosaurs were spacelike to our discovery of their fossils, it wouldn't work to take these events as belonging to regions I and II, but we *can* associate the fossils to region III, once it has been introduced into the scheme.

Conditional Preclusion

Both in science and in daily life, the predictions we make rest on presuppositions about the past, presuppositions which in practice derive partly from knowledge obtained through observation and partly from the more or less definite assumptions we make about initial conditions (which we might imagine as being at the time of the big-bang or the immediately preceding quantum era). If all valid assertions of preclusion could be founded on the initial conditions alone, then there would be no need for a separate criterion for conditional preclusion, since the latter would just be a special case of absolute preclusion. (Conditional preclusion of an event A given the (earlier) event B, would just signify absolute preclusion of their "conjunction" $A \cap B$.) However, there seems little prospect in practice of reducing the necessary input to cosmological initial conditions, and I suspect it could not even be done in principle. If so, we need a logically separate rule for conditional preclusion, and I offer here a very preliminary suggestion of how it should go.

No doubt it would be adequate, from a practical point of view, to employ the rule from classical probability theory that we incorporate our knowledge of the past just by *relativizing* the measure we use for future events to that knowledge; that is, in computing the measure $|S|$ of a future event we would restrict the sum in equation (1) to histories γ compatible with our knowledge. However satisfactory such a rule might be for practical purposes, though, it does not appear possible to restate it objectively without coming into conflict with the principle of the single world. Take two-slit interference, for example. We can certainly compute that arrival at a detector in a dark band is precluded if we apply our criterion at a

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recombiners in [21] (see also [22]). To exhibit the contradiction experimentally, one must set the analyzers to 117 different spin-axes during the course of the experiment. An analogous Gedankenexperiment involving three spin-1/2 particles but only two distinct settings for each analyzer was given subsequently in [23].

moment just after the electron leaves the source, but what about just after it has passed through one of the slits? At that stage the interfering alternative is no longer available, but by definition the preclusion cannot have gone away, even though the relativized measure of the event "arrival at the detector" is no longer zero. * In fact, if we were to specify the electron's world-line with full precision, and then relativize to the corresponding subset of histories, it would be as if we had forced the electron "by thought alone" into a position eigenstate, and our ensuing predictions would be completely erroneous.

To state the difficulty more generally, it seems that we should be able to make statements like: "If the actual history γ is an extension of the (particular) initial segment $\widehat{\gamma}$, then such and such a set S of possible future developments of $\widehat{\gamma}$ is precluded"; yet adopting as the criterion for such statements our earlier preclusion rule with the measure restricted to those histories sharing the common initial segment $\widehat{\gamma}$ yields the wrong results.

What seems to be needed is a way to bring some of the "bypassed" alternatives back into the picture, or in other language, to "fuzz out" the initial segment $\widehat{\gamma}$ on which we condition the definition of the relativized measure. It appears reasonable to take this fuzzing to be induced by a coarse-graining of the gravitational field †, since that plausibly would have the desired effect, in the two-slit case for example, of allowing a wide latitude to the electron trajectory while still keeping the diffraction grating well localized. If fuzzing via gravitational coarse-graining is the right approach, there still remains the question of how much fuzzing to perform, to which a reasonable answer might be that preclusions arrived at by *any* fixed degree of fuzzing are valid.

When Spacetime is Dynamical ...

When spacetime is dynamical (i.e. in quantum gravity) our preclusion rule becomes meaningless unless there is a way to specify in advance the triads of spacetime regions in terms of which it is phrased. Indeed, one might think to discern an unbreakable vicious circle here, because it seems impossible to locate a region not yet in being without having the kind of advance knowledge of the geometry which could

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* This is doubtless a major reason why people have felt driven to abandon realism in interpreting quantum mechanics.

† In causal set theory such coarse graining amounts to passing to a randomly chosen subset of the true causal set [24]

only be provided via preclusion statements, whose meaning relies on the ability to locate spacetime regions in advance. This circle is related to what is sometimes called "the problem of time", and one might try to resolve it by finding a way to identify the regions in question by means of invariant properties like curvature-invariants or their discrete analogs. Such an approach might or might not appear promising, but here I only want to point out, without trying to provide a definitive answer to the question, that *in connection with conditional preclusion*, there exists another, much more specific way to "locate" future regions, by "projecting forward from what already exists".
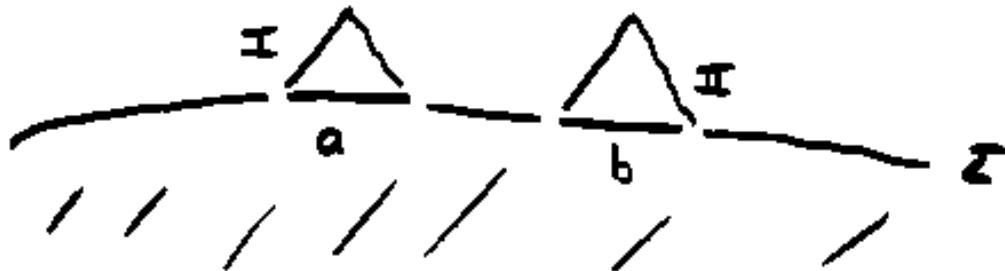

Figure 5 Locating future regions in a dynamical spacetime

As an illustration, let M (the history) be a Lorentzian manifold, and let us condition on the property that M contain some definite initial segment with future boundary Σ. (See Figure 5.) Then if a and b are any two disjoint regions on Σ, their future domains of dependence will necessarily be regions of M which are disjoint and spacelike separated. Hence they can be used for regions I and II of a triad, with region III then being defined—as always—in terms of I and II. In formulas, $I = D^+(a)$, $II = D^+(b)$, $III = \text{future}(I) \cap \text{future}(II)$. (Notice that in the causal set case, exactly the same definitions work if we interpret $D(a)$ as the "double causal complement" of a (cf. [25]), $D(a) = a'' = \{x \mid \forall y, y \natural a \Rightarrow x \natural y\}$, where $R' := \{x \mid \forall r \in R, r \natural x\}$.) Clearly, many other constructions of a similar type are possible, and would also suffice to specify "in advance" definite regions I and II to the future of Σ.

What More is Needed?

What more is needed in order to give us confidence that the interpretive scheme outlined above is truly adequate to its task? Several further steps seem called for.

First of all one must look for remaining contradictions which even our more restrictive preclusion criterion may still not have eliminated. I would conjecture that none will be found, but on the other hand, I have no good idea how one might go about providing a proof for this conjecture. Perhaps one can be satisfied if enough attempts to construct a contradiction fail, much as one has gained confidence in the consistency of Zermelo-Fraenkel set theory as the anticipated contradictions have failed to materialize. A related question is whether the introduction of a small number ϵ into our preclusion criterion is really needed, or whether it suffices to assert preclusion for sets whose measure vanishes exactly. Perhaps this question will prove easier to settle than that of consistency in general.

One should also think through a number of everyday situations (including measurement situations) to see if enough triads are present to justify the kinds of conclusions we normally draw, not only in quantum mechanical situations, but more generally. For example, can we conclude that the starlight we see when we look out at the sky was emitted by actual stars situated along our past light-cone? Here there do seem to be triads of the required type as illustrated in Figure 6, where our detection of the light occurs in region II, and region III contains, say, our memory of just having seen a star. A peculiarity of the preclusion rule in this situation is that event I cannot be taken to be the emission of the light itself, as that would not be spacelike to region II. Instead, we can choose region I just to the future of the light's path, with the corresponding event being the presence of the star there. What such a triad lets us conclude is that a star was present *just after* the earliest photon we received would have been emitted. Although the logic here is a bit different from what one is used to, the conclusion itself is indistinguishable in practice from what we normally obtain.
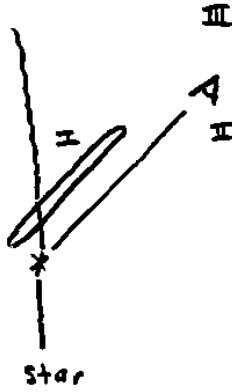
Figure 6 Stargazing

A third desideratum, of course, would be the sharpened conception of fuzziness needed to render the notion of conditional preclusion precise. And once this is found, it should be checked whether the resulting criterion for conditional preclusion is adequate to its task, which includes a requirement that it, too, be free of the kind of inconsistency we discussed above (of too many possibilities precluded).

Fourth, some further developments of quantum measure theory, though not logically necessary for the interpretive framework, would also be desirable. In particular, it would be nice to understand what extra conditions on the measure correspond to unitary evolution, i.e. what conditions would cause the measure to be expressible in the form (1) with $|S|$ independent of the choice of T.

Finally, if more tasks are needed, there is the one for which all of the above is just preparation: find the right space of histories and the appropriate measure on it to describe quantum gravity!

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