A SPECIMEN OF THEORY CONSTRUCTION FROM QUANTUM GRAVITY †

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Abstract

I describe the history of my attempts to arrive at a discrete substratum underlying the spacetime manifold, culminating in the hypothesis that the basic structure has the form of a partial-order (i.e. that it is a causal set).

Like the other speakers in this session, I too am here much more as a working scientist than as a philosopher. Of course it is good to remember Peter Bergmann's description of the physicist as "in many respects a philosopher in workingman's* clothes", but today I'm not going to change into a white shirt and attempt to draw philosophical lessons from the course of past work on quantum gravity. Instead I will merely try to recount a certain part of my own experience with this problem, explaining how I arrived at the idea of what I will call a causal set. This and similar structures have been proposed more than once as discrete replacements for spacetime. My excuse for not telling you also how others arrived at essentially the same idea [1] is naturally that my case is the only one I can hope to reconstruct with even minimal accuracy.

The background of the problem

Before describing the development I have just referred to, I should probably tell you what a causal set is. Before doing that, however, let me begin by saying a few words about the problem of quantum gravity itself. What people somewhat

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* The quotation comes from an earlier time. Today Peter would no doubt use 'working person', or some other non-sexist locution.

misleadingly call by this name is really the problem of restoring to physics the unified foundation it has lacked since the beginning of this century. If we adopt a slightly mythical view of how science progresses, we can imagine that a new theory begins to be constructed when too many experimental results accumulate in conflict with the old theory. A better theoretical understanding will then emerge, but it may take some time to put the pieces of this new understanding together in a coherent manner. It may even happen that these pieces cannot be mutually reconciled at all without some fundamental extension of theory that would allow the contradiction-among-the-parts to be dissolved within the context of a more comprehensive whole.

The present situation of "fundamental physics" is similar to that I have just described. Both Quantum Theory and General Relativity are consistent with the facts they were created to explain, but they are not consistent with each other. That this contradiction is purely internal to theory has meant until very recently that only people with a philosophical bent have taken the quantum gravity problem very seriously. (It is thus a very appropriate topic to be discussed at a philosophy of science conference.) Recently this neglect has given way to intense interest; but it still cannot be said that we have any direct conflict between experiment and accepted theory to guide us.

Why is it that quantum gravity suffers from such a lack of clearly relevant experimental data, and what kind of experiment or observation *could* be expected to provide such data? Historically you could say that Quantum Theory deals with the very small and General Relativity with the very large, but the essence of the distinction is not really one of size. Rather, "the quantum of action" is in general important whenever no more than a few degrees of freedom are excited,* while gravity – or in other words General Relativity – is important whenever a large enough amount of energy is compressed into a small enough space. More specifically, gravity is important when the ratio Gm/rc^2 is of order unity, where m is the total mass-energy, r is the radius of the region into which it has been compressed, and G and c are respectively the gravitational constant and the speed of light. Actually we can sometimes notice gravity in less extreme conditions than this, but to do so takes very precise measurements, or a very long time, such as the time it takes a satellite to circle the earth (which is indeed huge compared to the radius of the earth in light units).

* This generally, but by no means always, means that only a few particles are involved.

In any case, a typical object for which gravity always *will* be important is a black hole. Now for this object, we can count the number of its states N using the known value of its entropy S and the basic formula (or definition if you will) $S = k \log N$. The result is that N is gigantic for an astrophysical black hole, but of order 1 when the black hole's radius approaches the so-called Planck length of about 10^{-32}cm. If we could directly observe nature at this length-scale, we would expect to see quantum black holes, and more generally to see everything which occurs exhibiting both quantum and gravitational features. However, since the smallest lengths to which we have so far managed to penetrate by means of particle accelerators are around 10^{-16}cm., there is little hope of doing laboratory experiments in quantum gravity for a long time to come.

The problem, then, is not that we make wrong predictions about processes which we haven't seen yet anyway, but that we fail to make any predictions at all. The dynamical principles learned from quantum mechanics just seem to be incompatible with the idea that gravity is described by a metric field on a continuous manifold. When we try to combine these elements in a way similar to how we have "quantized" non-gravitational field theories, we run into apparently insurmountable technical and conceptual problems, of which I will mention only three.

First the quantum amplitudes resulting from such a "quantization" turn out to be "non-renormalizable", which means in effect that the theory they define ceases to make sense at short distances – very likely just at those distances where we expect to see quantum gravitational effects in the first place! Moreover the standard formulations of quantum field theories rely on the existence of a "background" notion of time with respect to which dynamical evolution can be defined, whereas Relativity makes time itself part of the dynamics. This leads both to difficulties in interpreting the formalism, and to technical problems in setting up what is called the Hilbert space metric. Finally the quantum Uncertainty Principle seems to combine with the General Relativistic connection between mass and spacetime-curvature in such a way that any Gedanken-experiment attempting to measure the metric at short distances gets trapped in a vicious circle: the more accuracy you try for, the greater the uncontrollable disturbance you induce in the geometry you are trying to measure.

The difficulties just mentioned arise when you attempt to unite Quantum Field Theory with General Relativity, but actually each of these two theories already has its own internal contradictions. Unquantized gravity gives rise to singularities where the Einstein equations must break down (inside black holes for example),

and quantum field theory in flat spacetime produces infinite amplitudes which, in the view of many workers, are only partly explained away by renormalization.

Taken together, all these difficulties and incompatibilities have suggested to many people that either Relativity theory or Quantum theory or both will have to be fundamentally modified before a successful union of the two will be achieved.

The causal set idea

At present my main hopes for quantum gravity center on an idea (the causal set idea) which by now has been around for a while, even if most people haven't taken it too seriously. I imagine that one reason for this neglect is that it is very hard to come up with plausible "laws of motion" for causal sets. Conversely, one of the things that encouraged me to begin to champion causal sets more enthusiastically was that I did finally get a glimpse of a possible dynamics for them. Equally important however, was the influence of M. Taketani's writings, which convinced me that there is nothing wrong with taking a long time to understand a structure "kinematically" before you have a real handle on its dynamics. In fact I think that Taketani's recognition of the importance of what he calls the "substantial" stage in the development of scientific understanding, allows him to put forward an analysis [2] of theory construction which is "non-trivial" in a way that other analyses I have seen are not. I might even have devoted my talk to an exposition of his ideas, had I not been asked to speak on something relating directly to quantum gravity. Anyhow, let me return to the topic that I am discussing, and tell you in the first place what the causal set concept actually is.

The idea [3] is that in the "deep quantum regime" of very small distances, gravity is no longer described by a tensor field living on a continuous spacetime manifold (the metric field). Rather, the notions of length and time disappear as fundamental concepts, and the manifold itself dissolves into a discrete collection of elements related to each other only by a microscopic ordering that corresponds to the macroscopic notion of before and after. Because of this correspondence the order may be called 'causal', and the structure it describes a 'causal set'. It is a "discrete manifold" (to use Riemann's term), and its defining order carries in particular all the information showing up at larger scales as the geometry of continuous spacetime: the topology, the differentiable structure, * and the metric.

* A differentiable structure on a manifold is a technical notion of "smoothness"; without one, a manifold can not support a metric field (or any other tensor field).

Mathematically a causal set may be defined as a locally finite partially ordered set, or in other words a set C provided with a "precedence" relation, $\prec$, subject to the following axioms:

(1) if $x \prec y$ and $y \prec z$ then $x \prec z$ (transitivity);
(2) if $x \prec y$ and $y \prec x$ then $x = y$ (non-circularity);
(3) for any pair of fixed elements x and z of C, the set $\{y |\, x \prec y \prec z\}$ of elements lying between x and z is finite;
(4) $x \prec x$ for any element x of C (reflexivity).

Of these axioms the first and second say that $\prec$ is a partial ordering, the third expresses local finiteness, and the fourth is a standard convention made for convenience. Instead of saying that $\prec$ is a partial ordering, one also says that C is a "partially ordered set", or "poset", for short.

In the figure I have shown three examples of rudimentary causal sets, represented graphically in a way suggested by the spacetime diagrams of Relativity theory. In these "Hasse diagrams" each "vertex" represents an element of the causal set, and each rising "edge" represents a relation. For clarity, not all of the relations are shown explicitly, but only those not implied via transitivity (axiom 1) by other relations. Thus, for example, the lowest element precedes the highest element in the second poset even though no direct line is shown joining them.

Of course, a causal set underlying even a very small portion of spacetime would be immeasurably larger than those shown, but the third picture is meant to give some flavor, at least, of how a realistic causal set might look. In contrast, the first and second posets (as well as being too small) are probably too regular to be realistic, but they do give some idea of how dimensional information can be present in a causal order. The second is clearly laid out like a two-dimensional checkerboard, and it can in fact be embedded as a subset of two-dimensional Minkowski space. The first has dimension three in a certain sense, since it can be embedded in a flat spacetime only if the latter has a dimension of three or higher.

One thing that the pictures do not show, is that macroscopic spacetime volume is supposed to be a measure of the number of elements in the corresponding region of the causal set. This is a crucial part of the physical interpretation, and a relationship that makes sense only because of the intrinsic discreteness of the causal set.

There is much more to be said on how a causal set can manage to determine an approximate spacetime metric, and also why a theory based on causal sets can be expected to bypass some of the difficulties of quantum gravity that I referred to earlier. [4] Since this is a meeting on theory *construction*, however, I will not further

describe or argue for the causal set idea as such. Instead I will try to reconstruct for you the chain of thought which, in my case, led from certain general expectations and desires to the particular proposal for quantum gravity that I have just sketched. Unfortunately some of this account will be rather technical, but I hope the general development will be clear, even if the meaning of certain concepts and issues remains partly obscure.

Initial expectations

The ideas from which I started were, I think, discreteness (or "finitarity"), operationality, and a desire to negate the manifold as the substratum of spacetime physics.

That spacetime might ultimately be discrete rather than continuous is an idea that goes back at least to the time of Zeno. In the last century it was clearly enunciated by Riemann and Boltzmann [5], and it has plainly been "in the air" for the last several years. One big reason for its recent currency is certainly the problem of infinite amplitudes in Quantum Field Theory that I referred to earlier, and to a lesser extent the problem in General Relativity of singularities at which the spacetime curvature becomes infinite. (And here I would add an infinity which, I think, is unduly overlooked: the infinite *entropy* that a black hole horizon would possess if arbitrarily fine variations in its shape, or arbitrarily fine fluctuations of matter fields in its neighborhood, were to contribute.) These "ultraviolet" infinities arise at infinitely short distances, and consequently would be immediately converted to finite quantities if there were some lower limit to the physical distances that actually exist.

Such a potential resolution of the problem has become much more real for physicists with the advent of so-called "lattice gauge theories"[6], which allow actual computations to be made on the basis of discrete (albeit artificially constructed) spacetimes. In fact, these discrete spacetimes are just transpositions to four dimensions of the atomic lattices that ordinary solids form. And conversely, when people adapted field-theoretical methods to the understanding of ordinary solid matter, they obtained quantum field theories with divergences which are manifestly no more than an artifact of the continuum approximation being employed. In this case, the "cutoff" that removes the infinities has a physical meaning which is transparent and uncontroversial.

Thus, the atomic structure of matter has suggested to physicists a like character for spacetime. In a similar way, the historically unexpected discreteness (of energy, volume in phase space, angular momentum, ...) from which quantum mechanics gets

its name also has intimated that an underlying granularity of apparently continuous quantities is a universal feature of nature. And finally there are digital computers. Without them, the lattice gauge computations I just mentioned could never have been done. But beyond that, their broader influence on scientific culture clearly reinforces a belief in the ultimately "finitary" nature of the microscopic world.

The prestige of "operationality" as a guiding principle is another fact of scientific culture whose roots are probably too deep to be fully unearthed. Being a scientific form of positivism, its presuppositions might have been merely transferred to physicists from bourgeois philosophy, where I think a positivistic approach has tended to dominate in this century. Within science itself, the strongest arguments have been based on the fact that – for whatever reasons – the unfolding of the quantum and relativity revolutions of this century has commonly been (mis)represented as a triumph of the operationalist method.

As applied to gravity, operationalism would require that the fundamental variables be things with "direct experimental meaning", like the components of the metric tensor. And it would tend to construe this tensor as merely a *summary* of the behavior of idealized clocks and measuring rods, rather than as an independent substance, whose *interaction with* our instruments gives rise to clock-readings, etc.

Finally, there is the desire to transcend the manifold concept, which has also been felt by many people, particularly those with a strong interest in gravity. Of course this desire is connected with the urge toward finitarity, but it also has independent roots, which unfortunately seem to be more technical in nature than the issues we have just been considering. For me, I think the strongest reason for dissatisfaction with the manifold concept had to do with quantum fluctuations in the microscopic topology. The occurrence of such tiny fluctuations is strongly suggested by the form of the Einstein-Hilbert lagrangian, but the resulting picture of ever-changing topological complexity on infinitely small scales (the so-called 'foam') is not one that can be painted in manifold colors. Indeed I did not even see how, within the manifold framework, you could express consistently the notion that topological fluctuations of *finite* complexity can "average out" to produce an uncomplicated and smooth structure on larger scales. The replacing of a manifold by something more fundamental, I felt, might allow such a picture to make technical sense. It might also provide a route to answering a related question that many people have hoped quantum gravity would be able to address, namely the old question of why there are precisely three spatial dimensions and one temporal dimension, rather than some other number of each. Such a possibility looked particularly attractive

in light of the revived interest in Kaluza-Klein theories, which posit that spacetime at sufficiently small distances actually does have a different dimensionality than that of our daily experience. *

Simplicial gravity

Led by the expectations and prejudices I have just described, I looked for some finitary model of gravity with an operational flavor. The only one I found – indeed the only discrete model I found at all – was one in which spacetime is represented as a simplicial complex.[7] In this so-called "Regge Calculus", which I encountered while a graduate student, curved spacetime is replaced by an assemblage of flat simplicial blocks, a simplex being the higher-dimensional analog of a triangle or tetrahedron. Such an assemblage is finitary in the sense that its geometry is fully determined by giving only a discrete list of real numbers: one length per simplex edge. Indeed Regge calculus is nothing but what engineers would call a "finite element" description of spacetime. The flat simplexes approximate a curving manifold in just the same way that a geodesic dome approximates the surface of a sphere. (A finitary purist would complain that even a single real number already contains an infinite amount of information, but in any case the structure is discrete in the sense that there are only a finite number of simplexes in any bounded region of the complex.)

A description in terms of simplicial complexes also has an operational flavor. Imagine that what we call a spacetime point is merely an ideal limit of finer and finer experimental "determinations of location" (measurements). Since the actual measurements are imperfect, they will determine not individual points but "fuzzy" ones corresponding roughly to the topological concept of an open set. Now if you cover a manifold by a finite number of open sets, and if you keep track of which of these sets (or determinations) overlap with each other, then you get what is called the 'nerve' of the covering, and this nerve is a simplicial complex! Thus one could

* Since the time I have been talking about, my study of so-called topological geons has convinced me even more strongly that topology-change is an unavoidable feature of quantum gravity. Also further evidence has accumulated that this phenomenon at least stretches, and probably bursts through, the manifold framework. However what I have described here is meant only as a static sketch of my thinking at the time when the train of thought leading toward causal sets got underway. For clarity, I have tried to exclude from my account any supporting considerations that arose later.

view the use of simplicial complexes in Regge calculus as a kind of formalization of "what we actually do to produce spacetime by our measurements".

At that time, however, my adherence to the discrete camp was not complete. I still believed in a *potential* continuum existing as an ideal limit of the actual discrete, or more specifically as a limit of finer and finer position determinations. There would thus be no bound in principle to how precisely we could measure spacetime location, and therefore no unbreechable minimum length in nature. Accordingly, I thought of spacetime not as a single simplicial complex, but rather as a sequence of them, each refining the previous one, with the whole sequence converging in the limit to some topological space that need not be just a manifold. This framework (described almost in these terms in an old Dover reprint [8] by Pavel Alexandrov) promised, with its built-in possibility of different simplicial complexes on different scales, to give a precise meaning to the intuitive picture of topological fluctuations that I referred to before.

But this promise was one I was never able to redeem. The simplicial complex of Regge calculus seemed in the end to be a useful tool for approximating the continuum theory, but not, after all, a finitary structure that could serve as the physical underpinning of the continuum. In Regge calculus the dynamics (or "equation of motion") is given by varying the discrete analog of a lagrangian, but this analog ceased to be meaningful as soon as you took the complex to be more-than-slightly more general than a manifold. For this reason also, the dimension had to be chosen in advance, and therefore seemed to be no more explicable in the simplicial framework than in ordinary General Relativity. In addition there seemed to be no natural way to include Fermi fields in the framework, although gauge fields did find their natural place.

However all these difficulties were ones which you could imagine removing with greater ingenuity. The failing that carried the greatest weight in my mind was actually a technical problem. It turned out that the successive complexes did not really converge to their putative limit as the determinations defining them became finer and finer! To say precisely what this means would be too much of a mathematical aside, but the basic problem was this. You could start with a given continuous space (say a manifold) and cover it by a finite collection of open sets, each representing a fuzzy point, perhaps. By adding finer and finer open sets, you got a sequence of simplicial complexes which did indeed have a well-defined limit (the so-called "inverse limit"), but the simplexes of the complex were not all getting small when regarded as subsets of this limiting space. Thus there was convergence in a certain

mathematical sense, but not in the physical sense that successive approximations were corresponding to successively smaller scales of physical size.

The finitary topological space

The next step beyond the continuum was to discard the simplicial complex as the basic structure, and try instead the finite topological space. In grappling with the limit problem I just told you about, I had noticed that the nerve of a finite open covering does not actually encode all the information about how the sets of the covering overlap each other; it only keeps track of their mutual intersections. If you do keep all the overlap information, then you end up not with a simplicial complex, but with a different mathematical structure: a finite topological space. * Like the simplicial complex, this structure also carries information of a topological nature. (In fact it *is* literally a topology, as its name says.) But unlike before, the sequence of finite topological spaces corresponding to a sequence of finite open coverings *does* converge properly to the continuous space being approximated. This seemed to open the way for a true negation of the spacetime manifold, something whose discrete/combinatorial character was more thoroughgoing than that of the Regge calculus had proved to be. It thus seemed possible that, of all the structures defining a smooth manifold, it would turn out to be the topology itself that bears the greatest structural similarity to the underlying discrete reality. [9]

As the correspondence between open coverings and finite topologies was becoming clear to me, I also realized that a finite topological space has a very different, yet entirely equivalent, description as a partial ordering. This intriguing correspondence between topology and order struck me as deep in itself; but it also resonated in my mind with a tradition in physics and philosophy of wanting to base the analysis of spacetime structure on the properties of a quite distinct order-relation – the so-called causal order of "past" and "future", which in standard General Relativity tells you which events are able to signal to (or more generally to influence) which other events. Still, the order inhering in the finite topological space seemed to be very different from the order defining past and future. It had only a topological meaning but not (directly anyway) a causal one.

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* Actually, a strictly finite covering is appropriate only for a bounded region of spacetime. In the more general case of a locally finite covering, as would be needed for an infinitely extended region of spacetime, the structure you get is what might be called a "finit*ary*" topological space, one fulfilling a certain natural condition of *local* finiteness.

In fact the big problem with the finite topological space was that it seemed to lack the kind of information which would allow it to give rise to the continuum in all its aspects, not just in its topological aspect, but with its metrical (and therefore its causal) properties as well. Could it be, then, that everything is ultimately topological – that even the notions of length and time emerge somehow from more fundamental relations of adjacency and convergence? To address this question I tried (maybe not very hard) to make a theory of dynamical topology alone (i.e. I tried to find a quantum law of motion for the finite topological space), but I got nowhere. On the other hand, the only way I could see to put metrical information back in explicitly, was to use a certain correspondence that exists between finite topological spaces and simplicial complexes, and then appeal to Regge calculus. But this would put us back where we started, not having gotten essentially beyond the manifold concept.

The causal set

The way out of the impasse involved a conceptual jump in which the formal mathematical structure remained constant, but its physical interpretation changed from a topological to a causal one. Although, unfortunately, I can no longer recall the inner development of this jump in any detail, it is easy to see it in retrospect as a natural step.

I wrote above that the mathematical structure "finite topological space" is equivalent * to the structure "partially ordered set"; and in fact I normally thought about finite topologies in the latter language, since it seemed to provide the more convenient representation in most cases. I was thus already thinking of the fundamental discrete structure as an order (poset), but an order with a topological meaning. The essential realization then was that, although order interpreted as topology seemed to lack the metric information needed to describe gravity, the very same order reinterpreted as a causal relationship, did possess metric information in a quite straightforward sense.

Or rather, it possessed the necessary information if you abandoned operationalism and took the causal set to be a real substratum, existing independently of any experimental activity on our part. This meant accepting an actual minimum length in nature, and it made possible the key hypothesis that I referred to earlier of a

* Strictly speaking, this equivalence presupposes that distinct elements of the topological space possess distinct neighborhood systems (that the topology is what is called T_0).

direct proportionality between number and volume. By itself, the choice of a causal ordering as basic could have been more than justified in operational terms, but there is nothing in what we do when we measure spacetime volume that phenomenologically has the character of counting. To believe such a relationship, you have to accept that the elements of the causal set are real, and that volume measurements "count" them in much the same way that weighing a copper ingot "counts" the number of atoms it comprises.

[Indeed, weighing is not precisely the same as counting atoms; and I would not view as *exactly* true either half of the compound hypothesis that microscopic number shows up macroscopically as spacetime volume, and microscopic order shows up macroscopically as causality. Rather I like to think of these basic assumptions as analogous to the hypothesis in classical General Relativity that bodies move along spacetime geodesics. Viewed thusly, they would belong to what Taketani would call the "substantialistic" stage of understanding of quantum gravity, being essentially approximate relationships, which will be corrected and more fully understood only in the higher theoretical stage when an exact dynamics is available.]

The result of these changes was that now you no longer needed to add anything to the combinatorial data, in order to recover the metrical aspects of the continuum. Everything necessary for gravity was already present in the unadorned causal set, whose discreteness, moreover, was now intrinsic to the physical interpretation (thereby realizing Riemann's claim, that for a discrete manifold, metrical properties do not need to be added in by hand.) Potentially unified now, in terms of a single notion of microscopic order, were all the basic structures going into the General Relativistic conception of the continuum − its topology, its differentiable structure, its metric and its causal structure. In addition, the Lorentzian signature of the metric tensor − in other words, the fact that precisely one of the dimensions is timelike with all the other ones being spacelike − became inevitable, whereas in itself it appears mathematically unnatural and inconvenient.

As I just said, I am not sure exactly how these changes in my thinking took place, but one stimulus for the transition from topological order to causal order may have been my going to Chicago, where David Malament had just emphasized how much information the causal ordering actually contains in the continuum case. [10] This also may have highlighted for me what the continuum order *fails* to contain − namely information on spacetime volume − and may thereby have prepared a sudden realization that my discrete order could make up for this lack if reinterpreted in a causal way. My retreat from operationalism, on the other hand, was definitely

not sudden. It was part of a much slower evolution with causes partly inside and partly outside of physics proper.

The reasons for accepting the causal set as the right structure sound convincing to me now, but for a long time I remained in some doubt. In fact it took me several years to definitively give up the idea of order-as-topology and adopt the causal set alternative as the one I had been searching for. Whether the resulting hypothesis is true, can of course be decided only after a lot of further work.

Other threads: fermions, geons, the sum-over-histories, ...

In essence this is the end of the story, but there remains at least the question that Dirac was said to have always asked when he woke up at the end of a seminar: "but what about the muon" – or in this case, what about fermions in general? Earlier I mentioned that one of the difficulties of the Regge calculus was that it did not appear to be able to accommodate fermi fields in a natural way, whereas fermions certainly exist in the real world. As far as I know, the situation is no better with causal sets, but in the meantime I have found out that I was wrong in thinking that fermions must occur at the fundamental level of any theory in which they occur at all. In principle they can all emerge at a higher level, as composite particles, or as objects derived in some less obvious manner from the fundamental structures. In fact you don't even need to go beyond pure gravity to get fermions since they can occur as topological excitations ("geons") in four dimensions [11], or more exotically, on the basis of the higher dimensional manifolds of Kaluza-Klein theory (Kaluza-Klein geons). This was a parallel development, and to some extent a hidden thread in the story I have been recounting.

Other hidden threads in the story concern the so-called sum-over-histories (or "path integral") interpretation of quantum mechanics in general [12], the conflict in my thinking between a more "algebraic/logical" and a more "geometrical/set theoretic" approach to the quantum gravity problem, and the question of whether some conceptual descendant of the pre-Relativistic notion of time will continue to play a role in the dynamical "law of motion" of quantum gravity. These threads intertwine with each other and with the fermion issue as well, but there is a limit to how tangled a history you can tell, or even begin to reconstruct in your own mind .....

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