Light, Links and Causal Sets★

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Abstract

After sketching a context in which to seek observable signals of spatio-temporal discreteness, I briefly review the status of the causal set program for quantum gravity, concluding with a simple model for the field produced by a moving charge in a background causal set.

1. How might discreteness show up?

Riemann had a nice phrase somewhere that talked about "the reality that underlies spacetime." ★ What we know of quantum gravity suggests that that reality is more in the nature of a discrete structure than a continuous one. If so then it's important to think about phenomena that could reveal this underlying discreteness. Several such possibilities come to mind, some of them present by definition, others suggested by analogy with the discreteness of ordinary material, and still others specific to one or another of the deep structures of spacetime that have been proposed by workers on quantum gravity.

Almost by definition, discreteness implies a "cutoff" in energy and wavelength. However (unless the idea of "large extra dimensions" turns out to be correct, cf. [1]) the expected Planckian scale of this effect places it beyond *direct* observation for the present. Moreover (and this is why I included the qualification "Almost" just above) the concept

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★ Of course he would have said "space" rather than "spacetime" of a cutoff takes on a more subtle meaning in the context of a discreteness that respects Lorentz invariance, like that of the causal set. One cannot naively translate such a granularity into the statement that an upper limit to frequency would exist in any given frame of reference. Such a claim would conflict with the fact, that, for example, a plane-wave solution of the massless wave equation can have any frequency at all, depending on which frame it is referred to.† This subtlety does not mean that a Lorentz-respecting cutoff must remain forever elusive, of course, only that it can be detected solely by effects whose nature is intrinsically frame-independent. For example, the notion of center of mass energy is invariant, and correspondingly, one can easily see in the context of (background) causal sets that scattering cross sections must fall to zero at Planckian energies. One could in principle accelerate particles to arbitrarily high energy (limited only by infrared cutoffs of cosmological origin), but if such "transplanckian" particles were to collide, the associated wave packets would simply "pass through" each other. They would not "see each other" because they would be supported on disjoint sets of causet elements.

Interesting as this "asymptotic freedom" effect is in principle, it seems far removed from observability in practice, and people have sought elsewhere for the phenomenology of spatio-temporal discreteness. By way of inspiration, one can call to mind some of the analogous phenomena that testify to the atomicity of ordinary matter. Within this ambit, the effect that has received the most attention is perhaps the possible breaking of Lorentz invariance. Here the condensed-matter analogs are the partial breaking of translational and rotational invariance by a crystal (resulting for example in Umklapp scattering processes and the Mössbauer effect) and the related high frequency modification of dispersion relations for waves traveling through the crystal. However, the observational and experimental evidence to date speaks against any such failure of Lorentz invariance, suggesting that we should limit our search to phenomenology consistent with this symmetry. Indeed, the vista from causal sets suggests that this negative verdict will persist indefinitely, because it seems to be impossible to design a causet that would single out any frame of reference without coming into contradiction with the basic hypothesis that "number

† Here I am of course identifying the physical symmetry group of Minkowski spacetime with the Poincaré transformations as normally defined. This ignores the possible extension of the concept of symmetry to a Hopf algebra that is not a group algebra, as essayed in "deformed special relativity" and non-commutative geometry more generally. [2]

= volume". (More specifically, it seems that the most uniform distribution of points in Minkowski spacetime $\mathbb{M}^4$ is given by a Poisson distribution; and one knows that Poisson distributions are frame-independent in a strong sense [3].)

But, not all matter is crystalline, and dispersion relations are not the only — or even the best — messengers from the realm of atoms. In gases and liquids, another whole family of phenomena springs from the irregular arrangement and chaotic motions of the constituent molecules. Some familiar instances of these "fluctuation phenomena" are the Brownian motion of microscopic particles suspended in a medium and the scattering and extinction of light propagating through air. By way of analogy people have asked whether particles moving freely through the vacuum might deviate from geodesic motion, and whether light from cosmologically distant sources might be attenuated or deviated or otherwise affected by a Planck-scale discreteness [4].

To fully answer these questions is not possible outside of a completed theory of "quantum gravity", but that need not stop us from seeking partial answers based on plausible models of an incomplete and/or partly phenomenological nature. Below I will briefly present one such model [5] that describes the propagation of light (more precisely, a massless scalar field) through a causal set as a process of "direct transmission from source to sink." Limited though it is, this model lets us appreciate that neither modified dispersion relations nor loss of coherence is a necessary consequence of discreteness. It will also allow one to estimate the size of some of the fluctuations in signal strength that do result from the discreteness. Before getting to that, however, I'd like to recall two or three more types of phenomena that have been suggested as potential "signals" of a quantum gravity founded on an atomistic deep structure of spacetime.

The first of these is the causal set counterpart of Brownian motion, but a similar effect could be expected in any Lorentz-symmetric discrete theory. Precisely because of the absence of a distinguished frame, a direct analog of the Wiener process is ruled out in the sense that fluctuations in the position cannot not be Markovian; but fluctuations of velocity or momentum pose no such difficulty. In fact, one can define a spacetime analog of the "Ornstein-Uhlenbeck" version of Brownian motion, in which the particle's worldline is differentiable but its tangent-vector is not, the acceleration being subject to a sort of Lorentz-invariant white noise. For a massive particle the resulting stochastic process [6] [7] [8] is characterized by a single phenomenological parameter, a diffusion constant with the dimensions of inverse proper time.[♭] For a massless particle, a second free parameter enters (a drift coefficient in energy), while for a photon (massless with polarization), still other parameters become relevant [8]. Since we have no estimate of any of these phenomenological parameters from first principles, the best that one can do is to seek observational bounds on them. Such bounds have been deduced from the undistorted Planckian shape of the cosmic microwave background (CMB) spectrum [8] and from cosmological bounds on the kinetic energy carried by relic neutrinos [9].

The second type of phenomenon stemming from the discreteness of the causet is a potential failure of locality at sufficiently high energies. At the root of this possibility is an inner contradiction among discreteness, Lorentz invariance, and locality: any two can coexist but not all three together.[⋆] Some models of discrete spatio-temporal structure maintain an (approximate) locality at the expense of Lorentz invariance. But with causal sets, this option is difficult or impossible to implement (even if it were desirable), and the problem becomes how to recover at least the appearance of locality at sufficiently great length scales, where a continuum spacetime offers a good approximation to the underlying causet. Work on this problem has illustrated concretely how a coarse-grained locality on mesoscopic scales can emerge from microscopic interactions that are radically nonlocal, but it also suggests that the nonlocality might persist at length scales much greater than the fundamental discreteness scale (Planck scale) [10]. Were evidence of such nonlocality discovered, that would open up an entirely new phenomenological area for quantum gravity. However I don't know that anyone has proposed a clear experimental signal of nonlocality that one could easily look for.

A final phenomenon that deserves incidental mention here — concerning the cosmological constant Λ — differs from the others I've been referring to in that it pertains to full quantum gravity, as opposed to the more limited setting of the propagation of fields and

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♭ These are the dimensions if the diffusion is referred to velocity-space. If it is referred to momentum-space the dimensions are $mass^2/time$.

⋆ To get a feel for this contradiction, observe that two "infinitesimally nearby" point-events are "infinitesimally nearby" in all frames, but two points at a small but finite timelike distance in a given frame will be separated by arbitrarily great times in other frames.

particles on a fixed, non-dynamical background. In conjunction with the quantum mechanical conjugacy between Λ and spacetime volume V, Poisson fluctuations in the "density of causet elements" imply fluctuations in Λ with an order of magnitude $1/\sqrt{V}$. This heuristic argument [11] led to the prediction, which later came true, that the contemporary value of Λ would be found to be comparable in magnitude (but not necessarily in sign) to the density of baryonic and other "matter". A simple model built on the above idea can be found in [12], but a fully consistent phenomenological theory of such fluctuations remains to be devised.

2. The causal set program

In the preceding introduction, I have sketchily reviewed some possible phenomenology corresponding to the hypothesis that a discrete structure supersedes the Lorentzian manifold at Planckian scales. Partly to provide background for those remarks (and for the remarks which follow), and partly in obedience to a request from the conference organizer, I will try in the present section to review — even more sketchily — the overall status of the causal set program itself[†]

At the root of this program is the recognition that, when it is combined with volume information, causal structure alone suffices to reproduce fully the geometry of spacetime. In a continuum — what Riemann called a "continuous manifold" — volume information is lacking. But what Riemann called a "discrete manifold" does carry an obvious volume-measure obtained simply by counting elements. In this case "less is more, as well as less". The kinematic meaning of the causet hypothesis, then, is that macroscopic spacetime M is an effective description of (an approximation to) a causal set C. Mathematically C is a partial order or "poset". Its defining order relation $\prec$ gives rise to the light-cone structure of spacetime, while its innate volume measure gives rise to the continuum volume element $\sqrt{-g}d^4x$. In brief "Number = Volume", and thus, "Order + Number = Geometry".

[†] For some general references to this program see [13].

A more precise notion of the causet-spacetime correspondence has been based on the concept of "faithful embedding". [♭] One says that $f : C \to M$ is faithful if the point-set $f[C]$ "could have been obtained by Poisson-sprinkling points into M at unit density". It it this notion of correspondence that is employed in the phenomenological applications discussed in the previous and following sections.

What would it take to turn the causet hypothesis into a theory of quantum gravity? For a physicist, it seems natural to follow the scheme advocated by Taketani that views a theory as comprised of three components or "stages", which one may call kinematical, dynamical and phenomenological. For causets, kinematics refers first of all to the *kind* of structure one has, i.e. to the definition of a causet as a locally finite partial order, and more broadly to the development of the mathematical ideas proper to causets (the mathematics that would play the same role for causet theory that differential geometry plays for general relativity). Of special importance is the development of a "dictionary" allowing one to translate back and forth between order theoretic and geometric concepts. By dynamics I mean what one might describe as the "equations of motion" of the causet. But in addition to this (which pertains to full quantum gravity) there are also questions analogous to those belonging to the theory of quantum fields on a background spacetime. With the exception of the above remarks on Λ, all the various effects mooted above fit into this second category. The word phenomenology, finally, needs no definition and no separate discussion in this section, since the other two sections of the paper are devoted to it. [⋆]

Kinematics

At the level of kinematics, much is known, although some important questions still remain open. Some of the necessary concepts can be borrowed from the mathematical

[♭] With Lorentzian geometry, the concept of "closeness" between two spacetimes M or between a spacetime and causet C turns out to be much more delicate than one would have been led to believe by the example of Euclidean signature.

[⋆] A "phenomenological" application of causets not mentioned in those sections concerns black hole entropy. Unfortunately the available accounts are rather outdated by now [14].

discipline of combinatorics (for example the "height of an interval"), others can be copied from the "global causal analysis" of continuum general relativity, while still others (like Myrheim's "ordering fraction", or "coarse-graining" by random selection of a suborder) seem not to have been studied before. As things stand now, one has a growing dictionary allowing one to "read out" geometrical and topological information from order information. [†] Corresponding to the timelike distance between two points of spacetime is the length of the longest (not shortest) chain between two causet elements. [♭] (As its name suggests, a *chain* in a partial order is a sequence of causet elements, $e_1 \prec e_2 \prec e_3, \cdots \prec e_n$, each an "ancestor" of the next one in the sequence.) Corresponding to spacetime dimensionality we have a number of different estimators. Here I will just mention three. For a given interval I we have first of all the Myrheim-Meyer dimension, which seems to be the most accurate so far [15]. It is a certain function of the ordering fraction $R/\binom{N}{2}$, R being the number of related pairs of elements of I and $N = |I|$ being simply the number of elements of I. (An *interval* in a poset is the set of elements causally between two given elements a and b: $I = \{x \in C \,|\, a \prec x \prec b\}$.) A second estimator, the "midpoint scaling dimension" of an interval is analogous to a Hausdorff or fractal dimension. A third estimator suggested by Eitan Bachmat [16], can be applied directly to the causet as a whole, not just to intervals within it. It also is a kind of scaling dimension, based in this case on how the "height" of C (the length of the longest chain it contains) scales under coarse-graining. All these estimators (for timelike distance and dimension) have been tested in a range of examples, but rigorous proofs of their validity exist only in special cases, like that of Minkowski spacetime.

In principle, if one knows all the distance relations in a Lorentzian manifold C, one know everything there is to know about M, including its topology. Thus, one can expect that something similar would be true of a causet well approximated by M: one could deduce the topology approximately by counting longest chains within C. However, no

[†] Of course this type of translation only makes sense when the causet in question is in fact well approximated by some Lorentzian manifold.

[♭] This estimator turns out to be remarkably accurate. There is some evidence that its fluctuations grow only *logarithmically* with the number of elements, in dimensions $3 + 1$ and higher!

technique for doing this is known. Nonetheless, there has been recent progress in recovering algebraic-topological information directly, more specifically in assigning homology groups to "slices" (the discrete analogs of spacelike hypersurfaces) [17] [18]. There also exist proposed definitions for homology groups pertaining to C as a whole (as opposed to slices within it), but they remain untested.

Probably the most exciting of the recent kinematical developments, is the prospect of an estimator for the Ricci scalar R. Were this prospect to pan out, it would furnish a causal set counterpart to the Lagrangian of continuum gravity, and therefore a direct route to setting up a "path integral" for causet dynamics. The estimator in question grew out of the attempt, referred to earlier, to formulate an approximately local analog of the scalar wave equation for a field ϕ defined on a causet [10]. The resulting "discrete D'Alembertian operator" $\Box$ does yield such an analog, and it can indeed be used to study propagation of waves in a causet. But once one has it, $\Box$ can also be combined with the "Synge world function" σ to obtain a discrete analog of R, by forming (purely at a kinematical level) the expression $\Box\,\Box\,\sigma$. In the continuum, this combination is known to reduce to (a multiple of) R in the coincidence limit. The question now is whether it will do likewise in the discrete case.

A final kinematical result that bears mention here relates back to the question of Lorentz breaking and the causet-spacetime correspondence. I asserted earlier that discreteness of the causet type respects the full symmetry group of $\mathbb{M}^4$, including the Lorentz boosts. This claim rests on the fact that the Poisson "sprinkling" process is Lorentz invariant (which invariance rests in turn on the fact that 4-volume is invariant). What this really signifies is that, given a large number of sprinklings into $\mathbb{M}^4$, one could not deduce a distinguished reference frame from the entire collection. Logically this still leaves open the possibility that even though the *process* of sprinkling is invariant an *individual* sprinkling would still determine a frame, so that invariance could return only in some sort of "sum over causets". Were things to turn out this way, the project of studying wave or particle propagation in a *single* causet, taken as typical, would be misguided. In effect, one would have to go all the way to full quantum gravity in order to analyze the effects of discreteness on propagation. Fortunately, one can prove that (with probability 1) even a single sprinkling fails to determine a frame [3]. Accordingly, it makes sense, as a first effort, to study propagation in a single sprinkled causet, and this is what we will do in Section 3.

Dynamics

In relation to causal sets, the word dynamics refers most fundamentally to the laws or principles that would govern the structure of the causal set itself, the quantum causet replacement for the Einstein equation. But dynamics can also be taken in a more limited sense, to refer to any process of evolution of fields, particles, or other forms of "matter" within a fixed causal set, treated as a non-dynamical background or "arena" (in other words a discrete analog of quantum field theory in curved spacetime). As discussed in the introduction, most of our phenomenological insights so far — with the exception of those relating to Λ — stem from the study of dynamics in this second sense.

I have already spoken about work on the propagation of classical particles and of classical fields through a causet, and in the next section I will summarize some results on the special case of the latter where the field is a massless scalar. However, first I would like to dwell briefly on the dynamics of the causet itself.

In addition, let me call attention here to some results on the propagation of *quantum* particles in a causet that relate to the topic of the next section [5] and that are described more fully in the contribution of Steven Johnston to this volume [19]. In a certain sense, the Green function G of the next section already has the character of a "path integral", not for fields but for particles. That is, it can be expressed as a sum over paths, each path contributing a single "field amplitude". The only thing is that these "paths" verge on triviality: each is a chain of just two linked elements of the causet. That it is nonetheless appropriate to regard such chains as discrete worldlines becomes clearer when one generalizes to the massive case. In the continuum, the massive Green function has support in the interior the light cone, not just on its surface, and one finds correspondingly that the "discrete worldlines" now comprise far more than two elements [19].

Let us proceed now to causet dynamics proper. Perhaps the first question one should ask in this connection is how such a dynamics could even be formulated, given that Hamiltonian evolution can get no foothold when time as well as space is discrete, and when there is no background manifold to support any sort of analog of a metric-field operator. What springs to mind, of course, is the path integral, but even this is problematic if one limits its role to furnishing an "evolution operator" between fixed boundaries. Rather, one must (or so it seems) replace the "path integral qua propagator" by the path integral qua decoherence functional or quantal measure. ⋆ When expressed in such measure-theoretic language, quantum theory resembles not so much a deterministic theory like classical mechanics as a classical probabilistic theory like that of of the random walk. It appears in fact as a generalization of the theory of stochastic processes that replaces the Kolmogorov sum-rule with a weaker sum-rule that allows for pairwise interference between alternatives [20].

In light of this resemblance, it appeared natural to seek a quantal "law of motion" for causal sets by first setting oneself the preliminary task of finding a classically stochastic theory of causet structure or development. In fact what emerged were not "laws of structure" as such, but rather a dynamics based on *growth* or accretion, in other words a birth process. The resulting Markov processes are known as classical sequential growth (CSG) models, and they follow almost uniquely from two or three general principles, including in particular a principle of "Bell causality". (Aside from some exceptional solutions, all of these models belong to the so called "generalized percolation" family, and as such are closely related to what are called "random graph orders" in the mathematical literature [21].)

Although the CSG models were initially intended only as stepping stones to a full quantum dynamics — a dynamics of "*quantal* sequential growth" — one can ask to what extent any of them can produce causal sets that resemble spacetimes (ie that are well approximated by a Lorentzian manifold, preferably a spacetime that also satisfies the Einstein equations!) The answer seems to depend on how strong a resemblance one is looking for. On one hand there are good reasons to believe that fully manifold-like causets will never be produced by any of the CSG models (more precisely, will be produced with negligible probability). On the other hand, for certain choices of the parameters, one obtains causets that resemble manifolds in important ways, e.g. deSitter space [22].

Thanks to this resemblance, the CSG models can bring to life possibilities inherent in the causal set hypothesis, that might be realized more fully (i.e. more realistically) in some

⋆ See [20] for definitions of the decoherence functional and the quantal measure, these being essentially equivalent objects.

future theory of *quantal* sequential growth. One such example concerns a "Boltzmann-Tolman universe" scenario that could solve some of the "large number riddles" of cosmology [23]. It turns out that for a large range of their parameters or "coupling constants", the CSG models lead to a qualitatively similar overall behavior of the growing causet C. It begins life as a single element (or as the empty set if you prefer to start there), expands rapidly to a large size, stabilizes, and after some time (depending on the parameter values chosen initially) collapses back to a single element (called a *post* in the combinatorial literature). There ensues a succession of cycles of expansion and contraction, separated by posts, each of which appears to its descendants as a new "big bang". In each successive cycle the initial expansion looks at first exactly the same, beginning with a "tree phase" in which each new element is born from a single "parent" chosen at random from the previously born elements of the growing tree.[†] The subsequent phases of the expansion do depend on the specific values of the parameters, but what is interesting is this, that the parameters governing the evolution change from cycle to cycle. As one cycle ends and the next begins, the effective parameters are transformed by the action of a certain "cosmic renormalization group" [24] that drives them toward the values characteristic of the particular CSG model known as *transitive percolation*, which contains a single free parameter p that, among other things, governs the size of maximum expansion. The value of this "renormalized" p decreases from cycle to cycle, and when it is much less than unity, C expands to a very large and homogeneous configuration. Overall the picture seems remarkably lifelike, given the limitations of classical growth. In such a scenario, the large numbers of cosmology, like the very great value of the spatial radius of curvature in natural units, would owe their size to the large age of the cosmos (measured in number of elements), rather than to any special "fine tuning".[♭]

The classical sequential growth models have also led to important progress of a conceptual nature by showing how the so called "problems of time" can be resolved in a way that applies equally well to the classical and quantum cases [25]. Without any appeal

[†] A causet is a *tree* iff each element has exactly one parent (or none in the case of the "root" element of the tree.)

[♭] The puzzle of why the radius of curvature is so much greater that the Planck length is often called the "flatness problem".

to "clocks" or other surrogates for coordinate systems (like asymptotic regions where an S-matrix can be defined), one has identified "observables" which first of all carry a clear physical meaning and second of all are complete in the sense that all other covariant (label independent) quantities can be formed from them in a well defined sense. Because this completeness theorem rests only lightly on the details of the CSG models, there is every reason to expect that it will go through unchanged in the case of quantal sequential growth (ie quantum gravity à la causet).

Of course this expectation can only be tested after we have a quantum dynamics to work with. How might one arrive at such a dynamics? The example of classical gravity suggests two very different but complementary ways to proceed, one based on general principles the other on imitation of the continuum theory.

The first approach was the one that brought the CSG models to light, the analogy here being with how the Einstein equation was (or might have been) deduced from the twin principles of general covariance and locality. As I stressed above, locality is not to be met with in a causal set, and covariance by itself (in the sense of labeling independence) did not seem to offer adequate guidance. But it turned out that a condition that came to be called "Bell causality" could take over where locality left off. The resulting CSG models are almost uniquely selected by these two principles, together with a so-called principle of "internal temporality". (Full details of the derivation can be found in [26].) To derive in the same manner a dynamics of "quantal sequential growth" has so far not been possible, however, because one lacks a satisfactory generalization of the condition of Bell causality to the quantum case.

The more general issue lurking here has nothing to do with quantum gravity as such, and raises questions that are worthy of study in their own right. Can one formulate the requirement of relativistic causality without invoking external agents like "observers"? Can one do so in the language of histories? And if so how? The answers, if we had them [27] [28], would help to remove the main obstruction blocking the first path to a theory of quantal sequential growth. (The quest for a suitable criterion of relativistic causality lies in the background of the work on quantum foundations reported on in Petros Wallden's article in this volume [29].)

An approach based on imitation of the continuum theory might seem to be more straghtforward than an approach based on first principles. Along this second way, one would try to discover a quantity in the causet that would go over to the scalar curvature in the continuum limit. Having found such an expression, one would then try to incorporate it into a discrete analog of the gravitational functional-integral. For a long time it was difficult to get started on this path because no discrete counterpart of the curvature was known. Now that we do have a candidate expression (based on the Synge function as described earlier), the next step would be to confirm that it really does have the Ricci scalar as its continuum limit.*

3. direct transmission along causal links

Earlier, I promised to return in this section to the model of propagation of a scalar field by direct transmission along causal links from source to sink. In my actual talk at the DICE conference, I devoted most of the time to this model, but since the details will appear soon in another place [5], I will limit myself here to a thumbnail sketch.

In the continuum, the field emitted by a specified source can be computed in two ways, either by solving the field equations with suitable boundary conditions or (for a free field) by folding the source in with a suitable Green function. Ordinarily one adopts the "thermodynamic" boundary condition of no incoming radiation and correspondingly employs the retarded Green function. What the model in question does is to adapt the second of these two methods to the causet.

Now for a massless scalar field ϕ in $\mathbb{M}^4$, G is particularly simple, being given (up to normalization) just by the delta-function on the future light cone, i.e. by $\delta(x^\mu x_\mu)$ in Cartesian coordinates ("Huygens' principle"). In this sense, ϕ propagates strictly between pairs of points joined by a null line segment. Now let C be a causet well approximated by (or by a portion of) $\mathbb{M}^4$. The nearest discrete analog of a null segment is perhaps a *link* of C, that is, a pair of elements $x \prec y$ related as parent and child, or in other words a pair

* A more "implicit" way to arrive at a counterpart of the Einstein-Hilbert action has been suggested recently by Bombelli and Sverdlov [30]. It is based on a kind of "dynamical selection of a local frame" that would allow one to deploy known estimators for certain components of the Ricci *tensor* in a causal set [31] [32].

such that the interval delimited by x and y is empty.[†] If, then, we define $G(x, y) = 1$ or 0 according as x is or is not a child of y, we obtain a natural analog of the continuum Green function; and it is perhaps not too surprising that the continuum limit of $G(x, y)$ coincides (up to normalization) with the retarded Green function in $\mathbb{M}^4$.

To complete the model, we can represent the source as simply (though not as realistically) as possible as a so called saturated chain or *path* within C,[♭] and we assume that the value of $\phi(x)$ due to the source is, modulo a prefactor of the source's charge, just the sum of $G(x, y)$ over all elements y of the source. Clearly this is just the number of source elements y linked to x. Finally we can imagine that the detector is active in a specified region of spacetime (it has a certain spatial volume and is turned on for a certain time) and just outputs the integral of the field over the region. Translated into causet terms, this just means (because "volume = number") that the signal is the *sum* of $\phi(x)$ over those $x \in C$ where the detector is active.

Given this setup, it becomes relatively easy to compute the expectation value of the detector's response to a given source, and also the leading correction thereto in powers of the Planck length. At leading order, one recovers exactly the continuum expression,[⋆] while the first correction involves an integral over the boundary of the source region and is completely negligible for any realistic detector and source. Thus as expected, there is no change to the dispersion relations, and no loss of coherence. (Nor is there any scattering, but that conclusion seems less certain to persist in the context of a more complete model that attributes degrees of freedom to ϕ itself, and then deduces the radiated field by solving the corresponding discrete field equations.)

[†] A slightly more general analog would be a pair $x \prec y$ such that the interval between x and y is a chain. In the (flat) continuum, this precisely characterizes a light ray.

[♭] A chain is saturated when no further elements can be interpolated between its endpoints. In other words it is "made exclusively of links".

[⋆] In particular, (the AC part of) the field of an oscillating charge falls off, as it should, like $1/R$ at large distances, not as $1/R^2$ as one might have concluded naively from the fact that the DC part goes like $1/R$. Interestingly, it is the Doppler shift in the number of contributing source elements that invalidates the naive reasoning.

There remains the question of the signal fluctuations induced by the discreteness of the underlying causet — the fact that the signal is actually a sum rather than an integral. The fluctuations in ϕ are less easy to compute than its expectation-value, and not every detail has been pinned down yet. However, it seems fairly clear that — unfortunately — the fluctuations are also too tiny to show up in observations of any realistic source using any realistic detector, even if the source is at cosmological distances from us.

In summary, the model is one in which the field spreads via a process of direct transmission along causal links. It seems remarkable that this law of propagation, which is probably the simplest one could have imagined had one thought directly in terms of the causal set, turns out to reproduce all the main features of the relationship between the field and its source in the continuum.

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