# Quantum Gravity Phenomenology, Lorentz Invariance and Discreteness

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**Abstract**

Contrary to what is often stated, a fundamental spacetime discreteness need not contradict Lorentz invariance. A causal set's discreteness is in fact locally Lorentz invariant, and we recall the reasons why. For illustration, we introduce a phenomenological model of massive particles propagating in a Minkowski spacetime which arises from an underlying causal set. The particles undergo a Lorentz invariant diffusion in phase space, and we speculate on whether this could have any bearing on the origin of high energy cosmic rays.

In discrete approaches to quantum gravity, the fundamental description of spacetime is not taken to be a manifold, but some discrete structure to which the manifold is only an approximation. The scale of this discreteness is usually assumed to be Planckian. It is often asserted any such theory must violate local Lorentz invariance (LLI) and a new area of research – LLI violating phenomenological effects of quantum gravity – has grown up around this idea. The purpose of this letter is to emphasize that causal set theory [1] respects LLI and to open a new phenomenological window on this approach to quantum gravity.

What does it mean to say that a discrete theory respects Lorentz invariance? It is difficult to give a precise answer, but intuitively the import is clear. Whenever a continuum is a good approximation to the underlying structure (and assuming specifically that the approximating continuum is a Lorentzian manifold $M$), *the underlying*

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*discreteness must not, in and of itself, suffice to distinguish a local Lorentz frame at any point of $M$.* In consequence, no phenomenological theory in $M$ derived from such a scheme can involve a local (or global) Lorentz frame either.[1]

Of course the above presupposes an answer to the question: "How is the approximating continuum related to the discrete entity that underlies it?". Whether or not a particular discrete theory respects LLI cannot be settled until this question is answered in the context of that theory. Luckily, in causal set theory, there is a clear proposal for an answer, and we will show that LLI is indeed respected.

A causal set (causet for short) is a locally finite, partially ordered set (for reviews and motivation for causal set theory see [2] [3]). This is a set, $C$, endowed with a binary relation $\prec$ such that elements of the set satisfy the conditions (i) $(x \prec y)$ and $(y \prec z) \implies (x \prec z)$ (transitivity), (ii) $x \not\prec x$ (acyclicity), and (iii) all *intervals* $\{x : y \prec x \prec z\}$ are finite. The relation $\prec$ gives rise, in the continuum limit, to the causal order on spacetime points, and the number of elements in a subcauset yields the volume of the corresponding region of the continuum in Planck units.

In the continuum context, the causal order and volume information suffice to specify a (causally reasonable) Lorentzian manifold [4, 5]. It is therefore reasonable to regard a Lorentzian manifold as an approximation to a causet if that causet is a discrete "sampling" of the continuum causal order with uniform density.[2] More specifically, we may say that a Lorentzian manifold $M$ approximates a causet $C$ ($M \approx C$) if $C$ could have arisen, with relatively high probability, via a random process of "sprinkling into $M$", at Planck density,[3] with the causet relations induced by the spacetime causal structure.[4]

A "sprinkling" is more properly described as a Poisson process. To see what this means, imagine dividing $M$, using any local coordinate systems, into small boxes of volume $V$, and then placing a "sprinkled point" independently into each box with probability $V/V_{fund}$, where $V_{fund}$ is the fundamental volume (of order the Planck volume). The Poisson process is the limit of this procedure as $V$ tends to zero. Because spacetime volume is an invariant, the limiting process is independent of the coordinate systems used to define the boxes. It follows that one cannot tell which frame was used to produce the sprinkling: the approximation is "equally good in all

[1] Naturally, there can be no question of a literal action of the entire Lorentz group on an individual discrete structure. Rather such a structure can only be Lorentz invariant in the same sense that a fluid is translation invariant. This should not detract from the fact that a fluid is indeed translation invariant in an important sense, whereas a crystalline solid is not.

[2] More generally, one would only require that some coarse-graining of the causet approximate $M$ in this sense; but we ignore this distinction here. For a somewhat different approach to defining a relation of closeness between a manifold and a causet see [6].

[3] By "Planck density", we really mean "density unity in fundamental units". One expects fundamental units to be equal in order of magnitude to Planck units.

[4] Taking the order relation of the causet to be induced strictly from that of the spacetime is only the simplest possibility. Other rules could be considered, but they would not affect anything in this paper.

frames."

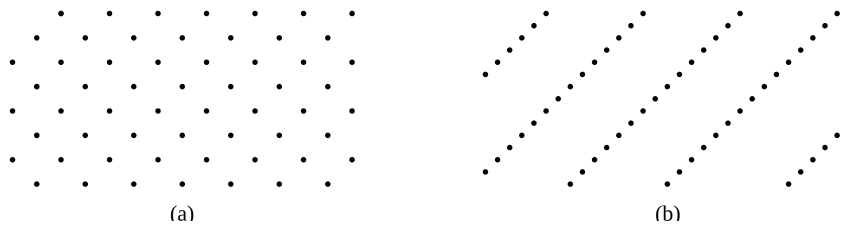


Figure 1: A regular lattice of spacetime points in two different Lorentz frames. Normal conventions for spacetime axes are used. While in (a) the lattice appears to have a regular density of elements, in the boosted frame shown in (b) the density of points is revealed not to be uniform.

Why is the randomness crucial? Let us take the example of 1+1 dimensional Minkowski space. One obvious way to try to discretize it is to choose a frame and use a "diamond lattice" with respect to that frame, i.e. the points with coordinates $(t,x) = (\epsilon(r+s), \epsilon(r-s))$, where $r$ and $s$ are integers and $\epsilon$ is some fixed length, as in figure (1a). In this frame the lattice appears to be a good approximation; all "nicely shaped" large regions have a similar density of elements. In a frame boosted at velocity $V$ in the positive direction, however, the elements are at $(\gamma\epsilon[(1-V)r + (1+V)s], \gamma\epsilon[(1-V)r - (1+V)s])$. Figure (1b) shows this lattice with $\gamma = 1.25$. Now it becomes clear that, if the boost is large enough, there will be "nice" big regions containing no elements at all, and others containing far too many elements. The approximation only looks good in the original frame, and so it breaks Lorentz invariance by preferring this frame. In light of this example, it seems likely that the same problem would affect any non-random discretization of a spacetime. Thus, for example, a Regge-type triangulation whose simplices look "fat" in one frame will look "long and skinny" in a relatively highly boosted frame.

We want to emphasise that not only is the *process* of sprinkling Lorentz invariant but so also are almost all of the individual causets that are generated. An objection that often comes up in this connection concerns the necessary occurrence of voids in any given Poisson sprinkling. While it is true that voids must occur, this does not cause a problem for Lorentz invariance (or any other problem that we know of). However, one may still feel uneasy about the voids, and some people seem to believe that they necessarily *would* break Lorentz invariance in some manner. To put such qualms in perspective, let us estimate the probability that there is at least one void of nuclear dimensions in the history of the observable universe since the Big Bang. More precisely, we will bound the probability that a sprinkling would leave empty any interval whose height is of the order of one Fermi. (An interval in spacetime is a "double light cone" or "Alexandrov neighborhood". We do not require the "axis" of the interval to be aligned with the cosmic rest frame. Hence our bound will apply to the probability of "finding a void in any frame".)

All the numbers in what follows are "of the order of". Consider as a model of the universe a portion $P$ of Minkowski spacetime, the size of the observable universe and defined by $0 \leq t \leq T$, $0 \leq x^i \leq T$, $i = 1,2,3$ in some frame. If $T$ is 13 billion years, the spacetime volume is $10^{240}$ in fundamental units. An interval of nuclear size has spacetime volume $10^{80}$. This means that the probability that any *particular* nuclear sized interval will be a void is $e^{-10^{80}}$. But we want the probability $q$ that at least one interval (any one) in $P$ will be void. This can't be calculated easily, because the intervals overlap and the probabilities for them to be void are not independent. However, we can put an upper bound on $q$ without much difficulty.

Let us fill $P$ with coordinate balls (with respect to the "defining frame" of $P$) of small enough radius that any "upright" interval of nuclear size is guaranteed to contain at least one complete ball. (Upright means that the top and bottom points have the same spatial coordinates.) A radius of one hundredth nuclear size will do, and we will have $10^{168}$ of these balls packed into $P$. The probability that at least one of them is void is $10^{168}e^{-10^{72}}$. These same balls will also suffice for intervals of nuclear volume that are slightly boosted or "tilted" from the upright. They will certainly do for all $\gamma$ factors less than or equal to $\gamma = 5/4$, that is for a region of the Lorentz group with volume of order 1. For each such cell of the Lorentz group we choose a set of coordinate balls in spacetime (in the corresponding frame). The relevant region of the Lorentz group is bounded by the maximum relevant boost in $P$, which corresponds to a $\gamma$ factor of $10^{42}$. (Any larger boost would produce a ball that could not fit into $P$. In fact the maximum $\gamma$ is actually smaller, since the boosted nucleus would meet the boundary of $P$ before the smaller ball would.) The number of cells needed to cover this region of the Lorentz group is $10^{84}$. The probability of getting any nuclear sized void is less than the probability that any one of the coordinate balls from any of the boosted sets will be void. This in turn is less than $10^{84} \times 10^{168} \times e^{-10^{72}}$, a number so tiny that the two prefactors have no impact whatsoever on its value.

Given that causal sets respect Lorentz invariance, what conclusions can be drawn? Most obviously, we predict that no violation of LLI will be observed at the phenomenological level, so that, if any of the experiments currently planned or under way did find such a violation, the causal set hypothesis would be disfavored. But this is only a negative prediction. Are there also positive signatures? Since the causal set hypothesis makes such a definite statement about the underlying structure for spacetime and how it is related to the continuum we actually experience, it is not difficult to devise concrete models predicting potentially observable effects of the underlying discreteness. We give one such model below which we call "swerving", after Lucretius [7]:

"The atoms must a little swerve at times – but only the least, lest we should seem to feign motions oblique, and fact refute us there."

In the continuum, massive particles travel on timelike geodesics. However, an underlying discreteness might induce small fluctuations in the particle's worldline, and the causal set picture naturally suggests models in which this effect would be Lorentz invariant. Though it might be too small to observe on everyday scales, such an effect might be detectable in sensitive laboratory experiments, or by astronomical observations if the particle were travelling over cosmic distances. Here is a model of this type.

Consider a hypothetical point-particle of mass $m$ moving through a causet $C$ derived by sprinkling Minkowski space $\mathbb{M}^4$. We will take its trajectory to be a *chain* of elements of $C$ (i.e. a totally ordered subset of $C$). No such chain can correspond perfectly to a straight line in $\mathbb{M}^4$. So, given an "initial segment" of the trajectory up to some element $e_n$ of $C$, how could its future continuation be determined? If we assume that the trajectory's past determines its future, but that only a certain proper time $\tau_f$ into the past of the trajectory is relevant, then we are led to the following Lorentz invariant rule as a particularly simple discrete analog of geodesic motion.

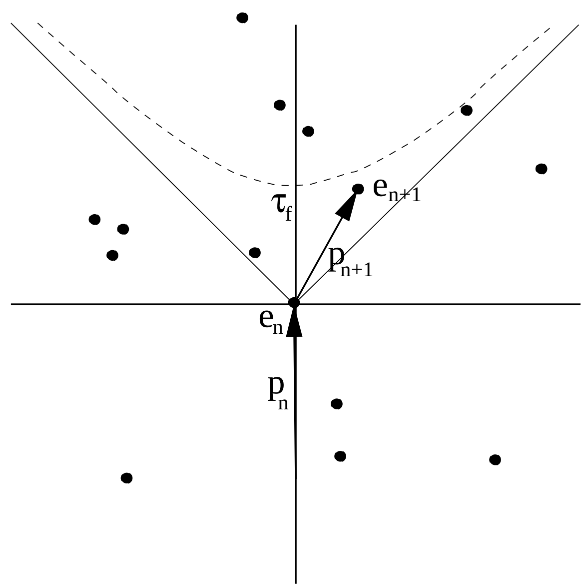


Figure 2: A portion of $1+1$ Minkowski space where the dots represent elements of a causet sprinkled into it. The trajectory of the particle has reached $e_n$ with momentum $p_n$ (the frame having been chosen so that the three-momentum is zero). The dotted line is the hyperbola of points a proper time $\tau_f$ to the future of $e_n$. The element within proper time $\tau_f$ of $e_n$ that best preserves the momentum is $e_{n+1}$. The ratio of $\tau_f^{-2}$ to the density of sprinkling has been exaggerated here to emphasize the momentum change in one step. In a more realistic model $\tau_f$ would be larger.

The future trajectory is constructed inductively, as illustrated in fig. 2. Starting from an element $e_n$ and a momentum $p_n$, the next element in the trajectory, $e_{n+1}$, must be chosen. For convenience we have drawn the diagram in the rest frame at $e_n$. The new momentum $p_{n+1}$ is defined to be proportional to the vector between $e_n$ and $e_{n+1}$. The selection of $e_{n+1}$ is made such that $e_{n+1}$ is in the causal future of $e_n$ and is within proper time $\tau_f$ of $e_n$ and so that $|p_{n+1} - p_n|$ is minimized. In the figure, this means that $e_{n+1}$ is within the future light cone of $e_n$, below the dotted hyperbola, and such that the vector from $e_n$ to $e_{n+1}$ is as close to the vertical as possible. (That there exists such an element in that region is guaranteed by the infinite spacetime volume of the region, the Poisson distribution and the local finiteness condition.) This realizes the ideas that the trajectory should be as close to a straight line as possible and that the dynamics should be approximately Markovian if the "forgetting time" $\tau_f$ is small. (There is ample room for it to be small and yet much bigger than the discreteness scale if the latter is Planckian.) The process is then repeated starting with $e_{n+1}$ and $p_{n+1}$.

Our model implies random fluctuations in the momentum of the particle. For any numbers $\delta_1 > \delta_2 > 0$ at stage $n$ there is a finite volume within proper time $\tau_f$ to the future of $e_n$ such that, if element $e_{n+1}$ were in this volume, the momentum change $|p_{n+1} - p_n|$ would lie between $\delta_1$ and $\delta_2$. The probability of this happening is the probability that this volume is not empty of sprinkled points, while the volume leading to a smaller momentum change is empty. Both these probabilities are given precisely by the Poisson distribution: the probability that a volume $U$ is empty is $e^{-\varrho U}$, where $\varrho$ is the density of the sprinkling.

This simple model can be criticised on many grounds: it treats the particle as if it were of zero size, it is deterministic rather than quantum, *etc.*. Most seriously, it could not possibly be fundamental, since the law of motion of the trajectory is not formulated in terms of the causet, but refers also to the approximating Minkowski spacetime.[5] We present it in the spirit of [8]: form a concrete model with testable consequences based on important aspects of the fundamental formalism and compare to observation.

Since the hypothesized Lucretius effect is supposed to occur on very small scales, it should be possible to approximate it over macroscopic distances by a diffusion equation (hydrodynamic limit). This is analogous to how the ordinary diffusion equation describes the long time behaviour of a random walk. In our case however, the diffusion is not in physical space $\mathbb{R}^3$, but on the phase space $\mathbb{H}^3 \times \mathbb{M}^4$, where $\mathbb{H}^3$ is the mass shell (Lobachevskii space), $\mathbb{M}^4$ is Minkowski spacetime, and the diffusion takes place in *proper time* $\tau$. The diffusion in spacetime is secondary and is driven by that in momentum space, in close analogy with the Ornstein-Uhlenbeck process.

Consider the scalar (not scalar density) probability distribution $\rho \equiv \rho(p^\nu, x^\mu; \tau)$ on $\mathbb{H}^3 \times \mathbb{M}^4$. It is a function of momentum, $p^\nu$, spacetime position, $x^\mu$, and proper time $\tau$. We write the full four-momentum as an argument with the understanding that on the mass shell there are only three independent components. With the condition that the process be Lorentz invariant, the following equation for $\rho$ can be derived by following the prescription set out in [9] for stochastic evolution on a manifold of states:

$$\frac{\partial \rho}{\partial \tau} = k \nabla_p^2 \rho - \frac{1}{mc^2} p^\mu \frac{\partial}{\partial x^\mu} \rho \tag{1}$$

where $\nabla_p^2$ is the Laplacian on $\mathbb{H}^3$, $m$ is the mass of the particle, and $k$ is a constant (that will depend on the parameters of the discrete process, such as the forgetting time $\tau_f$).

[5] This defect can be overcome fairly easily, however.

This equation defines a diffusion process in which the particle's proper time $\tau$ serves as time. It is the unique Markovian, Poincaré invariant, relativistically causal, diffusion law in vacuum that preserves $\rho \geq 0$.[6] For fixed mass, it has a single free parameter, the diffusion constant $k$. (In principle, the coefficient of the $p^\mu \frac{\partial}{\partial x^\mu}\rho$ term could be different, but that would mean that $p^\mu$ would not be the physical momentum.) Given these uniqueness properties, the equation should be insensitive to variations in the microscopic model that underlies it, so long as the latter is Lorentz invariant, causal and (approximately) Markovian. So although the above discrete swerve model might be wrong in detail, the macroscopic phenomenology of (1) transcends it.

If $\rho$ is a function of momentum alone and is initially (at $\tau = 0$) a delta function in momentum, then the (un-normalized) solution – adapted from the solution of the diffusion equation on $S^3$ [11] – is

$$\rho(p) = e^{-R^2/4\hat{k}\tau}(\hat{k}\tau)^{-\frac{3}{2}}e^{-\hat{k}\tau}\frac{R}{\sinh R} \tag{2}$$

where $\hat{k} = k/m^2c^2$, $R = \sinh^{-1}(p/mc)$, and $p \equiv |\mathbf{p}|$ is the norm of the three-momentum in the frame defined by the point in $\mathbb{H}^3$ at which the diffusion begins. $\sigma = mcR$ is then the geodesic distance from that point.

Equation (1) and its fundamental solution (2), expressed as they are in terms of proper time, are not well suited to comparison with experiment/observation, even though they exhibit the underlying Lorentz invariance very clearly. Instead, one needs a description of the same process with respect to cosmic or laboratory time. To this end, fix a preferred set of spacelike hypersurfaces $t \equiv x^0 =$ constant, and assume that the distribution on some initial hypersurface of the set is uniform in space (*i.e.* $\rho$ does not depend on $x^i$, where $i$ are spatial coordinates). The swerves, being spatially homogeneous and isotropic, will preserve this uniformity, and so if we assume that any additional frictional effects are also homogeneous and isotropic, the distribution will remain uniform. We require, under these conditions, an equation governing how the probability distribution evolves in cosmic time $t$.

Such an equation can be deduced from (1). (Details of the derivation and of the inhomogeneous case will appear elsewhere.) The result is:

$$\frac{\partial\rho}{\partial t} = k\nabla_p^2\left(\frac{\rho}{\sqrt{1+p^2/m^2}}\right) - \nabla_a(w^a\rho) \tag{3}$$

Here, the scalar function $\rho \equiv \rho(p^\mu; t)$ on $\mathbb{H}^3$, gives the momentum distribution, $\nabla_a$ is the covariant derivative on $\mathbb{H}^3$, and $p$ is the norm of the particle's three-momentum, $\mathbf{p}$ in the cosmic frame. (To make this equation plausible, notice that the factor

[6] The Ornstein-Uhlenbeck process for a particle diffusing in interaction with a relativistic fluid has been considered in [10]. In that case the rest frame of the fluid provides a preferred frame for the stochastic noise term driving the diffusion. The present process has no preferred frame and is fundamentally Lorentz invariant.

$\sqrt{1+p^2/m^2}$ is the boost factor $\gamma = dt/d\tau$.) The term involving the vector $w^a$ is a friction term added to represent the effect on the particle's momentum of, for example, the Hubble expansion and interactions with the CMBR (cosmic microwave background radiation). The specific form of $w$, which will in general be a function of the momentum, will depend on the type of friction involved.[7] For the cases mentioned, $w^a$ will have only a radial component (in the $p \equiv |\mathbf{p}|$ direction).

In the time-dependent case, (3) will probably have to be solved numerically. However, we can hope to analyze its equilibrium solutions analytically. For example, at large $p$, energy $E \sim p$, and if $w^p \sim -bE^n$ with $n \geq 1$, then the equilibrium solution will behave as $Ee^{-bE^n/n}$. If the dominant friction over any high-energy range is constant, *i.e.* if $w^p \sim dE/dt \sim$ constant, then the equilibrium distribution will be a power law in that range.

How might one observe diffusion of the above sort? Cosmological and astrophysical observations are the obvious places to look for consequences of a universal acceleration mechanism. But first, laboratory physics can put an upper bound on the diffusion constant $k$. Suppose the particles in question are protons. If $k$ were large enough, hydrogen gas would spontaneously heat up in a short time, and this has not been observed. In the laboratory regime, hydrogen is non-relativistic, so equation (3) can be approximated as

$$\frac{\partial \rho}{\partial t} = k \nabla^2 \rho \tag{4}$$

where $\nabla^2$ is now the standard Laplacian on $\mathbb{R}^3$. This is the standard diffusion equation and has the well known solution:

$$\rho = A(t) \exp(-\frac{p^2}{4kt}) \tag{5}$$

where $A(t)$ is a normalization factor. Usefully, this is also the form of the Maxwell distribution for a classical gas in thermal equilibrium:

$$\rho_{\text{Maxwell}} = A \exp(-\frac{p^2}{2mk_B T}) \tag{6}$$

where $m$ is the molecular mass, $T$ is the temperature, and $k_B$ is Boltzmann's constant. If a gas starts in a thermal state, it will therefore remain in a thermal state even if swerves are included. Moreover, the above two equations imply that the temperature will scale linearly with time, specifically:

$$\frac{dT}{dt} = \frac{2k}{mk_B} \tag{7}$$

Assuming, for the sake of argument, that a heating rate of a millionth of a degree per second would already have been detected in the laboratory, we obtain the approximate bound

$$k \leq 10^{-56} kg^2 m^2 s^{-3} \tag{8}$$

[7] In the case of violent momentum transfers, this would have to be generalized to a Boltzmann type collision term.

The maximum average energy gain due to swerves consistent with this rate of temperature gain can be obtained from the formula $\langle E \rangle = 3k_B T/2$:

$$\langle \Delta E \rangle / \Delta t \leq 4.3 \times 10^{-11} \mathrm{eVs}^{-1} \tag{9}$$

Now, let us turn to some possible astrophysical effects of swerves. One outstanding astronomical puzzle is the origin of high energy cosmic rays (see [12] for a recent review). Attention is often focused on the so-called "trans-GKZ" events, apparent detections of cosmic rays with energies above $5 \times 10^{19}$ eV. Such primaries, if they are protons, cannot have come from farther than about 20 Mpc (because they would have decayed due to photo-pion production with the CMB photons), but they have no obvious source in that distance range. But even for cosmic rays between $10^{15}$ eV and $10^{19}$ eV, there are only suggestions and no universally accepted acceleration mechanism for producing the observed energy distribution. The data (see e.g. fig 1 of [12]) seem to cry out for a universal cosmic acceleration mechanism that would inject protons, say, into the galaxy with a power law distribution of $E^{-a}$, where $2 < a < 3$, so that the observed variations in the power law and deviations from isotropy would be due to the dynamics of the protons in the galaxy. Could swerves provide such a cosmic mechanism?[8]

Swerves induce a "statistical acceleration" analogous to Fermi acceleration, and it is possible a priori that enough intergalactic hydrogen could be accelerated up to very high energies to explain the data. Unfortunately this degree of acceleration is inconsistent with the bound on $k$ already discussed. A "Lucretian" explanation of the cosmic ray data, assuming the primaries are protons, would require some protons to accelerate to $\sim 10^{20}$ eV from far lower energies on a timescale of the age of the universe. To produce a power law distribution in energy, a significant proportion of those protons reaching, say, $10^{18}$ eV would have to go on to double their energy and more. The rest frame of a proton with an energy of $10^{18}$ eV has a $\gamma$ factor of $10^9$ relative to the cosmic frame, so 10 billion years of cosmic time is only 10 years of proper time for such a proton. Doubling its energy in the cosmic frame would mean gaining around 250 MeV in its own frame. But from the inequality (9), in this frame the average energy gain in 10 years could be at most $1.4 \times 10^{-2}$ eV. (At these energies, we can trust the non-relativistic approximation.) Since the distribution of momentum is Gaussian, with such a low average energy gain, the probability of gaining 250 MeV is exponentially small. In other words, a proton has practically no chance of making it from $10^{18}$ eV to $2 \times 10^{18}$ eV in the age of the universe as a result of swerves. This calculation assumes that $k$ is roughly the same for an intergalactic hydrogen atom as it is for a proton, as it is for a $H_2$ molecule in a box of gas, but the argument is still valid even for a $k$ many orders of magnitude larger than has been assumed.

Proton swerves cannot explain trans-GKZ cosmic rays either (if indeed their apparent observation turns out to be correct). Swerves would accelerate some protons

[8] The idea that the rays might be the result of spontaneous acceleration, as a result of non-standard QFT, has been discussed in [13].

back up beyond $10^{20}$ eV after they entered the "GKZ sphere", $\sim 20$ Mpc from us; but this effect would not be significant. The argument for this is similar to that above. A proton with an energy of $10^{19}$ eV would reach us from the GKZ sphere in about 2 hours of proper time. This would not leave enough time for a non-negligible fraction of protons to, say, double their energy.

So the most direct application of the swerve idea to protons cannot explain the origin of high energy cosmic rays. However, more complicated scenarios can be considered. For example, in [14] the authors postulate homogeneously distributed sources producing (by some unknown mechanism) neutrinos with energies above $10^{22}$ eV which collide with a cosmological background of neutrinos (hot dark matter) to produce – amongst other things – protons and gamma rays that could be cosmic ray primaries. Perhaps swerves could provide the required acceleration in this case. Indeed neutrinos are more likely candidates than protons to be affected by the underlying discreteness as they are more point-like than protons, according to present beliefs. Moreover, we have few if any laboratory bounds on $k$ to contend with in the case of neutrinos.

More sophisticated models could also be developed. Our simple proton model assumes that the "diffusion constant" $k$ does not depend on local factors like average particle density, temperature etc. In a more realistic swerve model, perhaps a high particle density would lower the rate of diffusion, in which case the constraints from laboratory physics could be loosened. A second sort of generalization of the diffusion (or Fokker-Planck) equation (3) would relax the assumption of locality in *momentum* space. The "friction term" (3) captures the effect of many small momentum transfers due to (for example) CMBR scattering. The effect of large kicks would have to be described by a Boltzmann equation. A third improvement to our model would be to treat the particles quantum mechanically rather than classically, allowing one to take into account the finite size of the "wave packet". Such a change might be important, because it is conceivable that matter-induced decoherence would influence the value of $k$, making it different on earth than in interstellar space. Unfortunately, however, the type of over-arching framework that is available for classical diffusion seems to be lacking in the quantum case, so it is less obvious how to proceed. Finally, in a full theory of causal set quantum gravity, regions of continuum spacetime might be best described as a quantum superposition of many causets, and a better phenomenological model might have to reflect this aspect as well.

Let us return for a moment to possible observational evidence for swerves. Since the path of a particle would no longer be an exact geodesic, a certain amount of fuzzing of distant sources of particles would occur. Perhaps this could be revealed by highly directional detectors of some sort.

So far we have limited our discussion to the case of massive particles. A Lorentz invariant diffusion equation for massless particles can also be written down, although in this case we lack a concrete model of propagation on the underlying causet that could serve as motivation. [9] Rather than diffusion in $\mathbb{H}^3$, we have in this case diffusion on a "light" cone, since the 4-momentum of a massless particle is a null vector. We will describe this further in a future work. Lorentz invariant diffusion on the cone cannot alter the direction of the momentum, but it will cause its magnitude to fluctuate, so that a distribution peaked at a certain energy would spread with time. Accordingly, one might seek evidence for this kind of diffusion in the blurring of sharply peaked spectral features of distant sources (such as emission and absorption lines). This is in contrast to what has been proposed in Lorentz-violating models, where the speed of a photon is presumed to vary with its energy (see e.g. [15]).

Finally, just as tests of Lorentz invariance push special relativity to its limits, tests of unitarity would allow one to push the state-vector formalism of quantum mechanics to *its* limits. It seems only reasonable therefore that simple models of the possible effects of non-unitarity be formulated. A non-relativistic example is given in [16]. Perhaps a relativistic version of this could produce a similar effect to swerves.

To summarize, there is no reason that the assumption of an underlying spacetime discreteness must give rise to violations of local Lorentz invariance, because the causal set hypothesis does not. To illustrate the point that discreteness can nevertheless have observable effects, we have exhibited a Lorentz invariant momentum diffusion motivated by causal sets. If it is indeed the case that certain proposals for quantum gravity, such as loop quantum gravity and spin foams do predict violations of LLI, then we are in the happy situation of having a way to distinguish between different proposals experimentally. Be that as it may, the specificity of the models treated in this paper indicates that the causal set approach holds great potential for providing phenomenological theories of matter propagating in a discrete background. In an era of ever increasing sensitivity and power in cosmological observations, this potential to predict and detect the effects of a fundamental spacetime discreteness should be exploited.

**Note added:** The Poincaré-invariant diffusion process described above is constructed rigorously in [17] and [18].

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[9] We also lack an idea of how to describe such a diffusion in wave language as opposed to particle language. What would it mean in the case of Maxwell's equations for example?

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