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FORKS IN THE ROAD, ON THE WAY TO QUANTUM GRAVITY *

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Abstract

In seeking to arrive at a theory of "quantum gravity", one faces several choices among alternative approaches. I list some of these "forks in the road" and offer reasons for taking one alternative over the other. In particular, I advocate the following: the sum-over-histories framework for quantum dynamics over the "observable and state-vector" framework; relative probabilities over absolute ones; spacetime over space as the gravitational "substance" (4 over 3+1); a Lorentzian metric over a Riemannian ("Euclidean") one; a dynamical topology over an absolute one; degenerate metrics over closed timelike curves to mediate topology-change; "unimodular gravity" over the unrestricted functional integral; and taking a discrete underlying structure (the causal set) rather than the differentiable manifold as the basis of the theory.

In connection with these choices, I also mention some results from unimodular quantum cosmology, sketch an account of the origin of black hole entropy, summarize an argument that the quantum mechanical measurement scheme breaks down for quantum field theory, and offer a reason why the cosmological constant of the present epoch might have a magnitude of around 10^{-120} in natural units.

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I. A laundry list of alternatives concerning Quantum Gravity

The organizers have named this conference "Directions in General Relativity", and in that spirit I want to talk in some generality about directions in Quantum Gravity. On the way to a theory of quantum gravity there are many forks in the road, or in other words alternatives one must choose among, or questions one must answer, before proceeding farther. I will begin by listing some of those alternatives and questions which seem to me the most important, and then I will advocate answers in a manner that tries to place the choices in an overall context. I hope that along the way, a coherent approach to quantum gravity will be seen to emerge.

Also along the way, I will mention a few new or lesser-known results relevant to the alternatives we will be considering, including an interpretation of black hole entropy and a possible "non-unitarity" associated with "unimodular" quantum cosmology. But first the laundry list itself (including a personal selection of references to represent the alternatives *)

Is the signature of the spacetime metric *Lorentzian* [1] or *Euclidean* [2] (or both [3])?

Should we allow *degeneracies in the metric* [4] or *closed timelike curves* [5] (or possibly both)?

Should we *fix the 4-volume* in the gravitational Sum-over-histories [6] or extend the *sum over all 4-volumes*, as is normally assumed [7]?

Is the deep structure of spacetime *discrete* [8] or *continuous* [8]?

Which feature of spacetime is most basic, its *causal order* [9], its *metric* [10], or its *topology* [11] (or perhaps even the *algebra* [12] of functions on spacetime)?

Is topology *dynamical* [13] or is it *absolute* [14]?

Is the entropy of a black hole *outside* [15] or *inside* [16] its horizon?

What really exists, the *history* [17] or the *wave-function* [18]?

Should we approach the "quantization" of gravity via the *sum-over-histories* [19] or via *canonical quantization* [20]?

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* These references, like those in the rest of this paper, are meant to be indicative rather than comprehensive.

Is probability *absolute/unconditional* [21] or *relative/conditional* [22]? (do quantum probabilities make sense?)

Is the cosmological constant, Λ, *approximately* [23] or *exactly* [24] zero?

II. What is <u>Classical</u> Gravity?

In order to begin placing these alternatives in context, it is useful to go back to the classical theory we are trying to "quantize". Like the majority of physical theories, General Relativity has a threefold structure comprising a "kinematical (or substantial) part", answering the question "*What* is there—what 'substance' are we dealing with?"; a "dynamical part", answering the question "*How* does this substance behave?"; and a "phenomenological part", answering the question "How does this substance which is there *manifest itself* in a way accessible to us?".

In the case of General Relativity, the Kinematics comprises a *differentiable manifold* M of dimension four, a *Lorentzian metric* g_{ab} on M, and a structure which, although it is closely intertwined with the metric, I want to regard as distinct, namely the *causal order-relation* $\prec$. The Dynamics is then simply the Einstein equation $G_{ab} = T_{ab}$, or in case non-gravitational matter is absent, the purely geometrical statement that the metric is Ricci-flat. Finally, the Kinematics and Dynamics manifest themselves as the familiar Phenomena of length, time, inertia, gravity (in the narrow sense of the word), causality (for example the impossibility of signaling faster than light), etc.

III. Why quantize? (And what does this mean?)

According to classical General Relativity, the metric behaves deterministically, but of course this is inconsistent with the stochastic, quantum behavior of the matter to which the metric couples via the Einstein equation. Thus, a theory of quantum gravity in the broadest sense of those words would just be some theory having both classical gravity and quantum field theory in flat spacetime as limits (the latter being our best theory of non-gravitational matter to date). However, most of us who speak of *quantum* gravity mean, I think, something more specific than just this; and so a major question whose answer defines one's approach to quantum gravity is:

- In what sense do we expect this theory to be *quantum*?

To this I would add two further basic questions:

- Do we need a new kinematics (as well as the new dynamics which "quantization" entails)?; and
- What is the phenomenology of quantum gravity?

The rest of this talk is essentially an essay in answering these three questions, beginning with the first of them.

IV. What is a quantum theory? (two views)

There are of course many different viewpoints on how quantum mechanics is to be interpreted, but I will concentrate here on two broadly opposed attitudes, which I will call the Ψ-framework and the Sum-over-histories framework.

According to the former view the essence of quantum mechanics resides in its mathematical structure: a *Hilbert space*, an *algebra of operators* to be interpreted physically in terms of *measurements*; and a "*projection postulate*", which tells us how to take the results of measurements into account in predicting probabilities for future measurements. In this framework, the central object is the state-vector Ψ (which is why I am calling it the Ψ-framework), and the physical interpretation is made in terms of observables. (See almost any textbook, for example [25].)

Closely allied with the Ψ-framework is the *canonical quantization* approach to quantum gravity. Although different variants of this approach may employ different combinations of the basic dynamical variables, they all work solely with *space* (in the sense of a spacelike hypersurface), as opposed to spacetime. (For a review of such issues see [26] [27], for alternative choices of canonical variables see [28].)

From the Sum-over-histories point of view, quantum mechanics is understood quite differently, namely as *a modified stochastic dynamics characterized by a non-classical probability-calculus in which alternatives interfere.* To see the essence of quantum mechanics in this way goes back at least to Heisenberg's Chicago lectures [29], and of course is associated most closely with the name of Feynman [30]. Within this framework, the spacetime *history* itself is the central object. It exists in the same sense in which a history is taken to exist in classical physics, and the physical interpretation can thus be made directly in terms of properties of this history—what John Bell called [31] 'beables' (a word that I always thought was some kind of joke until I realized that he meant it to be pronounced "be-ables")—rather than indirectly in terms of "observables". (See also [17] for a statement of this view.)

Since the sum-over-histories is by nature a "spacetime approach", it naturally leads to a version of quantum gravity which works with *spacetime* as opposed to data on a hypersurface.[6]

In comparing these two attitudes, I think it is fair to say that the Ψ-framework is mathematically better developed (although this applies less to quantum field theory than it does to quantum mechanics in the narrow sense), whereas the sum-over-histories framework is, to my mind, more satisfactory philosophically, because it avoids the positivistic refusal to contemplate anything besides our generalized sense perceptions. *

Now why, aside from its philosophical advantages, do I favor the sum-over-histories/spacetime approach to quantum gravity over the Ψ-framework/canonical quantization approach? An important part of the answer has to do with what has been called "the problem of time" [32], though the plural 'problems' would probably be a more appropriate word in this connection.

One such problem which affects the Ψ-framework concerns the temporal meaning of the "logical ordering" required by the projection postulate. In employing that postulate, one writes the projections in a definite sequence determined by the order of the observations *in time*; but how can such a rule avoid leading to a vicious circle in a theory in which time itself is one of the things being "observed"?

A second, closely related difficulty concerns the "frozen formalism" that results when one applies the formal rules of canonical quantization to General Relativity (or to any generally covariant Lagrangian theory). In consequence of the Hamiltonian constraints, the "physical observables" are necessarily all time-independent (they are what Karel Kuchař [27] calls 'perennials'), and one seems forced into an attempt

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* Formally, the Ψ-framework may be seen as a special case of the sum-over-histories that arises when the amplitude is formed in a suitably local manner, allowing states $\Psi(t)$ associated with given moments of time to be introduced as convenient summaries of the past.

to “fix the time-gauge” in order to recover a semblance of spacetime from the disembodied spacelike hypersurface to which the formalism directly refers. Not only is such a procedure technically questionable, but it can be dangerous as well: one can easily smuggle arbitrary answers to important physical questions into the theory in the guise of a “gauge choice”, for example to the question whether collapse to a singularity is inevitable in “mini-superspace cosmology”.

Finally, in a framework based on “observables” rather than “beables”, how are we to speak about (say) the early universe, if there were no observers then and none in the offing for a long time to come? Since some of the most important applications of quantum gravity are likely to be precisely to the early universe, this also appears to present a serious difficulty.

None of these “problems of time” would seem to exist for the sum-over-histories/spacetime approach. Time itself doesn’t need to be recovered, because it is there from the very beginning as an aspect of the spacetime metric. The projection postulate is irrelevant, because there is no state-vector to be “reduced”. And the early universe existed just as much as we ourselves do here and now, even if from our vantage point it is relatively remote and inaccessible.

There is another point I want to mention here, which is presented more fully in my contribution to Dieter Brill’s Festschrift [33], and that is that—independently of any problems related to general covariance—the Ψ-framework starts to break down, in a certain sense, already for quantum field theory in flat spacetime. One of the seeming advantages of the Ψ-framework vis-à-vis the sum-over-histories is that it appears to possess a more comprehensive measurement scheme, telling us what in principle can be measured (every selfadjoint operator) and prescribing (at least formally) how to design an interaction-Hamiltonian to effect the corresponding measurement (see [34]). In contrast, there exists (so far) no equally comprehensive theory of measurements within the sum-over-histories framework. Now this lack is not the great disadvantage for the sum-over-histories which it would be for the Ψ-framework, because measurement is not a fundamental notion for the sum-over-histories. Nonetheless, the question of what can and cannot be measured is clearly an important one for any theory. However, it turns out that the simple measurement scheme which the Ψ-framework *appears* to possess is physically viable only for quantum mechanics in the narrow sense of non-relativistic point-particle mechanics.

The point is that in Quantum Field Theory, it proves inconsistent with causality to assume that every observable constructed from the field operators within a given spacetime region R can be measured by operations confined entirely to R; to be able to do so would lead to the possibility of superluminal signaling. There is no time here to repeat the argument [33] in detail, but it considers three spacetime regions A, B, and C arrayed so that communication is possible from A to B and from B to C, but not from A to C. Specifically one can choose B to be a thickened spacelike hyperplane, with A and C being spacelike separated points which are respectively to the past and future of B. Assuming that arbitrary localized ideal measurements were possible in these regions, the argument concludes that an experimenter stationed at A could transmit information to a colleague at C by deciding whether or not to perform a certain observation, given that both know that a certain other observation *will* be performed in the intervening region B.

Thus, one must reject the assumption of arbitrary localized measurements, and it becomes a priori unclear, for quantum field theory, which observables can be measured consistently with causality and which can't. This would seem to deprive the Ψ-framework for quantum field theory of any definite measurement theory, leaving the issue of what can actually be measured to (at best) a case-by-case analysis, just as it remains (so far) within the sum-over-histories framework. (Notice as well that most of the hypersurface observables with which a canonical formulation of gravity would presumably deal, are likely to run into locality troubles of this same sort.) As pointed out above, this actually puts the Ψ-framework at a disadvantage, because for it, the notion of measurement *is* fundamental.

Finally, I want to return to the more general comparison of the two opposed frameworks in order to stress what seems to me to be the great practical advantage of a spacetime approach vis à vis a purely spatial one. In fact the questions one wants to ask of a quantum gravity theory are—due ultimately to the diffeomorphism invariance of General Relativity—all of an unavoidably spacetime character. For example, one may want to study how the horizon area of a black hole responds to the emission of Hawking radiation. Or one may want to ask whether the cosmological expansion we are now experiencing was actually preceded by (say) nine previous cycles of expansion and recontraction. Both of these questions make perfect sense if one has access to an entire 4-geometry, but could one formulate them in terms of the kind of hypersurface data with which the canonical approach works?

Well, if we do go in the direction indicated by the signpost reading "sum-over-histories", we come a little way down the road to a secondary fork concerning the proper interpretation of the "quantum probabilities" which that formalism yields. In fact, the sum-over-histories will furnish a probabilistic answer to almost any question about the history you care to ask (more precisely, it will furnish relative probabilities for the elements of any partition of the set of all histories into mutually exclusive and exhaustive subsets [6]), but it is easy to see on physical grounds that most of these "quantum probabilities" cannot be very meaningful. A central question for this approach is therefore, under what circumstances such probabilities do acquire meaning (an issue symbolized in my initial laundry list by the question whether probability is relative or absolute).

Here there are two alternative points of view that I know of. According to the first view [35], probabilities are *absolute* and *unconditional* in the sense that their application rests on no assumption about the history other than a choice of a cosmological initial condition, but they have meaning only in the context of a fixed partition of history space * which obeys the condition that Jim Hartle and Murray Gell-Mann call 'decoherence'. In general however, there are very many decohering partitions, not all of whose probability-assignments are compatible with each other, cf. [36]. (An interesting example of such an incompatibility is that a given partition can have its decoherence destroyed by subsequent "observational activity" which has the effect of making the original alternatives interfere; or stated more generally and precisely: there can exist pairs of partitions P' and P'', based respectively on earlier and later properties of the history, such that P' and P'' each decohere, but their "union" $P' \vee P''$ does not. Here $P' \vee P''$ is the partition which

* In this sense, it is misleading to describe such probabilities as absolute: they are in fact relative to a choice of partition.

asks about both the earlier and the later properties together; its elements are the atoms of the lattice of sets generated by the elements of P' and P'' via intersection and complementation.) According to the second view [37], probabilities are *relative* to a split of the universe into subsystems, and *conditional* on possible assumptions about the behavior (or even existence) of these subsystems (cf. [38]). (For example one subsystem could be an electron and the other a collection of molecules in a cloud chamber.) The criterion for the "quantum probabilities" to be meaningful is then not that they necessarily decohere (and hence obey the sum-rules proper to classical probabilities), but that a sufficiently perfect *correlation* obtain between the two * subsystems (for example the correlation by which the track in the cloud chamber reflects the path of the electron) [37].

Incidentally, in arguing for the sum-over-histories/spacetime approach in preference to the Ψ-framework/canonical quantization one, I would not want to give the impression that I think that mathematical studies of the operator constraints are necessarily irrelevant. Indeed, some formulations of the sum-over-histories effectively employ something akin to a Hilbert space norm on wave-functions as a mathematical intermediary in computing quantum probabilities, and making such a formulation mathematically well-defined might still require a Hermitian inner product on the space of solutions ψ to the operator-constraints. On the other hand, a new kinematics in general *would* render the constraints irrelevant (except possibly in some approximate effective theory), and this brings us to the second basic question, the one about changing the kinematics.

* In the meantime, this criterion has evolved. It now seems that twofold correlations are not the whole story, but threefold ones might be [39]. Also, with respect to terminology, Jim Hartle has convinced me that the word "probability" should be reserved for a measure that obeys the standard "Kolmogorov" sum-rule; so I would now say "quantum measure" instead of "quantum probability". Finally, I think one could improve on the word "relative" used above in connection with the split of the universe into subsystems. Its meaning is not (as it might seem to be) that each possible split carries its own notion of probability, but rather that prediction on the basis of the quantum measure is possible only in relation to such splits.

V. A Modified Kinematics?

(a) *Lorentzian or Euclidean Signature?*

People have suggested several possible modifications of the kinematics of classical General Relativity, and the one I want to discuss first is perhaps the least radical—though it's radical enough. It proposes [2] that the sum-over-geometries be conducted with positive-definite (Riemannian) metrics instead of Lorentzian ones. Such a replacement carries less fundamental import if one interprets this sum within the Ψ-framework, where the histories have meaning only as intermediaries used to find a wave-function Ψ; but even so, it represents a significant modification.

The main motivation for altering the signature in this way seems to be that tunneling phenomena appear, in an "instanton" approximation, to occur via Riemannian solutions of the field equations. But something like this is already true in quantum *mechanics*, where one can use an imaginary-time path to compute the WKB approximation to barrier penetration, as in the classic problem of alpha-decay. Such a calculational technique can be interpreted as an infinite-dimensional saddle point approximation to the path-integral, and from this point of view the complex-time path, or saddle point, merely *summarizes* (to leading order in $\hbar$) the contribution of a large number of real-time paths. Certainly its use would not normally be taken to imply that physical time turns imaginary while the alpha particle is "under the barrier". In the same way, a gravitational instanton should presumably be understood as summarizing the contribution of a large number of Lorentzian histories.

On this view, the notorious ambiguities [40] in the choice of saddle point and contour which affect the so-called Euclidean functional integral will only be resolved (to the extent that this can be done at all without recourse to an underlying discrete theory) when one has succeeded in deriving the Euclidean-signature expression by analytic continuation from a Lorentzian starting point. Carefully observing (the appropriate infinite dimensional generalizations of) the rules for deformation of complex contours ought then to answer such questions as which saddle points contribute, and what are the relative signs of their contributions. For my own enlightenment, I actually went through the corresponding analysis in detail in the much simpler case of one-dimensional barrier penetration, and I can affirm that everything works out just as it should, including the fact that passage through the barrier results in damping rather than amplification. *

(b) *Should We Sum over Different Topologies?*

A kinematic question of another type is whether one should include more than one topology in the sum-over-histories. (Here, incidentally, I have posed the question in terms of the sum-over-histories, not only because that was the "fork" we followed earlier, but mainly because the $3+1$-framework does not lend itself to a dynamical topology in any known manner.) To this question, my answer would be: "yes, and again yes". Yes first of all, because it seems contrary to the spirit of Relativity to make of the topology the only absolute element, which "affects without being affected".

And yes secondly, because of a further reason which is perhaps more substantive, though not as widely appreciated. Namely, I want to claim that the study of topological geons (e.g. [41]) leads to the conclusion that *a dynamical metric requires a dynamical topology.* The basis of this claim is the observation that, for a type of particle such as a geon, whose existence is *defined* in terms of the spatial topology, any procedure of "second quantization" by definition forces the topology to change because the number of geons cannot vary if the topology remains constant. But experience shows that a relativistic particle which is only "first quantized" is not physically consistent (No matter how you slice things, you seem inevitably to meet with one or more of the following difficulties: negative energies, negative probability densities, non-conservation of probability in scattering, faster than light motion by the particle (the problem mentioned by Bob Wald yesterday [42]), inability to measure the particle's energy [43].), and there is no reason to expect geons to be immune from this imperative. In this sense, I believe one can say without too much exaggeration that quantum gravity without topology change is simply inconsistent physically [44] [45]. *

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* A sketch of the analysis may be found in the Appendix.

* This reasoning cannot be airtight, of course, because there might in theory be an absolute, "god-given" topology of spacetime for which no geons at all can exist, for example $S^3 \times R$.

A further consideration is that, without topology change, it is possible to (formally) quantize General Relativity so that certain geons violate the normal connection between spin and statistics [45] (for a not-only-formal treatment in $2+1$-dimensions see [46]). Considering that some process of pair-creation/annihilation seems always to underlie the known proofs of spin-statistics theorems, it is thus a natural guess that the correct incorporation of topology-change into quantum gravity would automatically set up a correlation between exchange and rotation which would exclude the spin-statistics violating possibilities [41]. Some recent evidence in favor of this idea comes from the finding by Fay Dowker that precisely such a correlation occurs for geons of certain types which have been pair created via "U-tube" cobordisms (this sort of cobordism being a universal mechanism of pair creation) [47]. If it is true that a correct incorporation of topology-change must reinstate the spin-statistics correlation for topological geons, then conversely, the requirement that this in fact occur will serve as an important test of any theory of quantum gravity, or to put it another way, as an important clue in the formulation of such a theory.

Remark Topology-change not only speaks against canonical quantization, but it also excludes what is sometimes called the "covariant" approach to quantum gravity, which works with an operator-valued metric field on a *fixed* spacetime manifold [48].

If we do accept that topology-change must be provided for, then we come still further down the road upon a fork corresponding to my earlier question whether it is the regularity of the metric or causality that should be sacrificed, the choice being forced upon us by a theorem of Geroch [49] according to which topology change entails either singularities or closed timelike curves.

The choice of allowing closed timelike curves in order to preserve regularity of the metric can be opposed on the grounds that it conflicts with the causal set idea, which I will come back to in a moment. More to the point, recent evidence is that closed timelike curves ("time loops") lead to trouble with the quantum fields living in such a (background) spacetime, specifically divergences in the stress-energy [5] and a breakdown of unitarity [50]. (See also Stanley Deser's paper in these proceedings [51].) An older, if lesser known difficulty concerns the pair-creation of monopoles in (5-dimensional) Kaluza-Klein theory. It turns out that such pair creation via a globally regular Lorentzian metric is impossible for topological reasons *even if* one allows time loops to occur (as long as time-*orientability* is maintained) [52]. To me, this strongly indicates that the imposition of strict global regularity on the metric is inappropriate, a view which is reinforced by considering how rough the metric is likely to be in any case, given the indications from quantum mechanics and quantum field theory.

In fact there does exist a mechanism of topology-change which preserves causality, at the cost of allowing the metric to degenerate to zero at isolated spacetime points [44] [53]. For any compact cobordism (interpolating manifold between two spacelike hypersurfaces of possibly different topology) one can find a metric which degenerates to zero at a finite number of points, in one of a finite number of predetermined ways (the number depending on the spacetime dimension), and for which no time-loops are present. With respect to such a metric, one can view the topology change as "happening" at the points of degeneration, and one might accordingly hope for simple dynamical rules to describe what takes place at such points. In this direction, there is an intriguing result in the two-dimensional case, where there are only two possible types of "elementary cobordism", corresponding to the "crotch-point" in a "trousers" spacetime, and the "crown-point" in a "yarmulke" or "big bang" spacetime. At the corresponding points of degeneration, the scalar-curvature Action becomes complex in such a way as to suppress the former type of topology change and enhance the latter [44] [54].

(c) *Is the Metric Fundamental, or Only an Effective Description of Something Deeper? (Causal Sets)*

A much more basic kinematical question than those dealt with so far is whether the spacetime metric should be replaced by some "deeper" structure of which it is only an approximate description. Now, String Theory [55] proposes one answer to this, but the answer I want to discuss follows from still another question: is spacetime ultimately continuous or discrete?

Here, I cannot resist quoting Einstein, who wrote in 1954,

> "The alternative continuum-discontinuum seems to me to be a real alternative; i.e., there is no compromise ... In a [discontinuum] theory space

> and time cannot occur ... It will be especially difficult to derive something like a spatio-temporal quasi-order (!) from such a schema ... But I hold it entirely possible that the development will lead there ..." [56]

(In this quotation the exclamation point is mine, put there because the words 'spatio-temporal quasi-order' seem so obviously to be calling for a theory based on causal sets!) Referring to the argument against the continuum, Einstein goes on to say "... This objection is not decisive only because one doesn't know, in the contemporary state of mathematics, in what way the demand for freedom from singularity (in the continuum theory) limits the manifold of solutions." Here, the objection was that quantum mechanics teaches that a bounded system can be described by a finite set of "quantum numbers", and such a description conflicts with the infinite number of degrees of freedom posited by a continuum theory. (The loophole referred to was the possibility that excluding singular solutions of the field equations might suppress these unwanted degrees of freedom (and reproduce all the characteristic quantum effects as well, all without leaving the domain of classical field theory)).

In addition to this argument for a fundamental discreteness there are several contradictions in existing theories which speak powerfully for the same conclusion. These contradictions, which I call "the three infinities" (or perhaps four depending on how you count them), include the divergences of Quantum Field Theory, the singularities of classical General Relativity, the apparent non-renormalizability of naively quantized gravity [57], and the apparently infinite value of the black hole entropy if no cutoff is present. The final item in this list rests on an interpretation of horizon entropy to which I will return below; to my mind it is the least adequately appreciated of the common arguments for discreteness.

If we accept all these indications, then we come immediately upon a subsequent multiple fork in the road corresponding to the question of what the discrete substratum actually is. To attempt an answer at this point would seem to be hopeless if there is not at least what I would call some sort of "structural bridge" between the continuum and the underlying discontinuum, i.e. some structural analogy which would allow one to understand how the former can "emerge" from the latter in appropriate circumstances. People have sought the source of such an analogy in at least three properties of the continuum, its *topology* [58], its *metric* [10] (cf. [59]), and its causal *order* [60]; and I personally have at one time or another been drawn to all of them before deciding, in connection with the causal set hypothesis, that it is the *order* or "causal structure" of the spacetime continuum which, together with one component of the metric (effectively its determinant), should be viewed as being its most fundamental property.

The causal set hypothesis which I have just alluded to, posits that the structure of the discrete substratum is that of a locally-finite partial-ordering (= causal set), and establishes the correspondence between this underlying structure and the overlying, "emergent" Lorentzian manifold by making the causal order and volume-measure of the latter correspond to the intrinsic order and "counting measure" of the former (so spacetime volume = number of elements).

For a general introduction to causal sets and a partial review of work on that idea see [23]. I will not discuss the subject further here, except to point to one last fork in the ensuing road which raises the possibility of allowing the analog of closed timelike curves to occur in the underlying discrete ordered set. Such a generalized structure (a "directed graph") would be a possible alternative to the causal set as presently defined, but it seems to me to be unnatural, because *even if* one does let such "cycles" occur in the substratum, it still seems impossible to broaden the rules of correspondence with the continuum in such a way as to allow a Lorentzian manifold having time loops to emerge as a valid approximation to a discrete directed graph. In this sense, one can *predict* that time loops must be absent from the continuum, whether or not their analog is admitted into the underlying ordered set (cf. [61]). *

* Even if at bottom, spacetime is discontinuous, this of course does not mean that every continuum theory of quantum gravity is necessarily useless, since such a theory might still apply at some intermediate level of approximation. If so, then one can anticipate that, from the point of view of the continuum theory, the cutoff coming from the discreteness would provide a regularizer, and also that an appeal to the discrete theory would serve to resolve the ambiguities of the continuum theory connected, in particular, with the effects of nontrivial spacetime topology (cf. [62]). Specifically, the deeper theory should be able to provide the rules that govern processes in which the topology changes.

VI. A final dynamical fork: should we constrain the four-volume in the sum over geometries ("unimodular gravity")?

The alternative "discrete versus continuous" was my last one concerning kinematics; but before turning to phenomenology, I want to raise one further dynamical question, which is more naturally discussed here than earlier. (I say "further" because the whole discussion of section IV was, of course, about dynamics.) Specifically, the question is whether one should hold the spacetime volume fixed in the gravitational sum-over-histories [6]. Unlike with topology-change, the phrasing of this question in sum-over-histories form is not a matter of principle; there is an equivalent formulation in canonical terms [63].

Now classically, fixing the volume before varying the Action makes essentially no difference; its only effect is to convert the cosmological constant from a free parameter in the Lagrangian into a free constant of integration of the resulting field equations. The physical significance of this "unimodular" constraint is therefore solely quantum mechanical. The motivation for adopting it comes, in my mind, first of all from causal set theory, where a constraint on the total number of elements seems necessary for the sum over causal sets to converge, and this constraint translates in the continuum into specifying the total spacetime volume. Also, independently of any discreteness, a direct constraint on the spacetime volume seems to ameliorate convergence problems with the continuum functional integral, as is especially noticeable in the limit corresponding to quantum field theory in curved spacetime. Most intriguingly, the manner in which the cosmological constant "becomes dynamical" in unimodular quantum gravity offers a new "mechanism" for producing a small or zero value for it.

It is therefore interesting to ask what difference the unimodular constraint would make in rudimentary models like the homogeneous universes of "mini-superspace quantum cosmology". Recently Jorma Louko and I have studied a couple of the simplest of such models with results that are peculiar enough to be interesting, but not so crazy as to become boring [64]. We find, specifically for the Friedmann universe $S^3 \times \mathbb{R}$, that adopting the analog for the unimodular theory of the "no-boundary boundary-condition", and computing the crudest saddle point approximation to the wave-function ψ (with the most obvious saddle point), that

ψ remains regular as a function of a, the radius of the universe, in both the limits $a \to 0$, and $a \to \infty$. (This is an example of the improved convergence I spoke of, since in "standard quantum cosmology", ψ diverges exponentially as $a \to \infty$.) On the other hand, ψ is now a function of the 4-volume, which serves as a kind of parameter-time T, and its "evolution" with T is non-unitary due to a flux of probability coming in from $a = 0$. One might interpret this effect as a "continuous creation of universes", or perhaps better, as an "induced emission of new branches of the universe, all stemming from a common root". *

VII. What is the phenomenology of Quantum Gravity?

The third and last set of "forks" I want to discuss concerns the phenomenology of quantum gravity, or more prosaically, the question of what observable consequences we might expect a theory of quantum gravity to possess. If in fact a new substance underlying the metrical field is the proper basis for such a theory, it becomes especially important to try to foresee how this new form of matter will manifest itself (or has already done so!); but even if only the dynamics of gravity is to be changed, one would expect some dramatic consequences to appear. In this connection, let me present a second laundry list of "phenomenological" questions whose answers some people have hoped would emerge from quantum gravity.

Why is there a metric? And why is it Lorentzian?

Why is Minkowski space a solution of the theory (with d=4)? Notice that this question includes also the question of why the cosmological constant is so small, since for $\Lambda \neq 0$, Minkowski space would not be a solution.

Why is the gravitational Lagrangian what it is?

Why does non-gravitational "matter" exist (fields and/or particles)?

What is origin of black hole entropy?

Why is the universe expanding?

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* Or maybe this is just the wrong saddle point. In an example of the ambiguity referred to earlier, Jorma has recently found a less obvious saddle point whose contribution leads to a ψ which is even better behaved as $a \to \infty$ but which dies out with T instead of blowing up, suggesting either unitary evolution or a flow of probability *out* through $a = 0$!

Is CPT broken?

What are the rules for topology change?

Unlike for my first laundry list, there will be time to address only a minority of these topics here, and I want to concentrate on two of them which also occurred in the earlier list, namely those concerning black hole entropy and the cosmological constant. Before getting to them, however, let me allude to part of the answer that causal set theory would give to the first question of why the spacetime metric is Lorentzian. The point is that no other metric signature, Riemannian or $(++--)$ or whatever, can lend a partial ordering to the events of spacetime, because only in the Lorentzian case do the light-cones provide a well-defined local distinction between before and after.

(a) *Is a Black Hole's Entropy Inside its Horizon?*

Although one might initially think that the entropy of a black hole must represent the number of its interior states, such a view is difficult to reconcile with the fact that the second law of thermodynamics pertains effectively to processes which proceed in ignorance of whatever is happening inside the horizon. Since it is by definition the autonomously developing ensemble of such exterior processes which are responsible for the entropy increase, it would seem most natural that the entropy itself be a property of the exterior region. In fact, if one adopts this view, and more particularly if one identifies the exterior entropy with $S = -\mathrm{Tr}\,\rho\log\rho$, where ρ is the effective Schrödinger-picture density-operator of a spacelike hypersurface in the exterior region, then there exists a schematic explanation of why S as so defined necessarily increases as the hypersurface to which ρ refers advances in time. * This explanation [65] rests on a certain theorem [66] concerning density-matrix evolution, and on the crucial fact (emphasized yesterday by Jimmy York [67]) that the total energy not only is conserved, but is meaningful as a property of the exterior region, since it can be read off from the behavior of the metric in the asymptotic region or on some suitable boundary surface.

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* In using this language I am presupposing, for example, a sufficiently classical approximate spacetime with respect to which a given hypersurface can be meaningfully located.

What is more, one can estimate the contribution to the above S from the zero-point fluctuations of a free scalar field in a black hole background, and one obtains a value which is proportional to the horizon area measured in units of the cutoff [68]. It is essentially this result to which I referred earlier in adducing the black-hole entropy as one of the "three infinities". Although the story is really more complicated than this (because the free-field approximation is probably wrong, and it is most likely the degrees of freedom of the horizon itself which account for its entropy) I believe that the conclusion that finite entropy requires a cutoff is correct (cf. Einstein's objection against the continuum quoted earlier), and that the sketch of an explanation cited above for the increase of the total entropy is fundamentally correct also. If so, then filling in the sketch so as to obtain a complete derivation of the increasing character of a well-defined total entropy which includes a horizon contribution of the correct magnitude will be a decisive test for any theory of quantum gravity.

(b) *Is the Cosmological Constant Exactly Zero?*

I want to conclude with a "prediction" about the cosmological constant, Λ, which draws together a few of the ideas advanced so far. From unimodular gravity let us take the idea that Λ is in some sense conjugate to the spacetime volume V (earlier called 'T'), and from causal set theory the idea that V is a measure of the number of elements N. From the former idea, we can write in the sense of the Uncertainty Principle,

$$\Delta V \, \Delta\Lambda \sim 1.$$

But since the correspondence between V and N has a probabilistic character, Poisson fluctuations in N of order of magnitude $\sqrt{N}$ translate into an uncertainty in V of

$$\Delta V \sim \Delta N \sim \sqrt{N} \sim \sqrt{V}.$$

Putting these two relationships together yields a minimum uncertainty in Λ of

$$\Delta\Lambda \sim \frac{1}{\sqrt{V}},$$

which, for the visible universe to date, is in order of magnitude, 10^{-120} in natural units. The prediction [23] is thus that whatever mechanism drives Λ to vanish *, will probably leave it with a small but non-zero value of this magnitude, which, intriguingly, is just barely large enough to be accessible to observation.

If this is correct then to test a prediction of Quantum Gravity, we might want to look outward rather than inward, and [69] we might even have the experimental answers in time for them to be presented at the next birthday celebration for Dieter and Charlie.

Finally, I would like to thank several colleagues who attended this talk for their stimulating questions and comments, and Sumati Surya for help with the references.

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Appendix: Tunneling and Analytic Continuation

In this appendix, I sketch the derivation of the WKB barrier penetration formula from the path integral. The purpose is not to derive the result as such, which of course is thoroughly well known, but only to expose the manner in which imaginary-time paths enter: not as fundamental integration variables, but only as saddle points of an analytically continued real-time path integral. By implication, the use of Euclidean signature metrics in quantum gravitational calculations is no more reason to doubt the Lorentzian signature of spacetime than the phenomenon of α-decay is reason to conclude that the physical time of non-relativistic quantum mechanics is really purely imaginary.

Let us imagine a source of frequency E that emits a particle at one end of a potential barrier (say at $x = a$) and a sink of the opposite frequency that absorbs it at the other end (say at $x = b$). The amplitude for propagation from emission to absorption is then

$$e^{iS_0(\gamma) - iE\Delta t(\gamma)},$$

* One possible mechanism is that only $\Lambda = 0$ is stable against non-manifold fluctuations of the causal set.

where

$$S_0(\gamma) = \int_\gamma \frac{m}{2}\frac{dx^2}{dt} - V(x)dt$$

and γ is a spacetime path that spends time Δt going from source to sink. The overall amplitude A is thus

$$A = \int d\mu(\gamma)\, e^{iS(\gamma)} \ , \tag{1}$$

where $d\mu(\gamma)$ is the "measure factor", $S(\gamma) = S_0(\gamma) - E\Delta t(\gamma)$, and the integral is over all spacetime paths γ that run (with $dt > 0$) from $(t,x) = (t_a, a)$ to $(t,x) = (t_b, b)$. Notice that $\Delta t = t_b - t_a$ is to be integrated over in (1), unlike the spatial endpoints a and b.

It is the path integral (1) that we wish to approximate. In doing so, I will follow the usual practice of supposing that manipulations that would be correct for finite dimensional integrals will also be valid here. Now as is common with such problems, we can either continue the integration "contour" or continue some parameters in the integrand itself. Here it seems clearer to do the latter by introducing a complex parameter ζ into the action integral as follows:

$$S \to S_\zeta = \int_\gamma \frac{mdx^2}{2\zeta dt} + (E - V)\zeta dt$$

(which formally is the same as the substitution $dt \to \zeta dt$).

Now the integral (1) is an oscillating "Fresnel integral". In order that it remain convergent as ζ is varied, it is necessary that the contribution to iS

$$\int_\gamma \frac{imdx^2}{2\zeta dt}$$

be negative definite. This implies (since $dt > 0$) that ζ can be continued freely into the lower half-plane, but not the upper. Let us continue it from $\zeta = 1$ to the negative imaginary axis and write there $\zeta = -ic$, with $c > 0$. The action integral then becomes

$$iS = \int \frac{-m}{2c}\frac{dx^2}{dt} - (V - E)cdt. \tag{2}$$

Unlike the original, this integrand has a saddle point (a maximum) within the domain of integration. Since iS is now real, we can perform a steepest descent approximation to A and (noting that dt can be varied freely, since Δt is not fixed) we find easily that (ignoring the prefactor)

$$A \sim e^{-I}, \tag{3}$$

where

$$I = \int_a^b dx \sqrt{2m(V-E)}\ . \tag{4}$$

Moreover, since this is independent of $c = i\zeta$, the analytic continuation back to $\zeta = 1$ is trivial and yields exactly the same answer (4), the familiar WKB result.

Now we have obtained I as the action of a path that extremizes the *analytically continued* action integral (2), a path that proceeds in "Lorentzian" time and belongs to the original integration domain of (1). But the integral (4) can also be interpreted (for $c = 1$) as the *original* action integral $S_{\zeta=1}$ evaluated at the complex saddle point γ_E, where γ_E is a path running along the positive imaginary axis in the complex t-plane; this interpretation would have resulted if we had chosen to deform the integration *contour* in (1) rather than the complex *parameter* ζ. But no matter which way we interpret (4), the right hand side of (3) is first and foremost an approximation to the "Lorentzian" integral (1). As such, it represents the sum of the amplitudes of all possible real-time paths from a to b, which, since none of them is a classical solution at energy E, interfere destructively, thereby suppressing the tunneling. From the sum-over-histories point of view, the tunneling particle follows one of these real-time trajectories through the barrier, although it is impossible to say which one it will be in any particular case. A complex-time path like γ_E, on the other hand, is not a possible history of the tunneling particle at all, but simply a mathematical device to help us express the superposition (1) more compactly.

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