Ten Theses on Black Hole Entropy ⋆

Rafael D. Sorkin

Perimeter Institute, Waterloo ON, N2L 2Y5, Canada
and
Department of Physics, Syracuse University, Syracuse, NY 13244-1130, U.S.A.

address for email: rsorkin@perimeterinstitute.ca

Abstract

I present a viewpoint on black hole thermodynamics according to which the entropy: derives from horizon "degrees of freedom"; is finite because the deep structure of spacetime is discrete; is "objective" thanks to the distinguished coarse graining provided by the horizon; and obeys the second law of thermodynamics precisely because the effective dynamics of the exterior region is not unitary.

Probably few people doubt that the twin phenomena of black hole entropy and evaporation hold important clues to the nature of quantum spacetime, but the agreement pretty much ends there. Starting from the same evidence, different workers have drawn very different, and partly contradictory, lessons. On one hand, there is perhaps broad agreement that the finiteness of the entropy points to an element of discreteness in the deep structure of spacetime. On the other hand there is sharp disagreement over whether the thermal nature of the Hawking radiation betokens an essential failure of unitarity in quantum gravity or whether it is instead betraying the need for a radical revision of the spacetime framework, as contemplated for instance in the "holographic principle". These alternatives are not necessarily in contradiction, of course, but in practice, the wish to retain unitarity has been one of the strongest motivations for taking seriously the latter type of possibility. My own belief is that non-unitarity is probably inevitable in connection with gravity and that,

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rather than shunning this prospect, we ought to welcome it because it offers a straightforward way to understand why the law of entropy increase continues to hold in the presence of event horizons. This conclusion is part of an overall viewpoint on black hole entropy that I believe to be, if not true, then at least coherent. In the following lines, I will try to convey this viewpoint as succinctly as possible by expressing it as a series of "theses". In other places, I have written on most of these points in more detail. Here I try only to bring them together and to indicate briefly the reasoning behind them. Following each thesis, a handful of relevant references is indicated in a footnote.

THESIS *1. The most natural explanation of the area law is that S resides on the horizon, at ≈ 1 bit per Planck area.*

By the area law, I mean of course the fact that the entropy that enters into the "generalized second law" for systems including black holes is proportional to the total area of all horizons which are present. (More correctly, the entropy is, in $3+1$-dimensions, associated with a spacelike or null 3-surface Σ, and the area in question is that of the 2-surface in which Σ meets the horizon(s).) I remember that when I first learned of this law three decades ago, its most natural interpretation seemed to me to be that the horizon carried some kind of information with a density of approximately one bit per unit area, the area being rendered dimensionless by setting to unity the rationalized gravitational constant $8\pi G$. (The precise formula for the entropy in these units is $S = 2\pi A$, where the coefficient 2π can be interpreted geometrically as the circumference of the unit circle in the Wick rotated time direction.) [†] In a simple-minded model, one might picture the associated horizon degrees of freedom as plaquettes of horizon-surface on which tiny '1's and '0's are engraved, leading to a total number of configurations of the order of $N \sim 2^A$, whose logarithm yields an entropy of $S = \log N \sim A \log 2$. No one would take such a picture literally, but it can serve to indicate the kind of thing one is looking for, and this brings me to my second thesis. [♭]

† These formulas also take Boltzmann's constant to be unity, of course.

♭ Some references for the first thesis are 35, 5, 41, 19, 45, 51, 1, 10, 18, 50.

Thesis *2. What these bits of information represent depends on the deep structure of spacetime, most naturally conceived of as discrete.*

I don't believe that this thesis requires much elaboration, but some reference to an analogous situation in condensed matter physics might be suggestive, namely a dilute gas at high temperature. In that case one knows that the entropy is in effect just counting the total number of constituents (molecules) of the gas. Indeed S is precisely this number modulo a logarithmic pre-factor that depends on the temperature, the pressure, the molecular mass, and $\hbar$. The deep structure in this case is just the atomic structure of matter, and the finiteness of the number of atoms is then what allows the entropy itself to be finite. One might expect something similar for the black hole, where however the "atoms" would be constituents of spacetime rather than constituents of ponderable matter. The horizon then could also have "constituents" which could play the role of "molecules" of the gas. (See [12] for how such an explanation might play out in causal set theory.)

It's worth remarking, however, that although the *ultimate* degrees of freedom responsible for the entropy of a gas are discrete, that does not necessarily prevent one from giving an approximate continuum account of its entropy in terms of effective fluid degrees of freedom like the local mass density. (I don't know whether anyone has tried to do this, but if not then I think it would make an interesting problem. See [44] for some preliminary results in that direction.) Similarly, it might be [35] [5] that fluctuations in horizon shape could be taken to be the main source of the entropy of a black hole. (See also the comments on entanglement entropy below.) *

Thesis *3. Two main tasks need to be accomplished then:*

- *Identify and count the "bits"*
- *Explain why S increases*

If we want to explain black hole thermodynamics in terms of some "informational bits" residing on the horizon, we have to identify their material basis and then use this knowledge to figure out how to count them. It is equally important, however, that we explain why the total entropy — that of the horizons plus that of gravity and matter in the exterior region — continues to respect the second law. Although this second task is

* Some references for the second thesis are 6, 41, 45, 44, 12, 49, 35, 31, 22.

often given short shrift, it is in my opinion the decisive one; for no entropy merits the name unless it can be shown to increase with time. †

THESIS *4. The idea that the degrees of freedom are inside the black hole is wrong*

In contradiction to thesis 1 above, it is sometimes maintained that the entropy of a black hole refers to degrees of freedom of the interior region, rather than its boundary, which by definition is the horizon. If this were so, however, it is difficult to see how one could ever hope to account for the generalized second law, since the familiar statistical mechanical derivations of entropy increase presuppose conditions which are very different from what one expects to find inside a black hole.

To start with, the form of the "first law" for stationary black holes is that appropriate to a subsystem in internal equilibrium, whose gross features can be characterized by a small number of thermodynamic parameters (for a Kerr black hole, just the energy and angular momentum). The entropy then counts the number of microstates that contribute to the given macroscopic equilibrium state. Now, such a characterization is completely appropriate to the black hole as viewed from the outside, but it is obviously a very poor match to the interior region, which is neither in equilibrium (since it is collapsing rapidly) nor described by a unique macroscopic state. Similarly, derivations of entropy increase usually appeal to some form of ergodicity (at least to justify counting all of the compatible microstates in the expression for the entropy), whereas conditions in the interior are no more ergodic than they are stationary. Rather, if the classical regime is any guide, the evolution has much more of a one-way than an ergodic nature. And the familiar derivations also take for granted that differences in the microstate of the subsystem will eventually make themselves felt in the environment, so that the *combined* system of subsystem plus environment can explore the entire "phase space" available to it. (For example, the loss of "phase space volume" associated with a decrease of the subsystem's energy has to be compensated by an increase in the number of microstates traversed by the environment).

In this connection, they also assume that the coupling between subsystem and environment is weak, so that the entropy will be approximately additive. Once again, the contrast with the black hole case could hardly be greater, since there the *outside* $\rightarrow$ *inside*

† Some references for the third thesis are 37, 5, 41, 45.

coupling is very strong, while the reciprocal coupling, *inside* $\rightarrow$ *outside*, is non-existent classically (and perhaps exponentially small or zero in the quantum theory). In the absence of such a coupling, it is hard to see how *anything* referring to the interior could play a role in the generalized second law, since the latter pertains entirely to features visible in the exterior region.

Path integral computations of the entropy also indicate, in their own way, that the source of the entropy should not be sought beyond the horizon. In fact, the Euclidean-signature black hole metrics that enter into these calculations do not even possess regions corresponding (under Wick rotation of the time direction) to the interior of the Lorentzian black hole.[♭]

THESIS 5. *The dissipative nature of the horizon precludes an appeal to unitarity in proving that the entropy increases.*

If we accept that S is a surface effect (not an attribute of the black hole interior), then the objections I have just recited lose much of their force, since the horizon *is* in equilibrium (for a stationary black hole) and its evolution is not so obviously non-ergodic. Moreover it is only "marginally decoupled" from the exterior classically, and so might in fact be weakly coupled in a suitable sense in the quantum theory. A traditional proof of the second law utilizing unitarity might then be logically possible. Unitarity implies "conservation of microstates", and this could lead to entropy increase much as it does for thermodynamic systems in flat spacetime (this being essentially the perspective adhered to by most string theorists.) In trying to implement such a derivation, one would have to come up with an appropriate notion of time, to which the unitary evolution could be referred; and it is far from clear how to do so in most black hole spacetimes. However, it seems to me that a more serious problem is posed by the highly dissipative nature of the horizon.

The horizon's dissipative character is seen first of all in the "no hair" theorems, according to which deviations from equilibrium (i.e. from the Kerr metric) die out rapidly on a time scale set by the diameter of black hole. In particular, this is true of the so called

[♭] Some references for the fourth thesis are 45, 5, 41, 19, 18, 34, 2, 52, 33, 14, 54, 22.

quasi-normal modes that describe the "ring down" of a newly formed black hole, as confirmed in numerical simulations of that process. (Indeed, the decay is so rapid that a bell does not make a very good metaphor; a marshmallow might be a better comparison.) One also recognizes dissipation in the horizon's effective "viscosity" and "electrical resistivity" that enter into formulas like those in [11].

Of course, one could imagine that this macroscopic relaxation to equilibrium was balanced by the excitation of microscopic degrees of freedom on the horizon that are invisible in the continuum picture, in such a manner that the overall dynamics (of horizon + exterior) remained unitary. The trouble with this idea is that semiclassical calculations of Hawking radiation portray the radiated quanta as strongly correlated with modes that fall into the singularity deep within the black hole. Unless something could *remove* these correlations, no amount of subtle, non-semiclassical correlations among the radiated quanta (or between the latter and any horizon degrees of freedom) could restore unitarity to the exterior region [45]. To those who dislike it, this conclusion is known as "the black hole information ⋆ paradox", but my own feeling is that, far from being paradoxical: †

Thesis *6. This non-unitarity is to be welcomed.*

In a semiclassical treatment, the breakdown of unitarity arises from tracing out the degrees of freedom inside the black hole; that is, it arises from a kind of *coarse-graining*. But coarse-graining is exactly what one needs to prove entropy increase. Indeed, all proofs of the second law that I know of invoke coarse-graining at some stage; they are forced to do so because evolution which is strictly unitary cannot change the statistical entropy. In the black hole situation, we are fortunate to have available an objectively determined way to coarse-grain, which therefore provides us with an objective definition of entropy.

⋆ The question is sometimes asked, If information is lost in black hole evaporation, then where does it go? To my mind, this way of putting things accords too much independent reality to the idea of information, which is not really a "substance" and does not really have a location in general. Perhaps the notion of information simply fails under the conditions one finds deep inside the black hole (i.e. in the vicinity of what classically would have been the singularity), where spacetime itself probably no longer makes sense and is superseded by some deeper, discrete structure.

† Some references for the fifth thesis are 33, 36, 38, 45, 39, 46, 33, 11, 15, 20, 30, 29.

If anything, this is a big improvement over the relatively *ad hoc* types of coarse-graining that one is usually forced to employ.

What is equally important, the one-way character of the horizon affords a relatively direct proof of entropy increase for the coarse-grained dynamics. Thus the non-unitarity not only gives us a natural definition of black hole entropy, it lets us see why the generalized second law must hold when entropy is defined in this way. Actually, instead of "proof", I should have said something like "proof scheme", to be filled in once the full theory of quantum gravity is available. This "scheme" relies on the assumption that the one-way character of the horizon persists in full quantum gravity, in the sense that the exterior region (including the horizon) evolves autonomously. If one adds the further assumptions that energy is conserved in the full theory and that the momentary state of the exterior region can still be defined by an effective density matrix (at least approximately), then one can derive an inequality that entails the non-decreasing character of the total entropy [37]. [♭]

THESIS 7. *Restricting to the semiclassical case we prove that the free energy always decreases.*

The proof scheme just sketched could only come to fruition in a theory of full quantum gravity. However, in the context of the semiclassical Einstein equation, it yields the more rigorously defined statement that the free energy, $F = E - ST$, on a hypersurface Σ decreases monotonically as Σ moves forward in time [45]. Here, E is the energy of exterior matter fields, S their entropy, and T the temperature of a quasi-stationary black hole. [⋆] In other words, one is doing quantum field theory in curved spacetime and in effect treating the black hole as a "reservoir" for the external matter. When combined with the assumption that the black hole adjusts its mass to keep the total energy constant, this

[♭] Some references for the sixth thesis are 28, 21, 7, 37, 5.

[⋆] For simplicity, I'm assuming a non-rotating, uncharged black hole. The energy E is defined with respect to the time-translation Killing vector of the black hole spacetime, normalized at infinity.

yields immediately the "generalized second law" for the combined system of black hole plus matter, in the form of the inequality:

$$dS_{outside} \geq d\langle E_{outside}\rangle / T_{BH} = -dS_{BH} \ .$$

This results covers almost all gedanken experiments people have "performed" to verify the generalized second law, e.g. those in which one lowers or drops "boxes" through the horizon. [†]

THESIS *8. The entropy would be infinite without a cutoff.*

If the above account is correct, then the entropy S_{total} of the exterior region, whose monotone increase is the content of the generalized second law, is that derived by coarse-graining away the black hole interior. This entropy must include both the entropy S_{matter} normally attributed to exterior matter [♭] and the entropy S_{BH} normally attributed to the black hole. Indeed, the latter must turn out to be (to a good approximation) $2\pi A$, according to the area law.

Now one contribution to S_{total} comes from the quantum fields inhabiting the spacetime (in which one can presumably include the "gravitons"), and an important component of this contribution is the "entanglement entropy" belonging to correlations between field values inside and outside the horizon. This includes first of all the gray-body entropy carried by the quanta of the Hawking radiation, which are entangled with modes that propagate toward the singularity, as we have already discussed. But it also includes a contribution from near-horizon modes that, even in their vacuum state, exhibit cross-horizon correlations. (Indeed, such a contribution was also present in the entropy $S_{outside}$ discussed in conjunction with thesis 7, but there it dropped out of consideration because only *changes* in S were in question. Notice that this near-horizon entropy may be thought of as belonging to the "thermal atmosphere" of the black hole.)

[†] Some references for the seventh thesis are 33, 36, 38, 40, 43, 26, 27, 45, 37, 24, 47, 4, 56, 13, 3, 53.

[♭] The definition of S_{matter} may involve a further coarse-graining unrelated to the presence of a horizon.

Now, in a semiclassical treatment, the entanglement entropy is easily shown to be infinite, thanks to the divergent number of near-horizon modes. If, then, S_{BH} is actually finite, this can only be due to some cutoff on the modes, or to some other type of failure of the semiclassical picture. If one introduces a cutoff at length scale l then the entanglement entropy in the leading approximation takes the form $S \sim A/l^2$, a formula which once again points to the Planck length as the fundamental discreteness scale, and confirms that, within the semiclassical framework, there is no escape from the need for a cutoff.

On the other hand, the semiclassical treatment ignores the "back reaction" of field fluctuations on the metric, and in particular on the horizon itself. There are indications [44] that this back reaction induces fluctuations in horizon shape on scales well above Planckian, and this might provide a loophole in the argument that a cutoff is necessary. Unfortunately, this potential loophole is difficult to assess since it brings us into the realm of quantum gravity proper. However, it would seem that the horizon fluctuations, even if they suppressed the entanglement entropy at short wavelengths, would only replace it with another infinite contribution, this time the infinite "geometrical" entropy of the shape fluctuations themselves.[⋆] Once again, one would need something like an underlying spacetime discreteness at around the Planck scale in order to avoid a divergent entropy.[†]

THESIS *9. To understand the generalized second law requires a spacetime approach, not a canonical one.*

My ninth thesis concerns not so much what the statistical mechanics of black holes can teach us about the micro-structure of spacetime, but rather what it can teach us about the status in quantum gravity of spacetime itself (of four-dimensionality as opposed to three). Specifically, I am claiming that an approach like canonical quantum gravity, which formulates its dynamics in terms of data on a purely spatial 3-manifold, cannot do justice to black hole thermodynamics, and in particular to the generalized second law.

⋆ In fact, it's tempting to believe that this geometrical entropy (suitably cut off at around l_{Planck}) could be viewed in an effective continuum description as the main source of the horizon entropy, in accord with the geometrical nature of the area law itself.

† Some references for the eighth thesis are 35, 5, 39, 42, 44, 48, 23, 4, 25.

Consider for example the proof of this law that was adumbrated in the discussion of thesis 6. For it to go through, we must be able to associate an effective density matrix $\widehat{\rho}$ to the portion of the hypersurface Σ that lies outside the horizon, and this in turn requires that one be able to *locate* the horizon. Can one do so if one has access only to data on a spatial slice, but not to the spacetime that contains it (or at least to a sufficiently great portion of that spacetime)? To me, the task looks hopeless.[♭] In contrast, a spacetime ("path integral") formulation is, relatively speaking, free from this difficulty (as rigorously illustrated in a somewhat different context in [8] and [9]).

The difficulty with locating the horizon given only hypersurface data shows up even more dramatically in connection with the analysis of claimed giant fluctuations in the entropy induced by quantum triggers to gravitational collapse. Here, the very presence or absence of a massive black hole depends on a future quantum event that, apparently, has no way even to "register" on the hypersurface in question. (The gedankenexperiment is described more fully in [46].)[⋆]

THESIS *10. Summary*

This last thesis is not an independent claim, but merely a tying together of the previous theses. If they are valid, then, taken together, they speak in favor of these conclusions: the inner basis of spacetime has a discrete structure; the effective degrees of freedom of the horizon, if we can identify them, will provide a clue to the nature of this discrete structure; a theory of quantum gravity built on this basis will necessarily be expressed in a language much closer to that of "histories" or "path integral" formulations of quantum mechanics than to the purely spatial ("$3+1$") language of canonical quantization.[†]

♭ Even for the apparent horizon, the difficulties would be similar, as illustrated by controversies over the interpretation of numerical simulations claimed to exhibit the formation of naked singularities.

⋆ Some references for the ninth thesis are 41, 46, 16, 8, 9, 43, 17, 32, 55, .

† A further conclusion, for which however I have offered no evidence here, is that black holes can also teach us something about statistical mechanics itself, specifically that the formula $-\operatorname{Tr}\widehat{\rho}\log\widehat{\rho}$ is singled out as the "correct" one for the Gibbs entropy [46].

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