A Sum-Over-Histories Account of an EPR(B) Experiment[1]

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ABSTRACT

We analyze a variant of the EPRB experiment within a framework for quantum mechanics that rests on a radical interpretation of the sum over histories. Within this framework, reality is (just as classically) a single "history", e.g. a definite collection of particles undergoing definite motions; and quantum dynamics appears as a kind of stochastic law of motion for that history, a law formulated in terms of non-classical probability-amplitudes. No state-vectors enter this framework, therefore the problem of their nonlocal collapse is absent as well.

[1]published in *Found. Phys. Lett.* **4:** 303–335 (1991)

1 Introduction

Near the end of the last century certain new forms of matter were discovered, some of them expected since ancient times, some hardly anticipated at all. Not only were atoms finally discerned, but particles from a deeper stratum, including electrons and protons, showed up almost simultaneously. Although the kinematics of these new particles proved for a time[2] to be relatively mundane, their dynamics was from the outset much more puzzling. In grappling with that puzzle, scientists were led to a new type of dynamical framework — one in which probability was involved at the most basic level, but in which the rules of the classical probability calculus did not apply. In fact the new quantum theory did not refer directly to probabilities at all, but rather to other constructs, such as wave functions, whose relation to "observable" probabilities is only indirect. Ever since their appearance, the proper role of these constructs has been the subject of continuing controversy, as has the closely related question whether pre-quantum concepts such as the trajectory of a particle continue to have a place in subatomic physics.

Now the phenomena leading to this new type of dynamics all emerged in laboratory experiments; and as long as theoretical inquiry remained focused on the understanding of experiments for which the quantum formalism continued to produce adequate predictions, the attendant conceptual controversy did not need to be resolved in order for physics to progress. Today, however, things are different for two related reasons. In the first place electrons and protons were not the only new "substances" found around 1900. The spacetime metric had also been recognized as a field with a life of its own; but General Relativity had only provided a classical dynamics for it, not one consistent with the quantum behavior of the particles and/or other fields with which the metric is entwined. These days, the "quantum gravity" problem of finding such a dynamics is in everyone's mind, and for it the existing quantum framework has at least to be broadened, if not replaced. In the second place, one of the most important applications of a theory of quantum gravity would be to understanding the epoch of the cosmological "big bang". In that setting there is no real laboratory and certainly no experimenter, so that the limited set of interpretive rules on which everyone can agree is inadequate to help us connect up any quantum gravity formalism with the real world, even supposing that one could be found. A resolution of the philosophi-

[2] i.e., before pair creations and annihilations were discovered. (The electronic and nuclear spins might also be regarded as new aspects of their kinematics. But perhaps spin is better construed, within the sum-over-histories framework, as a quality of a more dynamical character, namely as a generalized sort of probability-amplitude.)

cal issues raised by the quantum principle could thus help us in two ways. It might single out certain approaches to quantum gravity as being more natural than others, and it might suggest the kind of dynamical questions that will then have meaning in unfamiliar physical contexts, such as the very early universe.

Now, the central issue in the controversy over the meaning of quantum mechanics would no doubt be identified differently by different participants. In our view it is the question, "what is the physical reality underlying the quantum formalism?". One extreme response to this question has always been the positivist (or "operationalist") dictum that there *is* no underlying reality, the formalism being no more than an elaborate algorithm for predicting what outcomes will be obtained if one performs certain suitably designed laboratory experiments. However any such view threatens to trap us immediately in a vicious circle: since our instruments are themselves composed of subatomic particles, they would thus be composed of symbols whose ultimate interpretation must be sought in the properties of the instruments themselves (cf. [12]). On the side of practice the operationalist view is also lacking. Even in a laboratory setting it would be exceedingly difficult to design, construct and calibrate new instruments in ignorance of their atomic structure and intended function in interacting with definite subatomic objects. But how, in reference to quantum cosmology (say), are we to formulate fertile questions and heuristic guidelines concerning events such as galaxy formation while dealing not with "the events themselves", but instead with the very indirect "traces" they may have left on complicated recording instruments fabricated billions of years later? Similarly the operationalist dictum is simply mute on important technical questions like whether quantum gravity should prefer to rest on the notion of spacetime (as does classical General Relativity) or on the 3+1-notion of "spacelike hypersurface", as many workers have believed. Overall, the positivistic refusal to try to identify any reality underlying "pointer readings" seems to be not only self-contradictory, but—what is worse—sterile for further scientific progress.

An attitude which is at the other extreme from positivism, in the sense that it takes the accepted quantum formalism as seriously as possible, holds that the state-vector ψ really exists, or even that it (or perhaps the density-operator ρ) is the sole reality. But if you make ψ real then you must admit that its inherently non-local features are real as well. These features of ψ include first of all, its built-in non-local correlations of the EPR sort, and secondly, the closely related process of its "collapse". Not only is this latter process beyond the comprehension of quantum mechanics as it now stands,[3] but it is hard to imagine how any future account of

[3] A possible escape would be the so-called "Everett interpretation", in which the collapse

it could reconcile its “instantaneous global” character with the Special Relativistic exclusion of superluminal propagation. In fact the clash with Minkowskian locality seems to be a very broad one: if ψ is real then “where is it?”, or at least, “how does the logical ordering implicit in its sequential collapses relate to the causal ordering of events in Relativistic spacetime?”. While some of these questions may perhaps admit consistent answers in the context of flat- (or even curved-) space Quantum Field Theory with external observers,[4] they seem to become hopelessly entangled as soon as the causal structure itself acquires dynamics (as in General Relativity), especially when external observers must be given up (as in cosmology).

As is often remarked, this kind of issue shows up with particular clarity in (gedanken- or actual) experiments of the EPR-Bohm type, which also have the advantage of proceeding entirely within the realm of experimentally established physics. Any interpretation of quantum mechanics aiming to be adequate for applications such as quantum gravity can therefore find a natural test in how it explains the non-local correlations that are observed in EPR-type settings.

Now the interpretation for which ψ is real and really collapses runs into trouble here for the reasons just mentioned.[5] Firstly, the mathematical nonlocality by means of which ψ encodes the EPR correlations must now be admitted as present in reality, even though it can never be detected directly since ψ is not supposed to be measurable. In addition a further (though related) difficulty appears in connection with collapse. Not only does a local measurement on *one* of the particles produce a nonlocal effect on the entire wave function, but this effect turns out to be observationally undetectable by any apparatus interacting locally with the second particle. Whether such behavior can be ultimately compatible with physical locality or Lorentz invariance is open to doubt. At the very least the imputation of nonlocal effects which can't actually be

never occurs, but its effects are supposed to be recovered via a more careful analysis of closed systems in which “measurement-like processes” take place. Among other things, this approach tends to lead either to the view that “nothing really happens” [1] or to the view that “everything really happens” [2] (which perhaps is not that different from the former view).

[4]For example, the rule, “collapse occurs along the past light cone (in the Heisenberg picture)”, appears to be consistent.

[5]And Bell's inequality shows that *any* theory formulated in terms of an instantaneous state evolving in time would encounter the same trouble. Indeed, the trouble shows up even more glaringly if one adapts Bell's argument to spin-1 systems, using the results of Kochen and Specker[10]. In order to use the Kochen-Specker results in the EPR manner, one needs a scheme for measuring the relevant observables, but this can be accomplished by means of suitably concatenated Stern-Gerlach analyzers with recombining beams [13]. Then, as Allen Stairs has pointed out [14], even the perfect correlations become impossible to reproduce, and no reference to probability theory is needed to establish a contradiction with locality.

detected suggests that ψ has an odor of fictitiousness about it.

If, in order to escape these difficulties, we decide that ψ is not real after all, then we are again faced with the question of finding an interpretation which will identify microscopic matter with something more than just a collection of pointer readings. Perhaps the so-called Copenhagen interpretation is a step in this direction, since in Bohr's writings the microworld is taken to be real, even if an element of operationalism is also present. One may also envision entirely different possibilities, including non-local hidden variables or more subtle options also involving action at distance [3]. However the quantum framework which seems to us the most natural is one in which reality is taken to be precisely what it was in classical physics: a collection of particle world-lines, a spacetime field, or whatever, but in any case a "history".

Although this "sum-over-histories" interpretation of quantum mechanics has existed for a long time, it seems to have been largely ignored in philosophical discussions about quantum mechanics. Perhaps one reason is that its measurement theory remains in a rudimentary state [5]; another may be that it has difficulties with fields of "Grassmann type";[6] and even in a non-relativistic context, it is clumsy in application to spinning particles. Nevertheless, it has a definite answer to what is real, it suggests definite approaches to quantum gravity [6, 7], and it appears to overcome the principal conceptual difficulties of other interpretations. In particular, it obviates the problems with state vector collapse because for it there is no state vector *to* collapse. [17]

For all these reasons we believe the sum over histories interpretation deserves more attention than it has received from philosophers and from physicists concerned with the meaning of quantum mechanics. To help stimulate such attention, and to provide a point of reference for further discussion, we propose to analyze here a simple EPR experiment in detail. However the version of the experiment we will analyze is not the most well-known one involving a pair of spin-1/2 particles. This would have been possible, but would have required us first to extend the formal sum-over-histories framework to deal with particle spin, which in turn might have raised suspicions that some technical trick was being played to obtain the correct correlations. Fortunately, there exists [16] a version of the EPR experiment that employs only scalar particles,

[6] No technical problem obstructs an extension to fermionic fields (indeed the "functional integral" formalism for Quantum Field Theory is probably the most popular at present), but the realistic interpretation of the individual histories seems to get lost. One way out would be if all fermions were composites or collective excitations of fields quantized according to "bosonic" commutation relations. Another would be if the particle formulation were taken as basic, with the "complementary" field formulation being merely a mathematical artifice (at least for fermions).

which are precisely the type to which the sum over histories applies most naturally. Working with this version will allow us to show as clearly as possible, how the non-local correlations are accounted for without introducing any action at a distance or superluminal propagation into the formalism.

For reasons of space we will not try to introduce the sum over histories framework in detail, but only recall its principal features in the form they will be needed below. For further exposition of the basic ideas, we refer the reader to [4, 5, 6, 7, 8], not all of which, however, consistently adhere to the view that the particle world lines, and only the particle world lines, are real.

2 Sum Over Histories Framework

To introduce the sum over histories approach, let us consider the example of a single non-relativistic particle in one dimension. The basic element of physical reality in this picture is a "path" or history described by the particle in time, $\gamma = \gamma(t)$, and relevant questions refer to the properties of the path as a whole. To bring out the close correspondence between the sum over histories approach to quantum mechanics and classical stochastic processes we will start with the classical situation and then explicitly state the modifications needed to make a transition to quantum mechanics.

Let us consider the description of Brownian motion as a stochastic process. Consider the space of all possible histories $\gamma(t)$. One introduces dynamics by giving a rule for computing probabilities. This is done in the following manner. For example, we would like to know the probability for a certain class C of paths or histories. The rule is to attach a positive real number p to each history γ and assign to the class C the composite probability

$$P(C) = \sum_{\gamma \in C} p(\gamma) \tag{2. 1}$$

One point should be emphasized in anticipation of the quantum situation that will follow. In the above, since the paths $\gamma(t)$ are unbounded in time, the individual probabilities do not really exist. It is possible to make them well defined by truncating the histories at some final time. In this case, since we are interested only in class probabilities it is not necessary to do so, since, for appropriate classes, these are well defined in spite of the fact that the individual $p(\gamma)$ are not. Of course even a truncated individual history has measure zero in a continuous space of histories, and this is another reason why probabilities can only be sensibly assigned to classes. However this reason is independent of the truncation issue, which would be present even for a discrete random walk.

Let us now make a transition to the quantum mechanical sum-over-histories framework. As before, the basic objects are taken to be the paths $\gamma(t)$. The "only" difference is in the modification of the rule (2.1) for computing probabilities. This involves associating complex amplitudes with individual paths rather than positive probability weights. Also, unlike the classical case, to get agreement with standard quantum mechanical probabilities, one *must* work with paths truncated at some final time T, which we can call the "collapse time". Given the collapse time T, one can state the quantum mechanical rule for computing probabilities as follows:

1. To every truncated path assign a complex amplitude $A(\gamma)$

2. The total amplitude for all truncated paths in a class C that arrive at q at T is given by

$$A(q,T;C) = \sum_{\substack{\gamma\in C \\ \gamma(T)=q}} A(\gamma) \qquad (2.\ 2)$$

3. The (un-normalized) probability of the class C is given by

$$P(C) = \sum_q \Big| \sum_{\substack{\gamma\in C \\ \gamma(T)=q}} A(\gamma)\Big|^2 \qquad (2.\ 3)$$

Notice that (2.3) consists of a combination of coherent and incoherent summation. The amplitudes are further restricted by the consistency condition that $P(C)$ must be independent of T, which can be interpreted as a generalized unitarity condition.

One can also get the probabilities directly without having to perform two separate summations by using a slightly modified rule for assigning amplitudes. As before one considers histories which are limited in time; but at $t = T$, rather than simply terminating, they "turn around and proceed backward in time". Such a "two-way path", γ is effectively a *pair* of histories, $\gamma'(t), \gamma''(t)$, with the property that $\gamma'(T) = \gamma''(T)$. In terms of two-way histories the basic probability-rule appears as follows.

1. To each forward directed path between the initial time and T attach an amplitude A.

2. To the time reverse of each such path associate the conjugate amplitude A^*.

3. To each two-way path assign the product of the amplitudes of its forward and backward parts.

The rule for obtaining the probability of the class C of (one-way) histories is then

$$P(C) = \sum_{\gamma \in \widetilde{C}} A(\gamma) \tag{2. 4}$$

where $\widetilde{C}$ is the class of all two-way paths whose "upward" and "downward" portions both fulfill the conditions defining C. The total amplitude $P(C)$ itself gives the probability and does not need to be squared because the amplitudes of the two-way paths already contain appropriate products of one way amplitudes.

Note that in the analysis using two-way paths it is not necessary that the two-way amplitudes be complex. In fact, it would be sufficient to replace them by their real parts in the probability rules to get the correct probabilities. Thus, from this point of view the existence of quantum interference effects traces back to A being allowed to be negative, but its being complex is not essential.

In the framework just described, the characteristic quantum "interference effects" result from the fact that the probabilities (2.3) [or (2.4)] do not obey the classical sum rules. (Indeed, if they did obey them, they could always be re-expressed in the form (2.1) — say for a finite sample space $\{\gamma\}$, where a discrete sum makes sense.) In order to emphasize this fact, one should perhaps refer to the P's as "quantum probabilities", as suggested to us by Jim Hartle.

Because the classical sum rules no longer obtain, one can not interpret quantum probabilities as frequencies with respect to a hypothetical "ensemble of universes", as one sometimes does with classical probabilities. This intensifies some important conceptual puzzles concerning the meaning of probability in general, and in particular concerning its experimental meaning in the face of the dialectical limitations on knowledge implied by the uncertainty principle. Perhaps the central question here is "what is the physical meaning ("material basis") of the specific partition into classes contemplated in (2.1)-(2.4)?". From a definite answer to this question would flow answers to other important questions, like the question whether probabilities must be taken relative to some "observer" (who properly would be an *internal* "subsystem", rather than the external observer of von Neumann's paradigm); the question whether probabilities corresponding to arbitrary partitions need be "experimentally realizable" (cf. [5])[7]; and the question whether one must exclude interference between macroscopically distinct histories of subsystems like human beings (to which we would answer 'no').

[7]In the approach of Gell-Mann-Hartle and Griffiths for example, only a small subset of the possible partitions is granted meaning, in such a way that all interference terms are suppressed and quantum probabilities reduce to classical ones.

It is probably in connection with such issues that the main philosophical interest of the sum-over-histories interpretation lies, but for present purposes we will be able to make do with a simpler set of rules, which any more comprehensive resolution of the above issues will surely preserve. Thus, in the situation we will consider, we can have effectively external "observers" who, in the end, need only determine which beam a given photon has emerged in (e.g. by employing the detectors provided in figure 2 below); also, the experiment can be repeated many times and then (it is easy to see) the frequency interpretation of the computed probabilities will apply. This, together with the usual rule (which follows from it in this case) about relativizing probabilities to new knowledge, will suffice for interpreting all the quantum probabilities we will compute in section four.

3 EPR Experiment (Bohm version)

Before attempting to analyze the EPR Gedankenexperiment using sum over histories, let us first recall Bohm's spin-$\frac{1}{2}$ version of the EPR paradox and review the problems that arise in understanding this using the state vector picture. Consider a pair of spin-$\frac{1}{2}$ particles formed somehow in the singlet state ($S = 0$). In the experimental setup shown in Fig.1, measurements can be made, say by Stern Gerlach magnets D_L and D_R on selected components of the spin σ_L and σ_R. D_L and D_R perform dichotomic measurements, i.e, a spin-$\frac{1}{2}$ particle can be found in one of the exit channels labeled u or d. The labels u and d refer to the orientations of the spin along and opposite to the magnetic field of the analyzer. Let us take the analyzer D_L to be oriented such that the magnetic field is in the direction of the unit vector $\vec{a}$ and the analyzer D_R with a magnetic field in the direction $\vec{b}$. It is assumed that $\vec{a}$ and $\vec{b}$ can be chosen freely at the experimenter's will.

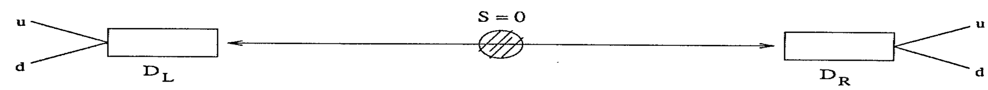

Fig. 1

Figure 1 *Experimental set-up for Bohm's version of the EPR experiment. Two spin-$\frac{1}{2}$ particles in the singlet state are emitted from the source at the center. D_R and D_L are Stern-Gerlach analyzers capable of measuring selected components of spin.*

Writing the singlet state vector as

$$\Psi(L,R) = \frac{1}{\sqrt{2}}[|\uparrow>_L|\downarrow>_R - |\downarrow>_L|\uparrow>_R] \tag{3. 1}$$

where $\uparrow$ and $\downarrow$ refer to spin up (down) with respect to an arbitrary axis, we can calculate the probabilities of various outcomes of measurements according to quantum mechanics. The results turn out to be the following:
For single measurements with D_L and D_R in orientations $\vec{a}$ and $\vec{b}$ respectively,

$$P_{u_L}(\vec{a}) = P_{d_L}(\vec{a}) = \tfrac{1}{2} \tag{3. 2}$$
$$P_{u_R}(\vec{b}) = P_{d_R}(\vec{b}) = \tfrac{1}{2} \tag{3. 3}$$

where $P_{u(d)}$ refers to the probability of a single particle emerging in the upper (lower) beam. For joint measurements,the quantum mechanical predictions are

$$P_{uu}(\vec{a},\vec{b}) = P_{dd}(\vec{a},\vec{b}) = \tfrac{1}{2}\sin^2\frac{\theta}{2} \tag{3. 4}$$

$$P_{ud}(\vec{a},\vec{b}) = P_{du}(\vec{a},\vec{b}) = \tfrac{1}{2}\cos^2\frac{\theta}{2} \tag{3. 5}$$

where the first(second) subscript on P indicates the outcome of the left (right) detector. For example, P_{ud} represents the joint probability for the photons to emerge in the upper beam at the left detector and the lower beam at the right detector respectively. θ is the angle between the orientations of the analyzers. In particular,for $\theta = 0$ (same direction for analyzers)

$$P_{uu} = P_{dd} = 0 \tag{3. 6}$$

$$P_{ud} = P_{du} = \tfrac{1}{2} \tag{3. 7}$$

This implies a strong correlation between the results of measurement of the two spin $\frac{1}{2}$ particles. If particle L is in the upper beam, we are sure to find particle R in the lower beam and vice versa.

According to "standard quantum mechanics" the complete specification of the two particle system is given by the state vector ψ, which is constructed so that the combined system is in the singlet state. It is meaningless to talk about either particle possessing a definite spin unless a measurement is performed on it to determine the spin. All one can say is that, given the orientations $\vec{a}$ and $\vec{b}$ of the analyzers, the probability of either particle registering spin up (down) with respect to its analyzer's axis is 50%.

Let us consider the special case where both the analyzers have the same orientation $(\theta = 0)$, though $\vec{a}$ itself can be chosen arbitrarily. Let us also imagine that the encounters of the left and right particles with their respective analyzers are spacelike separated events. Now if a measurement is performed on (say) the left particle and it is detected in (say) the upper beam, then from (3.5), (3.7) one knows that the result of measuring the right particle will be that it will emerge in the lower beam at D_R. In the language of state vectors one describes this situation as follows. On performing a measurement on the left particle , the combined wavefunction collapses, assigning a definite value to the spin of the right particle compatible with the result of measurement on the left particle in accordance with the correlations given by (3.5), (3.7). The wavefunction collapse process is thus totally insensitive to the fact that the particles are spacelike separated and is therefore nonlocal, as, indeed, is the wave-function itself, even before it collapses.

This experiment was originally proposed to argue that quantum mechanics is incomplete as a theory, and should perhaps be supplemented by classical hidden variables that would lend an underlying deterministic structure. Statistical averaging over these hidden variables was to yield the usual probabilities of quantum mechanics. It was also hoped that this would restore locality in the description. However, in 1964, Bell [9] demonstrated, using precisely the example of the spin-$\frac{1}{2}$ EPR experiment, that no local hidden variable theory can correctly reproduce all the statistical correlations predicted by quantum mechanics; and for spin-1 systems there is an even more drastic conflict between the predicted correlations and locality (see footnote 4). In other words any theory which replaces the state-vector by other variables of state, and yet reproduces quantum mechanical predictions, must still contain non-local interactions among these variables or among them and the analyzers.

In the next section we will describe a slightly different version of the EPRB experiment, which does not involve the notion of intrinsic particle spin. Working with that other version, we will then show how the sum-over-histories account of the EPRB experiment is able to produce the required nonlocal correlations without hidden variables, and without nonlocal interactions.

4 Calculation of Probabilities using Sum Over Histories

The modified version of the EPR experiment with photons is described here. [16] It must be emphasized, as we have stated before that the spin of the photon plays no role here. For example, we can assume that the photons are linearly polarized perpendicular to the plane of the paper. The experimental arrangement is shown in Figure 2, wherein S is a photon source, the M's are mirrors (with M_0, M_5, and M_6 being only half-silvered); the D's are detectors; and the rectangles intercepting CE and $C'E'$ are adjustable phase-delayers P_1 and P_2. The source emits two photons at a time, one toward A and one toward B; and each emitted photon then follows one of the possible paths indicated. Ideally, one would like to have a source always emitting one photon to the right and one to the left. Since it is difficult to devise such a source, we use the arrangement with M_0 to simulate that situation by simply ignoring the events in which both photons go to the right or left. The detection of a photon in the upper or lower beam is analogous to a measurement with a Stern Gerlach detector in the spin-$\frac{1}{2}$ case.

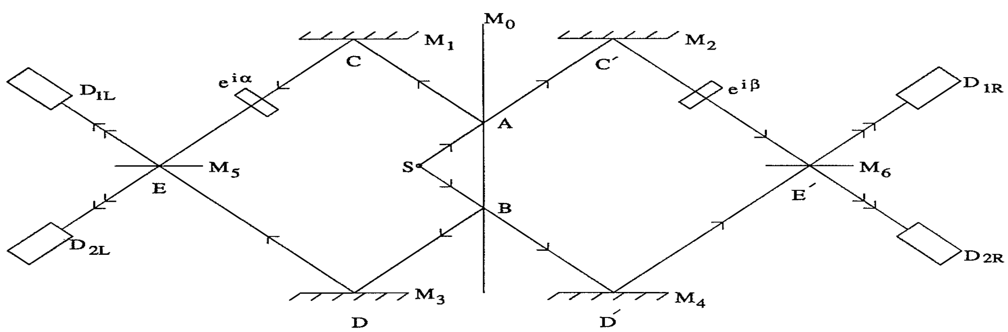

Fig. 2

Figure 2 *An experimental arrangement for the modified two photon version of the EPR experiment. S is a photon source, the M's are mirrors (with M_0, M_5, M_6 only half slivered). The D's are the detectors with the subscripts 1(2) denoting upper and lower beams, indicated by u and d in the text. The rectangles marked $e^{i\alpha}$ and $e^{i\beta}$ intercepting the beams indicate the phase delayers. The source emits two photons at a time that follow the possible paths indicated by arrows.*

Let us now calculate the joint probabilities for

1. both photons emerging in the upper beams, and hence being detected at D_{1R} and D_{1L}.

2. one photon emerging in the upper beam and hence being detected at D_{1R} (D_{1L}) and the other emerging at the lower beam at D_{2L} (D_{2R}).

The rules for assigning amplitudes for this particular experiment are the following:[8]

1. At a half silvered mirror, the amplitude attached to the reflected beam $= \frac{i}{\sqrt{2}}$. Amplitude attached to a transmitted beam $= \frac{1}{\sqrt{2}}$. The fact that (reflection amplitude) $= +i$(transmission amplitude) follows from the requirement that the evolution be unitary, given that the transmission probability is 1/2 (and the mirror is symmetric). We have chosen the amplitude for the reflected beam as $\frac{+i}{\sqrt{2}}$. One can also design mirrors for which we have $\frac{-i}{\sqrt{2}}$ instead, and that would give the same results.

2. Whenever the path of a photon passes through a phase shifting device P, attach an amplitude $e^{i\theta}$ to that particular path where θ is the phase appropriate to the device.

Using these rules and the prescription for sum over histories given by (2.2) and (2.3), we can calculate the probabilities for photons to arrive at D_{1R} and D_{1L}. There are two possible histories fulfilling this condition:
(i) the photon that goes to the left follows $SACED_{1L}$ and the photon going to the right follows $SBD'E'D_{1R}$.
(ii)The photon that goes to the left follows $SBDED_{1L}$, and the photon that goes to the right follows $SAC'E'D_{1R}$.

At this point one should mention that in applying rules (2.2) and (2.3) the outer summation in equation (2.3) is inoperative in this specific situation. The collapse time can be taken to coincide with detection at the detectors, and then each class C refers to (truncated) paths with definite final positions. Thus there is only one term in the incoherent sum that contributes. In the computation that follows we will also use the fact that the amplitude of a path is equal to the product of the amplitudes of its segments.

Amplitude for (i)

$$\begin{aligned} &= (amplitude\, for\, SACED_{1L})(amplitude\, for\, SBD'E'D_{1R}) \\ &= A_0(\frac{i}{\sqrt{2}}e^{i\alpha}\frac{i}{\sqrt{2}})(\frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}}) \end{aligned} \tag{4. 1}$$

[8]In stating these rules we consider an idealized situation in which the spatio-temporal indeterminacy of particle-location *within* a given one of our trajectories is ignored; or if you prefer, you can take the experiment as only a Gedanken one affording a simplified illustration of how EPR-like correlations are understood within the sum-over-histories framework. In this connection recall also that the semiclassical propagator is in fact exact for a free particle.

where A_0 is the amplitude common to all paths and (4.1) gives the un-normalized amplitude.
Amplitude for (ii)

$$\begin{aligned} &= (amplitude for SBDED_{1L})(amplitude for SAC'E'D_{1R}) \\ &= A_0(\frac{i}{\sqrt{2}}\frac{1}{\sqrt{2}})(\frac{1}{\sqrt{2}}e^{i\beta}\frac{i}{\sqrt{2}}) \end{aligned} \quad (4.\ 2)$$

The total amplitude according to (2.2) is the sum of (4.1) and (4.2).

$$\begin{aligned} A_{uu} &= A_0(\frac{i}{\sqrt{2}}e^{i\alpha}\frac{i}{\sqrt{2}})(\frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}}) + A_0(\frac{i}{\sqrt{2}}\frac{1}{\sqrt{2}})(\frac{1}{\sqrt{2}}e^{i\beta}\frac{i}{\sqrt{2}}) \\ &= \frac{-A_0}{4}(e^{i\alpha} + e^{i\beta}) \end{aligned} \quad (4.\ 3)$$

where as before we adhere to the convention that the first(second) subscript on A indicates the outcome of the left(right) detector. When we calculate the probabilities we normalize them so that the events where both photons go to left/right are ignored $(A_0 = \sqrt{2})$. [9] Probability of both photons emerging in the upper beam according to (2.3) is given by

$$P_{uu} = |A_{uu}|^2 = \frac{1}{4}[1 + \cos(\alpha - \beta)] = \tfrac{1}{2}\cos^2\frac{(\alpha - \beta)}{2} \quad (4.\ 4)$$

Similarly,

$$P_{dd} = \tfrac{1}{2}\cos^2\frac{(\alpha - \beta)}{2} \quad (4.\ 5)$$

For the case where one photon emerges in the upper beam and the other emerges in the lower beam, again there will be two paths contributing to the total amplitude. These are:
(A) the photon that goes to the left follows $SACED_{1L}$ and the photon that goes to the right follows $SBD'E'D_{2R}$.
(B) the photon that goes to the left follows $SBDED_{1L}$ and the photon going to the right follows $SAC'E'D_{2R}$.
As in the previous case amplitudes for (A) and (B) will combine giving the total amplitude.

$$\begin{aligned} A_{ud} &= A_0(\frac{i}{\sqrt{2}}e^{i\alpha}\frac{i}{\sqrt{2}})(\frac{1}{\sqrt{2}}\frac{i}{\sqrt{2}}) + A_0(\frac{i}{\sqrt{2}}\frac{1}{\sqrt{2}})(\frac{1}{\sqrt{2}}e^{i\beta}\frac{1}{\sqrt{2}}) \\ &= \frac{A_0 i}{4}(e^{i\beta} - e^{i\alpha}) \\ &= -A_{du} \end{aligned} \quad (4.\ 6)$$

[9]This can be interpreted either as part of the specification of the initial conditions, or (as suggested by a referee) merely as an example of relativization of probabilities.

The resulting probabilities are obtained as

$$P_{ud} \;=\; P_{du} \;=\; |A_{ud}|^2 \;=\; \frac{1}{4}[1-\cos(\alpha-\beta)] \;=\; \tfrac{1}{2}\sin^2\frac{(\alpha-\beta)}{2} \qquad (4.\ 7)$$

Equations (4.4), (4.5) and (4.7) give the correct quantum mechanical probabilities coinciding with those calculated using a state vector. Comparing these results with those of the spin-$\frac{1}{2}$ case mentioned earlier, we notice that they can be put in one to one correspondence by identifying θ with $\alpha - \beta + \pi$, and the probability of having both spins up with P_{uu}. However the correspondence between the two situations is not perfect, in the following sense. In the spin-$\frac{1}{2}$ case, the unit vector $\vec{a}(\vec{b})$ specifies the orientation of the analyzer $D_R(D_L)$. This is equivalent to specifying a point on the unit sphere given by two angles (θ, ϕ). However, in attempting to make the correspondence between the spin-$\frac{1}{2}$ case and the results of this section, we find that the orientation of the analyzers must be specified by the single angle $\alpha(\beta)$. This translates to specifying the orientations of the analyzers by two-dimensional unit vectors instead of three-dimensional ones. As a result, to make the appropriate identifications with the spin-$\frac{1}{2}$ case, one must consider the special case where the axes of the analyzers are restricted to the $x-y$ plane (say). This is however not too restrictive, since in the usual version of Bell's inequality, in order to demonstrate that the inequality is violated by quantum mechanics, it is sufficient to choose a set of orientations of the analyzers only in the $x-y$ plane.

5 Calculation of Probabilities Using Two-way Paths

One can get the probabilities (4.4), (4.5) and (4.7) directly, without having to square any partial amplitudes, by using amplitudes for "two-way histories" as specified earlier. A two-way history in this case is a pair of photon trajectories each starting from the source and reaching the detector then turning back at the detector and ending at the source. We will calculate P_{uu} by summing over amplitudes for two-way histories in accordance with (2.4). The class C refers to paths in which a photon emerges in the upper left beam and another emerges in the upper right beam. All two-way paths that are included in the sum must fulfill the condition C in both their "forward" and "backward" portions. We can see that there are four such paths.

(I)$\{(SAC'E'D_{1R})(SBDED_{1L})\}\{\overline{(D_{1R}E'C'AS)}(\overline{D_{1L}EDBS})\}$

(II) $\{(SAC'E'D_{1R})(SBDED_{1L})\}\{(\overline{D_{1R}E'D'BS})(\overline{D_{1L}ECAS})\}$

(III) $\{(SBD'E'D_{1R})(SACED_{1L})\}\{(\overline{D_{1R}E'D'BS})(\overline{D_{1L}ECAS})\}$

(IV) $\{(SBD'E'D_{1R})(SACED_{1L})\}\{(\overline{D_{1R}E'C'AS})(\overline{D_{1L}EDBS})\}$

The overbar denotes the "return" portion of the paths. The notation is more or less self explanatory, but to avoid any ambiguity, let us explain what we mean by the amplitude (I) in detail. The first factor { } corresponds to the amplitude for the photon transmitted to the right to be reflected at M_2 and then get reflected at M_6 and emerge in the upper right beam, and the photon reflected to the left to get reflected at M_3 and be transmitted at M_5 and emerge in the upper left beam. The second factor { } denotes the amplitude for the following backward portion of the path. The photon in the upper right beam returns after reflection at M_6 and M_2 and transmission at M_0 to the source S, and the photon in the upper left beam returns to S after getting transmitted through M_5 and reflected at M_3 and M_0.

We assign amplitudes in accordance with the rules stated before and compute probabilities using (2.4)
Amplitude for (I), A_1

$$= A_0\{(\frac{1}{\sqrt{2}}e^{i\beta}\frac{i}{\sqrt{2}})(\frac{i}{\sqrt{2}}\frac{1}{\sqrt{2}})\}A_0{}^*\{(\frac{-i}{\sqrt{2}}e^{-i\beta}\frac{1}{\sqrt{2}})(\frac{-i}{\sqrt{2}}\frac{1}{\sqrt{2}})\} \quad (5.\ 1)$$

Amplitude for (II), A_2

$$= A_0\{(\frac{1}{\sqrt{2}}e^{i\beta}\frac{i}{\sqrt{2}})(\frac{i}{\sqrt{2}}\frac{1}{\sqrt{2}})\}A_0{}^*\{(\frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}})(\frac{-i}{\sqrt{2}}e^{-i\alpha}\frac{-i}{\sqrt{2}})\} \quad (5.\ 2)$$

The factors corresponding to the component amplitudes are written in the same order as they appear in (I).
Amplitude for (III), A_3

$$= A_0\{(\frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}})(\frac{i}{\sqrt{2}}e^{i\alpha}\frac{i}{\sqrt{2}})\}A_0{}^*\{(\frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}})(\frac{-i}{\sqrt{2}}e^{-i\alpha}\frac{-i}{\sqrt{2}})\} \quad (5.\ 3)$$

Amplitude for (IV), A_4

$$\begin{aligned} &= A_0\{(\frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}})(\frac{i}{\sqrt{2}}e^{i\alpha}\frac{i}{\sqrt{2}})\}A_0{}^*\{(\frac{-i}{\sqrt{2}}e^{-i\beta}\frac{1}{\sqrt{2}})(\frac{1}{\sqrt{2}}\frac{-i}{\sqrt{2}})\} \\ &= A_2{}^* \end{aligned} \quad (5.\ 4)$$

Using (2.4), the un-normalized probability of finding the right and left photons both in the upper beams

$$\begin{aligned} &= A_1 + A_2 + A_3 + A_4 \\ &= \frac{|A_0|^2}{4}\cos^2\frac{(\alpha-\beta)}{2} \end{aligned} \quad (5.\ 5)$$

Normalizing as before, we get

$$P_{uu} = \tfrac{1}{2} \cos^2 \frac{(\alpha - \beta)}{2} \tag{5. 6}$$

which, not surprisingly, is the same result as obtained before. Note that in calculating the probabilities using two-way histories the individual amplitudes A_1, A_2, A_3, A_4 are in general complex, but their sum is always positive.

6 Conclusion

On its own terms our treatment of the EPR experiment is now complete. We have shown that the probabilities yielded by the sum-over-histories rules are the familiar ones which violate Bell's inequality, which entail perfect correlations for equal settings of the phase delays, and which manifest the "non-locality" that frameworks based on states evolving in time find so troublesome. In all of our analysis the fundamental notion was that of a history, which reduced in this case to a pair of photon trajectories. *The "events" whose probabilities we computed referred directly to these trajectories, and no notion of state-vector (or any other auxiliary variables) entered into the analysis at any stage.* The calculation itself was almost trivial, but we presented it in full in order to demonstrate that no hidden steps were needed where any sort of "spooky action at a distance" might have entered.

And yet the reader may still be tempted to ask, "where did the wave-function collapse sneak in?" While our main point is that, within the present framework, such a question would be asking about a mathematical fiction, it is nevertheless true that these fictions can sometimes be introduced as useful intermediaries in calculating probabilities, and we would like to describe in a moment how our analysis would look if this were done.

Before doing so, however, we would like to modify slightly the experimental situation we have been considering so far. Namely we will imagine the final detection processes to be displaced from each other in time, the better to address an important related issue we did not need to deal with in the above analysis, namely the issue of how, within the sum-over-histories framework, the acquisition of new knowledge modifies the prediction of future probabilities.

To render the correspondence with the Bohm version of EPR as close as possible, let us mentally divide the experimental apparatus of figure 2 into three parts: the effective source, consisting of the photon source proper together with the mirror M_0; the analyzer on the left, consisting of the mirrors M_1, M_3 and M_5 together with the phase-delayer P_1; and the similar analyzer on the right, comprising M_2, M_4, M_6, and P_2. To simplify the wording of the present discussion we omit the detectors indicated in figure 2 and let the photons be seen directly by human beings: "ourselves" on the left and "our friends" on the right. Finally, let us suppose—in order to frame the "collapse issue" as vividly as possible—that our friends postpone their observation of the photon on the right, while in the meantime we see the left photon emerge in the upper beam.

On the basis of this observation we would then of course predict probabilities for our friends' subsequent detection, which differ from the a priori odds of 50-50 that applied when the two photons left the effective source. This change comes about not for any mysterious reason, but merely because *we follow the standard rule of relativizing probabilities to new knowledge.* Having determined that the left photon is in the upper beam, we must recompute probabilities for the right photon, retaining only the contributions from those histories which are compatible with our new information. The resulting probability that the right photon emerges in the upper beam is then, according to equations (4.4) and (4.7), $\cos^2(\alpha - \beta)/2$, instead of 50%.

In all of this so far there was nothing involving the notion of state-vector. If we desire, however, we can arrange to describe the effects of the recomputation in such a language, though of course nothing forces to do so. A state vector (or "wave function") in the sum over histories framework is effectively a *summary of information about the past*, which retains precisely what is needed for computing probabilities pertaining to the future.[10] From the standpoint of the abstract sum over histories, the possibility of such a summary represents a partial decoupling of past from future closely analogous to the decoupling that occurs for non-quantum stochastic processes of Markov type (i.e. processes which fulfill the condition that the past be statistically independent of the future given the present). Such a decoupling need not occur (just as a stochastic process need not be Markov) and indeed there exist useful generalizations of the ordinary amplitude-rules for which no state-vector can be introduced at all.[11] If, on

[10] Thus a state-vector may be defined as an equivalence-class of sets of partial histories.

[11] One such generalization applies to open systems, for example to a particle in contact with a heat reservoir. For this example see [11], wherein the "two-way path" formalism of §5 above is used, and the influence of the reservoir results in an effective dynamics for the particle in which the "forward" and "backward" portions of its world-line are coupled to each other by a certain "interaction term" in the amplitude. In this type of situation a density-operator ρ (though not a state-vector ψ) can still be introduced, but it no longer summarizes all the relevant information about the past (and correspondingly its evolution lacks the "Markov" property that $\rho(t + dt)$ is determined by $\rho(t)$ alone). For quantum gravity, it may be that not even such a non-Markov ρ will be exactly definable, and only

the contrary, the amplitudes of familiar quantum theories do admit state-vectors, it is only because they are built up in a special way according to which the amplitude of a concatenation of two or more partial histories is the product of the amplitudes associated to the individual parts (multiplicativity).

In the present setting, the amplitudes are in fact multiplicative, and state-vectors can be introduced at several stages, the first one occurring immediately after the photons emerge from the effective source. At this stage there are only four possibilities, which we may denote as $|\uparrow>_L|\uparrow>_R$, $|\uparrow>_L|\downarrow>_R$, $|\downarrow>_L|\uparrow>_R$, $|\downarrow>_L|\downarrow>_R$; and the amplitudes for two of them vanish, leaving the effective state

$$|S>=\frac{i}{2}(|\uparrow>_L|\downarrow>_R+|\downarrow>_L|\uparrow>_R).$$

A second state-vector $|T>$ summarizes the partial amplitudes that apply just after the photons have emerged from their respective analyzers. Using a similar notation to indicate what are then the upper and lower beams, and referring to equations (4.3) and (4.6), we easily write down

$$\begin{aligned}|T> &= A_{uu}|\uparrow>_L|\uparrow>_R+A_{ud}|\uparrow>_L|\downarrow>_R \\ &\quad +A_{d_Lu}|\downarrow>_L|\uparrow>_R+A_{d_Ld}|\downarrow>_L|\downarrow>_R \\ &= \frac{A_0}{4}(e^{i\alpha}+e^{i\beta})|\uparrow>_L|\uparrow>_R-\frac{A_0 i}{4}(e^{i\alpha}-e^{i\beta})|\uparrow>_L|\downarrow>_R \\ &\quad +\frac{A_0 i}{4}(e^{i\alpha}-e^{i\beta})|\downarrow>_L|\uparrow>_R-\frac{A_0}{4}(e^{i\alpha}+e^{i\beta})|\downarrow>_L|\downarrow>_R\end{aligned}$$

which just combines into a single expression the four amplitudes we used in our earlier analysis. Finally there is the state vector $|V>$ which pertains to a time after we have seen the left photon in the upper beam, but before our friends have made their own observations. This vector involves only the photon on the right, and summarizes the amplitudes obtained by excluding those histories for which the left photon emerges in the lower beam. In light of equations (4.3) and (4.6) $|V>$ is immediately seen to be

$$|V>=-\frac{A_0}{4}(e^{i\alpha}+e^{i\beta})|\uparrow>-\frac{iA_0}{4}(e^{i\alpha}-e^{i\beta})|\downarrow>.$$

From this state-vector we trivially recover the same "relativized" probabilities for the right photon that we found above.

Clearly it is in the transition from $|T>$ to $|V>$ that the dreaded collapse has occurred; but it is equally clear that, within the present framework, this collapse is essentially a fictitious event. To illustrate what we mean by saying this, we may refer

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the global probabilities themselves will make sense.

to the analogous collapse which occurs in the context of *classical* stochastic processes. Take, for example, a point-particle undergoing Brownian motion in a homogeneous medium. Given that it is released at the origin at time $t = 0$, we can compute the positional probability distribution $f(x)$ appropriate to some later time $t = t_0$. This density f is analogous to ψ in the sense that all future probabilities follow from it independently of any knowledge pertaining to $t < t_0$. Now, if at $t = t_0$ we find the particle at $x = x_0$, f will suddenly "collapse" from a Gaussian distribution centered at the origin to a δ-function centered at x_0. From the sum over histories standpoint, the collapse of ψ from $|T>$ to $|V>$ is no more or less real (or nonlocal) than this collapse of f in the case of Brownian motion. In both cases it is the history itself that constitutes the physical reality, and the dynamics is most fundamentally expressed in terms of probabilities of sets of histories, not in terms of derived quantities like ψ or f whose usefulness is contingent on very special properties of those probabilities.

Before concluding, we should perhaps stress that we do not intend the last few paragraphs as a comprehensive exposition of how "von Neumann measurement theory" may be recovered in the sum over histories language. Indeed, our analyzers are not even "ideal" in the sense that they don't preserve the eigenstates they effectively look for (although it would be easy to modify them so that they did). Rather we only wanted to show how the notions of state-vectors and their collapse can be recovered (in certain situations) without the vectors thereby taking on any more physical reality than, for example, a one-particle distribution function has in pre-quantum kinetic theory.

Finally, we would like to return to the question of locality by commenting briefly on the sense in which this fundamental feature can be regarded as present in the sum over histories formulation we have been using. It is true, of course, that no blatantly nonlocal process like wave-function collapse enters this formulation, but on the other hand the essential quantity on which the whole framework rests — the probability amplitude — already has a global character since it pertains to entire (one-way or two-way) histories, which are spread out not only in space but in time as well. In such a framework it is not even clear a priori that anti-causal effects are excluded, let alone non-local ones (in the absence of Lorentz invariance, the two are not necessarily connected, of course.) If nonetheless, neither sort of effect is present, it can only be because the amplitudes take a special form, to wit that the amplitude of an entire history is the product of the amplitudes of its spacetime parts.[12] (In

[12] Ironically it is just this property of the amplitudes which, as mentioned above, makes possible the introduction of the state-vectors whose "collapse" then introduces such a

the non-relativistic theory this can't be exactly true, of course, because action at a distance is present. Thus locality can only be approximate in this case.) In the presence of certain other general features (cf. [5]) this locality of the amplitudes translates into the impossibility of superluminal signaling and signaling into the past.

Using our EPR setup we can illustrate the absence of superluminal signaling with a couple of familiar examples. In particular, when a measurement is performed on the right hand photon , the choice of the phase delay β cannot affect the probability of finding the left photon in the upper beam, for example. Thus, the probability of finding the left photon in the upper beam $= P_{uu}+P_{ud} = \frac{1}{2}\cos^2\frac{(\alpha-\beta)}{2}+\frac{1}{2}\sin^2\frac{(\alpha-\beta)}{2} = \frac{1}{2}$, which is independent of β. Similarly, it can be shown that removing the mirror M_6 altogether on the right will not affect the left hand probabilities.

Thus, locality maintains a rather uneasy presence in sum-over-histories quantum mechanics. It corresponds to a very definite feature of the amplitudes, but it tends to conflict with the global spirit of the entire framework. On one hand, this is reminiscent of classical stochastic processes, where probabilities apply to entire histories but causality can still be present in the specific form of these probabilities (e.g. if they are Markov). If seen in this light the tension between global amplitudes and locality might appear to be inconsequential. On the other hand Minkowskian locality loses much of its meaning in the presence of a dynamical spacetime metric, so maybe a more global framework which still has room for a certain kind of formal locality is exactly what we should have been looking for in connection with quantum gravity.

Be that as it may, it is perhaps fair to say that the sum-over-histories formulation goes a long way toward taking the "mystery" out of quantum mechanics, or at least reducing it to the mystery inherent in the notion of probability itself. No doubt that mystery is enhanced somewhat by the presence of non-positive amplitudes and references to two-way paths, but the fundamental idea of assigning weights to classes of histories remains the same, as does the identification of reality with a single (though in general imperfectly known) history or "path". We believe that this sum-over-histories way of understanding the quantum principle clears up some of the philosophical puzzles connected with other formulations; and for those it doesn't fully clear up, it at least recasts them in a way which promises to be fruitful for the further development of quantum theory, or of the more general forms of dynamics that will one day replace it.

In concluding, we would like to acknowledge the stimulus to the ideas presented herein of a continuing discussion over the years between R.D.S. and John Friedman

strong appearance of *non*locality into the theory.

concerning the possibility of making the sum-over-histories formulation the basis for a philosophy of quantum mechanics. We also would like to thank Jim Hartle for helpful criticism of the wording of an earlier draft of this paper. Finally, one of us (S.S) would like to thank the Physics Department of Syracuse University for hospitality during the period over which this research was done. This research was partly supported by NSF grants PHY-9005790 and PHY-8717155.

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