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Everpresent Λ

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A variety of observations indicate that the universe is dominated by "dark energy" with negative pressure, one possibility for which is a cosmological constant. If the dark energy is a cosmological constant, a fundamental question is: Why has it become relevant at so late an epoch, making today the only time in the history of the universe at which the cosmological constant is of order the ambient density. We explore an answer to this question drawing on ideas from unimodular gravity, which entails fluctuations in the cosmological constant, and causal set theory, which predicts a specific magnitude for the fluctuations. The resulting ansatz provides a cosmological "constant" which fluctuates about zero, remaining always comparable to the ambient energy density.

I. INTRODUCTION

The most startling discovery to emerge from the recent plethora of cosmological data is that the universe appears to be dominated by a so called dark energy [1–3]. We know that this dark energy accounts for roughly seventy percent of the effective energy density in the universe, does not cluster like ordinary matter, and has negative pressure. Otherwise, we are in the dark about the nature of this extraordinary phenomenon.

Perhaps the most popular explanation is that the dark energy is due to a cosmological constant, for such a parameter Λ was introduced into General Relativity at its birth [4] and has remained an important tool for cosmologists seeking to model the observed universe [5]. The strongest argument against the cosmological constant is that naively we expect it to contribute an effective energy density, $\rho_\Lambda = \Lambda$, of order $m_p^4 = \kappa^{-2}$, where $\kappa = 8\pi G$ is the rationalized Newton constant and m_p is the corresponding Planck Mass[1]. This estimate is some one hundred and twenty orders of magnitude larger than the observed value. An equivalent way of stating the problem is to note that, if we give Λ its observed value, then only today is it of order the ambient density in matter or radiation. At all past epochs, ρ_Λ would have been sub-dominant and immensely so. Many people have felt that no theory could naturally predict such a tiny value for Λ (or equivalently such a late epoch for it to become relevant) without predicting Λ to vanish entirely, and for this reason they have sought other explanations of the observations.

Many alternatives to the cosmological constant have been proposed. Most significant among these are "quintessence" models in which the dark energy is due to a homogeneous scalar field shifted away from the true minimum of its potential [6]. Like a simple cosmological constant, many of these suffer from the "Why Now?" problem: Why does the quintessence field come to dominate only recently? To make sense of this, they typically need to invoke an extremely small mass scale ($m \lesssim 10^{-33}$ eV). Even more disturbing, none of them are connected to realistic particle physics models. Perhaps then, instead of altering the energy content of the universe, we need to look in another direction and modify gravity in order to explain the appearance of dark energy.

The simulations reported here flesh out an old heuristic prediction [7, 8] of a fluctuating cosmological term arising from the basic tenets of causal set theory.[2] Intuitively, the words "cosmological term" refer to a contribution to the effective stress-energy-momentum tensor of the form $T_{\mu\nu} = \Lambda(x) g_{\mu\nu}$. However, in classical General Relativity (GR) such a $\Lambda(x)$ must be constant if the total energy momentum in other components of $T_{\mu\nu}$ is separately conserved. Here we consider a specific modification of GR motivated by the search for a theory of quantum gravity based on causal sets.

Although the ultimate status and precise interpretation of the prediction of a fluctuating Λ must await the development of a quantum dynamics for causal sets [9], the basic lines of the argument are simple and general enough that

[1] In this paper, we will use units in which $\hbar = c = m_p = 1$.
[2] For an introduction to the causal set hypothesis see [7, 10].

they have a certain independence of their own. In this paper we review the motivation for a fluctuating Λ from causal set theory, propose an ansatz for the form of these fluctuations, apply the latter to the Friedmann equation with time-dependent cosmological term, and find that we can have a viable cosmology for some fraction of the solutions. Finally, we address issues related to our choice of evolution equations.

II. CAUSET THEORY

Here, we will review the arguments leading to a fluctuating cosmological term and then describe the specific ansatz via which we have chosen to implement their main implication: that Λ can be expected to fluctuate, and with a magnitude that diminishes as the universe grows older.

In causal set ("causet") theory, the predicted fluctuations arise, as a kind of residual (and nonlocal) quantum effect, from the underlying spacetime discreteness. More specifically, the basic inputs to the argument are: spacetime discreteness leading to a finite number N of elements; the interpretation of spacetime volume $\mathcal{V}$ as a direct reflection of N; the conjugacy of Λ to spacetime volume $\mathcal{V}$; and the existence of fluctuations in $\mathcal{V}$ coming from Poisson fluctuations in N. (Of these four inputs, the first is not peculiar to causal sets, but the remaining ones all are to a greater or lesser extent.)

The two most basic tenets of causal set theory are first, that the causal ordering of macroscopic events reflects a more fundamental order relation among the elements of an underlying discrete structure to which continuous spacetime is only an approximation, and second, that the four-volume of a region of spacetime reflects the number of discrete elements of which the region is "composed". The hypothesized discrete substratum or *causal set* is taken to be a partially ordered set and its dynamics is conceived of as a kind of growth process in which elements come into being one at a time. Although a classically stochastic dynamics expressing these ideas is by now fairly well developed [11], a corresponding quantum dynamics is only just beginning to be sought. Any prediction of quantum fluctuations in Λ must therefore rest on an anticipation of certain features of this "new QCD" (quantum causet dynamics).

Let us begin by assuming that, at some level of approximation, this dynamics will correspond to a spacetime "path integral" in which one is summing over certain classes of four-geometries. At the deeper level however, this will still be a sum over causal sets. Then let us take from the already developed classical growth models for causal sets the feature that the ever growing number N of causet elements plays the role of a kind of parameter time – the time in which the stochastic process which mathematically represents the growth unfolds, and with respect to which the probabilities are normalized. Just as one does not sum over time in ordinary quantum mechanics, one would not expect to sum over causets with different values of N in the quantum theory. But, because number corresponds macroscopically to volume $\mathcal{V}$, this translates into the statement that one should hold $\mathcal{V}$ fixed in performing the gravitational path integral. Any wave function that arises will therefore depend not only on suitable boundary data (say a three-geometry) but also on a four-volume parameter $\mathcal{V}$. Such a restricted path integral may be called "unimodular".

Now the unimodular modification of ordinary GR has been fairly well studied [12], and it is understood that within it, Λ and $\mathcal{V}$ are conjugate in the same way that energy and time are conjugate in ordinary quantum mechanics. (Indeed, this is almost obvious from the fact that the Λ term in the general relativistic action is just the product $-\Lambda\mathcal{V}$.) In particular, this means that, to the extent that $\mathcal{V}$ is held fixed in the gravitational path integral, the effective cosmological constant will remain undetermined by the fundamental parameters of the theory. (Again this is almost obvious by reference to the classical limit of unimodular gravity, where the Lagrange multiplier used to implement the fixed $\mathcal{V}$ constraint combines with any "bare" Λ in such a way that the observed or "renormalized" Λ represents nothing more than a constant of integration.)

If this were the whole story, then our conclusion would be that Λ is subject to quantum fluctuations (just like energy E in ordinary quantum mechanics), but it would not be possible to say anything about their magnitude, nor about the magnitude of the mean Λ about which the fluctuations would occur.

But here there enters a second aspect of the causal set hypothesis that we have not mentioned earlier. In order to do justice to local Lorentz invariance, the correspondence between number and volume cannot be exact, but must be subject to Poisson type fluctuations[3], which, as is known, have a typical magnitude of $\sqrt{N}$. This means that, in holding N fixed at the fundamental level, we in effect fix $\mathcal{V}$ only up to fluctuations of magnitude $\pm\sqrt{\mathcal{V}}$. (Notice that these are not dynamical fluctuations. Rather they occur at a *kinematic* level: that of the correspondence between order theoretic and spatio-temporal variables.) Hence, we do end up integrating over some limited range of $\mathcal{V}$ after all, and correspondingly we do determine Λ to some degree — but only modulo fluctuations that grow smaller as $\mathcal{V}$

[3] The correspondence between the underlying causet and the approximating spacetime is defined via a Poisson process of "sprinkling" at unit density, see references [7, 13] for details.

grows larger. Specifically, we have

$$\Delta\Lambda \sim 1/\Delta\mathcal{V} \sim 1/\sqrt{\mathcal{V}}\,. \tag{1}$$

As any proper dilemma should, that of the cosmological constant has two horns: Why is Λ so nearly zero and Why is it not exactly zero? None of what we have said so far bears on the first question, only on the second. All we can conclude is that partially integrating over $\mathcal{V}$ in the effective gravitational path integral will drive us toward *some* value of Λ. We must assume, as the evidence overwhelmingly suggests, that this "target value" is zero, for reasons still to be understood.[4] Then we end up predicting fluctuations about zero of a magnitude given by (1).

Independent of specifics, the spacetime volume $\mathcal{V}$ should be roughly equal to the fourth power of the Hubble radius, H^{-1}. Therefore, at all times we expect the cosmological constant to be of order

$$\Lambda \sim \mathcal{V}^{-1/2} \sim H^2 \sim \rho_{\rm critical}\ , \tag{2}$$

the critical density (recall that we are setting $8\pi G = 1$). We thus obtain a prediction for today's Λ which agrees in order of magnitude with current fits to the astronomical data. And this argument is not limited to today: at all times we expect the energy density in the cosmological constant to be (in absolute value) of the order of the critical density.

This is the basic idea, but any attempt to implement it immediately raises questions whose answers we can at present only guess at, pending the development of a fuller quantum dynamics for causets. At a conceptual level there is first of all the question of precisely how to interpret the $\mathcal{V}$ that figures in equation (1) and second of all the question how to incorporate a fluctuating Λ into some suitable modification of the Einstein equations. At a more practical level, if we aim to understand, for example, how fluctuations in Λ would have affected structure formation, we need to know, not only their typical magnitude at each moment of cosmic time, but also how the fluctuations at one moment correlate with those at other moments.

Concerning the conceptual questions, we will, for present purposes, resolve them provisionally as follows. First we impose spatial homogeneity, so that the Einstein equations reduce to a pair of ordinary differential equations for the scale factor a. Of these two equations, the first, the so called Friedmann equation or Hamiltonian constraint, is first order in time and embodies the energy law in this setting, while the second involves $\ddot{a}$ and, in the case of a *non-fluctuating* Λ, adds no information to the first (except at moments when $\dot{a} = 0$). Now Λ enters the Friedmann equation in its role as an effective *energy density* $\rho_\Lambda = \Lambda$, whereas it enters the $\ddot{a}$ equation in its role as an effective *pressure* $p_\Lambda = -\Lambda$. Unfortunately, these equations taken together imply that Λ cannot vary with time.[5] Therefore, to allow Λ to fluctuate, we must modify at least one of the equations. We choose to maintain the Friedmann equation and the relationship $\rho_\Lambda = \Lambda$. We can then retain the second Einstein equation as well if we make the ansatz, $p_\Lambda = -\Lambda - \dot{\Lambda}/3H$, $H = \dot{a}/a$. (Equivalently, we ignore p_Λ and drop the second Einstein equation entirely.)

The quantity $\mathcal{V}$ which governs the magnitude of the fluctuations in Λ we will identify (up to an unknown factor of order unity) with the volume of the past light cone of any representative point on the hypersurface for which we want the value of $\mathcal{V}$, as illustrated in Figure 1. Although this interpretation is somewhat at odds with the meaning that $\mathcal{V}$ has in the unimodular context, it seems more in accord with causality, and it is the only number accessible to observation in any useful sense.

With these choices made, the only remaining question is what sort of random process we want to use to simulate our fluctuating Λ. Ideally perhaps, this would be some sort of "quantal stochastic process" (since the underlying physics is quantal), but here we do the simplest thing possible and let the fluctuations in Λ be driven by those of an unadorned random walk. In fact the ansatz we will use has some appeal in its own right as an independent "story" of why the cosmological constant might be expected to fluctuate in any discrete quantum gravity theory that incorporates the equality $N = \mathcal{V}$ between volume and number of elements.

With reference to the Einstein-Hilbert Lagrangian, one could describe the cosmological constant as the "action per unit spacetime volume which is due just to the existence of spacetime as such, independent of any excitations such as matter or gravitational waves". Re-interpreting volume as number of elements, we can say, then, that Λ is the "action per element". One would expect this to be of order unity in fundamental units, and if we identify the latter with Planck units, we get the old answer which is off by some 120 orders of magnitude. On the other hand, if we suppose that each element makes its own contribution and these contributions fluctuate in sign[6] then the relative

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[4] One possible mechanism is that only $\Lambda = 0$ is stable against the destructive interference induced by *non-manifold* fluctuations of the causal set.

[5] We assume here that whatever matter or radiation is present obeys its usual equation of motion so that its energy is separately conserved. This implies, via the two Einstein equations, that the Λ component of the effective energy is also conserved, which in turn forces Λ to be constant unless we modify its "equation of state", $\rho_\Lambda = -p_\Lambda$.

[6] It would probably be more suitable to speak not in terms of action S but rather $\exp(iS/\hbar)$ and say that the contributions (now multiplicative rather than additive) fluctuate in *phase*.

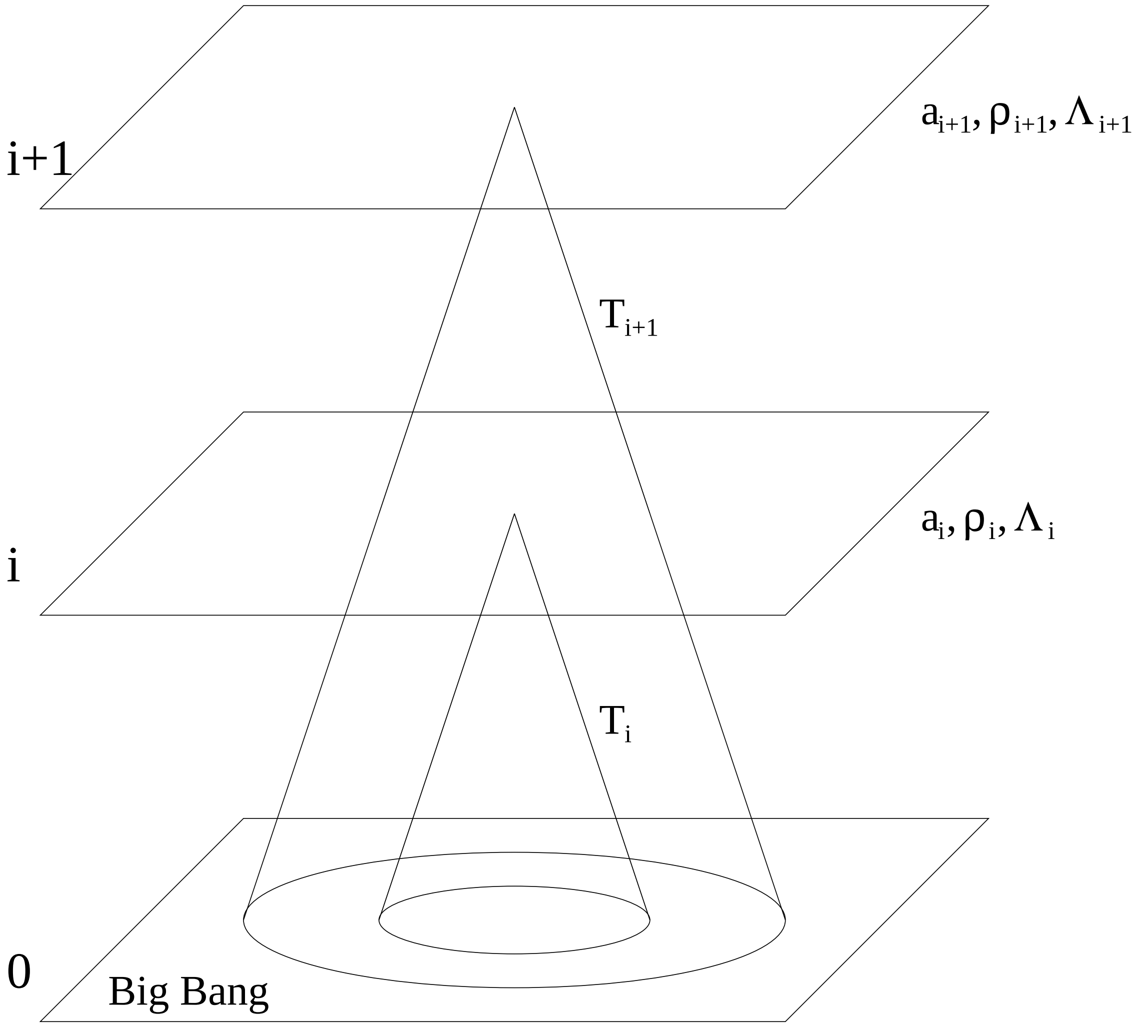

FIG. 1: Schematic representation of the backward light-cone at two different cosmic times. Evolution of the scale factor between the two time slices is determined by the Friedmann equation while Λ varies stochastically.

smallness of Λ will be explained; but one would also expect a residual $\sqrt{N}$ contribution to S to remain uncanceled. Consequently, there would remain a residual contribution to the action per element of $\sqrt{N}/N = 1/\sqrt{N}$, in agreement with our earlier argument.

To implement such an ansatz is now straightforward. What we need for the sake of the Friedmann equation is just Λ as a function of N (or equivalently of $\mathcal{V}$). To produce such a function we need only generate a string of random numbers of mean 0 and standard deviation 1 (say) and identify $\Lambda(N)$ with the ratio $S(N)/N$, where $S(N)$ is the sum of the first N of our random numbers. Modulo implementational details this is the scheme we have used in the simulations on which we report next.

III. SIMULATIONS

We take as the spacetime volume

$$\mathcal{V}(t) = \frac{4\pi}{3} \int_0^t dt' a(t')^3 \left[\int_{t'}^t dt''/a(t'') \right]^3 \tag{3}$$

where $a(t)$ is the scale factor of the universe at proper time t. Note from this formula that the backward light-cones depicted in Figure 1 are quite deceptive: because $a(t)$ was much smaller in the past and vanishes at the Big Bang, most of the four-volume $\mathcal{V}$ of these light cones accumulates recently. One consequence of this is that $\mathcal{V} \sim H^{-4}$ recently, even if there was a period of cosmic inflation in the early universe.

Our algorithm for calculating the cosmological constant at time-step $i+1$ is then to set

$$\delta N_i \equiv N_{i+1} - N_i = \mathcal{V}(t_{i+1}) - \mathcal{V}(t_i) \tag{4}$$

and then write

$$\begin{aligned} \Lambda_{i+1} &= \frac{S_{i+1}}{N_{i+1}} \\ &= \frac{S_i + \alpha \xi_{i+1}\sqrt{\delta N_i}}{N_i + \delta N_i}. \end{aligned} \tag{5}$$

Here α is an unknown dimensionless parameter which governs the dynamics of the theory; ξ_{i+1} is a random number with mean 0 and standard deviation 1; and S_0 is set to zero at some very early time t_0. We then expand the universe according to

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{1}{3}\left(\rho_{\text{matter}} + \rho_{\text{radiation}} + \rho_\Lambda\right), \tag{6}$$

recompute the new spacetime volume and repeat.

Figure 2 shows the evolution of the energy density in one such realization. During the radiation era, $|\rho_\Lambda|$ scales roughly as a^{-4}, while during the matter era it scales as a^{-3}. Thus at all times it is comparable to the ambient energy density. If the recipe we have devised for implementing the ideas of causal set theory and unimodular gravity is an accurate approximation to the ultimate quantum theory, then these modifications of GR do indeed lead to an *Everpresent* Λ, a cosmological term which is always with us [14].

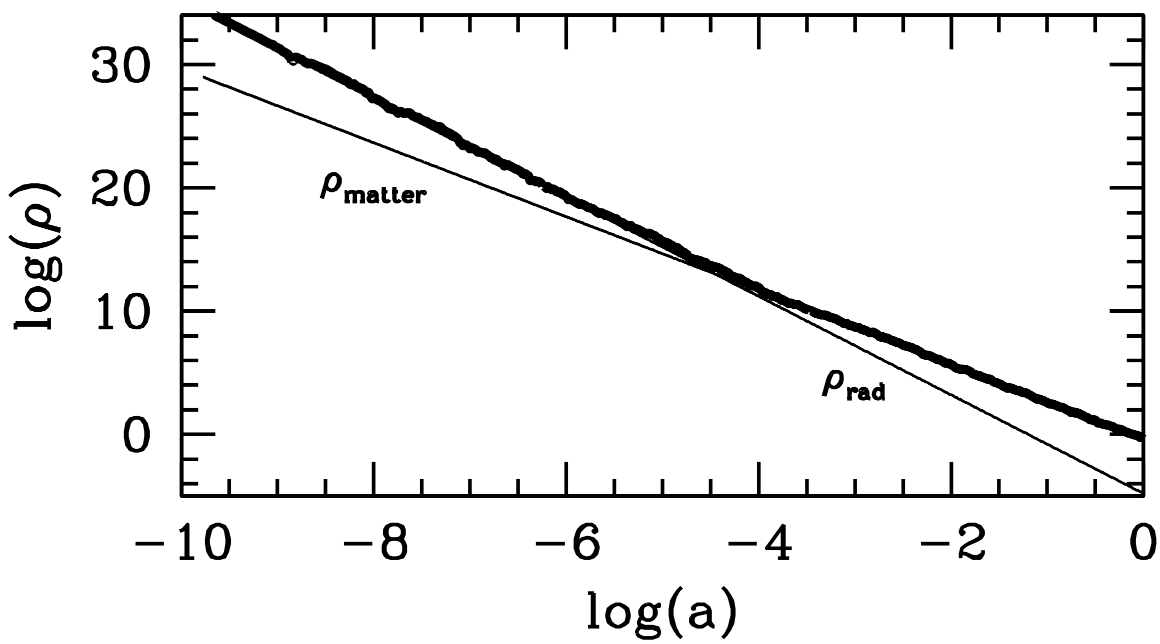

FIG. 2: Evolution of the energy densities in the universe. The thick curve is the absolute value of the energy density in the cosmological constant. The fluctuating ρ_Λ is always of order the ambient density, be it radiation (early on) or matter (later). Here the dimensionless parameter α which governs the amplitude of the fluctuations has been set to 0.01.

Hidden in the gross structure of Figure 2 are the fluctuations about this average scaling. These fluctuations are crucial if the theory is to describe the real universe for two reasons: First, there cannot be too much excess energy at $a \sim 10^{-9}$ or else the successful predictions of Big Bang Nucleosynthesis (BBN) will be destroyed. Second, if ρ_Λ scales exactly as matter today, it will not have the correct equation of state to account for the cosmological observations.

Figure 3 shows the ratio of the energy density in Λ to the total energy density as a function of the scale factor for another realization, this time with a slightly larger value of α. This ratio, Ω_Λ, fluctuates about zero with an amplitude of order 0.5 (as we will shortly see, this amplitude is a function of α). In this particular realization, Λ accounts for over fifty percent of the energy density today and changes very little going back to redshift $z = 1$ ($a = 0.5$); thus it behaves recently as a true cosmological constant, and therefore satisfies the observed cosmological constraints.

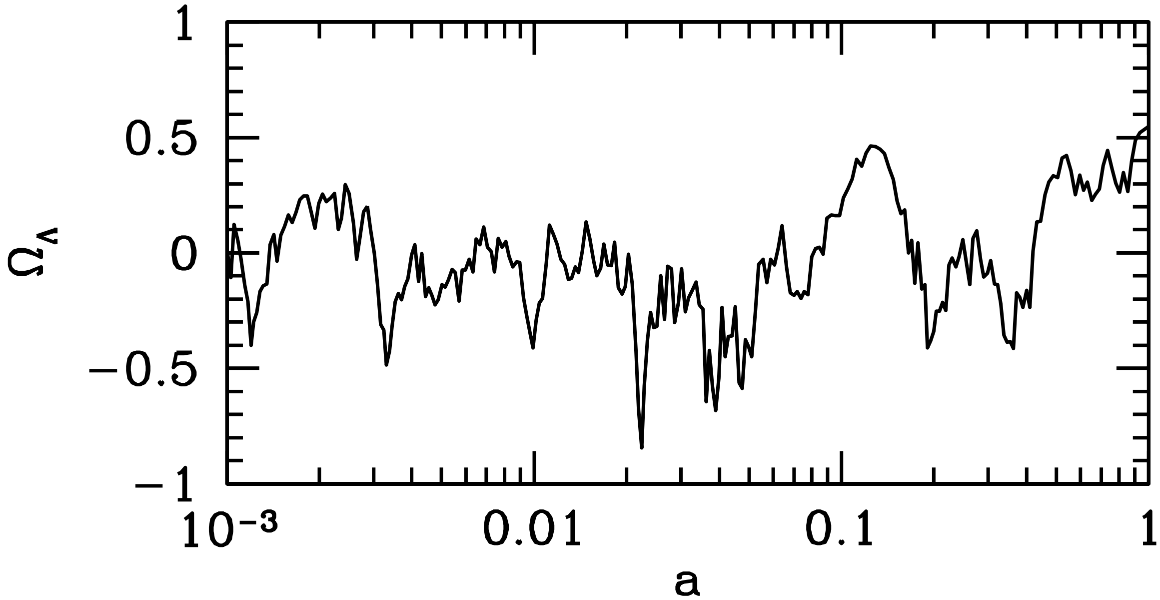

FIG. 3: The ratio of the energy density in cosmological constant to the total density as a function of scale factor. Here $\alpha = 0.02$.

In half the realizations, Λ will be positive today. Whether or not it is positive enough to explain the observations then becomes a question of probability. For $\alpha = 0.02$, it clearly is not that improbable (indeed in the same run, we see another spike in the energy density at $a \simeq 0.1$).

The same qualitative argument applies to the BBN constraint. In fact the situation there is even better. Half the time the extra energy density will be negative, thereby reducing the total energy density in the universe. This in turn will reduce the predicted abundance of ^{4}He. There is some disagreement at present as to whether the current observations agree with the standard cosmological model or not [15], with some cosmologists arguing that the observed abundances are too low. A negative ρ_Λ would fix this problem.

Why have we chosen α in the range 0.01-0.02? In part, our choice responds to a fundamental incompleteness in our implementation. If α exceeds 0.01 by very much, there will inevitably be times during which the total effective energy density, the sum of the terms in parenthesis on the right side of equation (6), goes negative, thereby invalidating the equation. (Whenever this happens, we terminate the run.) In the next section we offer some thoughts on this problem; here we simply spell it out.

Figure 3 shows a history for $\alpha = 0.02$ going back to the time of decoupling. If we had started earlier, say at the "Planck time" $a \sim 10^{-32}$, we would have had only about a 1 in 3 chance of completing the run without hitting a time at which $\rho_{\rm tot}$ went negative. Taking $\alpha = 0.01$ evades this problem; for that parameter choice, very few runs hit a time at which the total energy goes negative. However, for $\alpha = 0.01$, the fluctuations are smaller. Figure 4 shows a histogram of final values of Ω_Λ for 6000 realizations, each with $\alpha = 0.01$. Only rarely does the final value of Ω_Λ approach those necessary to explain the observations.[7]

There is therefore a tension: if we push α too low, it becomes very unlikely that ρ_Λ will be large enough today to agree with observations. If we push α too high, there inevitably comes a time at which the total effective energy density in the universe becomes negative and the simulation cannot continue. Of course we are dealing with probabilities, so for any value of α there is always the chance that the total energy density remains positive throughout the history of the universe and the final value of ρ_Λ is large enough to account for observations. Fortunately, this happens reasonably

[7] In saying this, we have naively equated the observational value quoted for Ω_Λ with the contemporary value of our fluctuating variable Ω_Λ. However, because neither Λ nor $w = p_\Lambda/\rho_\Lambda$ is constant in our model, a more careful comparison with observation is actually needed.

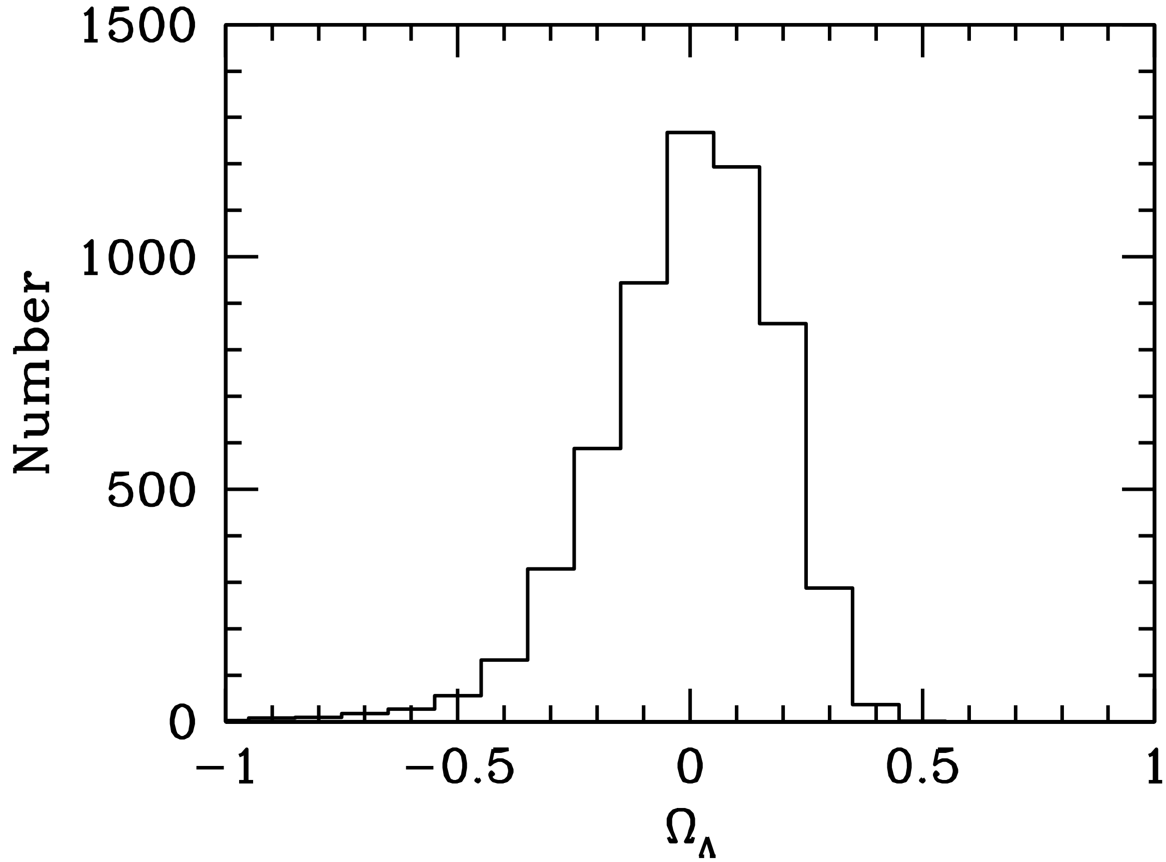

FIG. 4: A histogram of the final value of Ω_Λ, the ratio of ρ_Λ to the total density. The dimensionless parameter governing the fluctuations in Λ has been set to $\alpha = 0.01$.

often for α in the range $0.01 - 0.02$. Nonetheless, we suspect that we will ultimately have to deal more directly with the possibility that $\rho_{\rm tot}$ goes negative.

We would also like to comment here on the possible objection that an α around 0.01 is "unnaturally" small. Actually, α can be thought of as the ratio of two conceptually distinct magnitudes, one (call it s) being the "contribution to the action of a single causet element", and the other being l^4, where l is "the linear size of a causet element in Planck units". Presumably, "naturality" would then require that both s and l be of order unity. But, to obtain $\alpha = 0.01$ it would suffice, for example, to take $s = 1/2$ and $l = 50^{1/4} = 2.66$, neither of which is a particularly big number.

IV. COMPLICATIONS

We can think of two ways to deal with the possibility of a negative $\rho_{\rm tot}$ without having to terminate the simulation: change the implementation so that this never occurs or reinterpret $\rho_{\rm tot}$ going to zero (or negative) so as to give a viable cosmology.

One possibility would be to suppose that Λ fluctuates but is positive semi-definite. This is the position adopted by Ng and van Dam in reference [16]. There, they argue that the kernel for the Euclidean gravitational path integral over Λ histories takes the form

$$e^{-S_E} \propto \exp\left(\frac{24\pi^2}{\Lambda}\right), \tag{7}$$

in our units. From this, they argue that the most probable value of Λ is zero and that, if it is not zero, it must be positive. As they observe, however, this result is peculiar to the assumptions of Euclidean quantum gravity with all its uncertainties and controversy. In particular, this result does not seem to follow from causal set theory or unimodular gravity by themselves and we do not favour it.

Another possibility would be to suppose that the cosmological term comes from a decrease in the local energy of one of the matter fields or gravitational waves. This is the philosophy of, for example, Chen and Wu [17] who consider a non-fluctuating, but time dependent, cosmological term $\Lambda(t) \propto a^{-2}(t)$. As mentioned above, this supposition is forced upon us if all of Einstein's equations are to be simultaneously satisfied and if the standard equation of state for Λ is to obtain, because Einstein's equations—the contracted Bianchi identity in particular—require that total energy-momentum be conserved. Then, in classical GR, ρ_Λ cannot fluctuate without a compensating fluctuation in the energy-momentum density of one or more of the matter fields. Our approach has been to solve this problem by, in effect, modifying the equation of state for Λ. Nevertheless, let us consider briefly the possibility of instead adapting the solution above to our case. Suppose that some matter component, let us take gravitational waves as a concrete example, were somehow converted into the energy density of a cosmological term, while the energy density in every other component (dust, radiation, etc.) were separately covariantly conserved. Then the first law, $\frac{d}{dt}\left[(\rho_{\rm tot} + p_{\rm tot})\, a^3\right] = a^3 \frac{dp_{\rm tot}}{dt}$, applied to these two components becomes:

$$\dot\rho_{\rm grav} = -4H\rho_{\rm grav} - \dot\Lambda(t)\,, \tag{8}$$

where H is the Hubble parameter. As could be expected, an increase in Λ must lead to a decrease in the energy density in the gravity waves. However, for a generic fluctuating $\Lambda(t)$, the cosmological term now might increase enough that the energy density in gravity waves became negative. It's not clear how this could be interpreted and it appears that we would simply exchange one problem for another. (Note that, in the case of Chen and Wu [17], this is not a problem, since their cosmological term decreases monotonically with the expansion of the universe.) Thus, we see that this solution might work with a fluctuating Λ if we demand that $\dot\Lambda \leq 0$. In fact, relaxation processes of this sort have been considered for some time. The earliest we are aware of is that of Abbott [18]. Recently, there has been renewed interest in a similar suggestion of Brown and Teitelboim [19] where domains of four-form flux decay spontaneously, relaxing the effective local value of the cosmological term, see [20] for recent references. The difficulty with these proposals is again the "Why now?" problem: relaxation rates and/or boundary values must be tuned for any hope to obtain a viable cosmology.[8]

All in all, neither of these possible solutions really seems to come to grips with the central difficulty: Within the contexts of causal set theory and unimodular gravity, the sign of the total effective energy density is fundamentally not constrained. We see no good reason to assume either that Λ is positive definite or that Λ will always decrease. Thus, let us seek instead to understand what happens when $\rho_{\rm tot}$ approaches and perhaps goes through zero. Our guide will be the classical theory. Consider, for example, a dust filled, flat universe with a negative cosmological constant $\lambda_0 < 0$ (a true constant). The 0-0 component of Einstein's equations gives us the Friedmann equation: $3H^2 = \rho_{\rm tot}$. Meanwhile, the i-i component gives us the deceleration: $2\frac{\ddot a}{a} = -|\lambda_0| - H^2$. We see that, once the matter has red-shifted enough that the total effective energy density vanishes, the universe stops expanding and begins to contract. As it contracts, the energy density in matter once again begins to exceed the magnitude of the cosmological term and $\rho_{\rm tot}$ never becomes negative.

We expect that something like this phenomenology will carry over into our case except that, with a fluctuating cosmological term, it seems likely that the collapse can reverse itself if the cosmological term later becomes sufficiently negative a second time. This is in contrast with the classical example above where once the universe starts to collapse, the matter term always dominates and keeps $\ddot a/a$ negative. In our model, however, we would expect the amplitude of the cosmological term's fluctuations to track the matter or radiation energy density in the collapsing universe.

Of course, none of this follows from the simple evolution ansatz we have implemented in this paper. Such detailed dynamical understanding must await further developments. Nevertheless, it is reasonable to suppose that the complete theory will reduce in stages: a full theory with non-metric structures at the Planck scale and a semi-classical theory describing metric structures at larger scales. Furthermore, it is reasonable that the semi-classical theory will be describable as some sort of sum-over-histories, where the intermediate states are three-geometries. We can envisage the evolution equation we propose as some sort of classical approximation to propagation in a stochastic potential. In the sum-over-histories theory, we expect that $\rho_{\rm tot} < 0$ will correspond to a tunneling-type solution. Our difficulty in handling $\rho_{\rm tot} < 0$ here is, in this sense, no different from the more venerable problem of finding an effective, classical description of barrier penetration.

Beyond our inability to deal with "tunneling" behavior, a second, potentially more serious incompleteness of our implementation is its assumption by fiat of spatial homogeneity: we have allowed Λ to fluctuate in time but not in space. To overcome this limitation, one would have to generalize the ansatz we have made for the "Λ equation of state" to the case of a spatially varying cosmological term, something we do not know how to do. Were this to be done, an important question would be how the resulting spatial fluctuations in Λ would affect the isotropy of the

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[8] Of course, back when Abbott and Brown and Teitelboim first made their suggestions, there was no compelling evidence for a non-vanishing cosmological term. One needed only to make it small enough.

cosmic microwave background radiation and the power spectrum of matter fluctuations. Since the overlap of the pasts of regions in different parts of the sky could easily be very small, one might worry that too great a degree of spatial inhomogeneity would develop in both matter and radiation. On the other hand, the temporal fluctuations of Λ might tend to average out the effects of its spatial fluctuations; a recent positive Lambda might reduce an otherwise excessive "structure formation"; and the uneven expansion induced by the temporal Λ fluctuations might plausibly act to increase the overlap of the past light cones (enlarge the "horizon"). Finally, quantum effects might induce correlations between Λ in different regions, even without major overlap of their pasts, given that the fluctuations we are studying arise from an underlying mechanism that is both nonlocal and quantal. In the absence of further simulations or an extension of the theoretical framework, it is difficult to go beyond a mere statement of some of these possibilities.

V. CONCLUSION

It is still too early to understand the full implications of recent cosmic discoveries that point to dark energy in the universe. A number of possibilities have previously been explored in detail, including a non-zero (but truly constant) cosmological constant Λ, and zero Λ with dark energy hidden in a scalar field.

It is also possible, though, that the measurements are telling us that we need to modify our understanding of space and time. In particular, the notion that spacetime is continuous may be simply an approximation that breaks down on scales as small as the Planck scale. If so, drawing on ideas from causal set theory – which postulates a discrete spacetime – and unimodular gravity, we have shown that the cosmological "constant" need not be a fixed parameter. Rather, it arguably fluctuates about zero with a magnitude $1/\sqrt{\mathcal{V}}$, $\mathcal{V}$ being some measure of the past four volume. The amplitude of these fluctuations is then of the right order of magnitude to explain the appearance of dark energy in the universe. This argument is so general that it would apply at all times, and, indeed (as confirmed by our simulations), we expect the effective energy density in the cosmological "constant" to always be of order the ambient density in the universe.

In §IV, we presented a number of issues which inevitably will confront anyone wishing to implement this idea. Until these issues are resolved, it will be difficult to make unambiguous, robust predictions. Nevertheless, one can already see that this theory of a fluctuating Λ differs significantly from most other solutions to the dark energy problem. Most important for its testability is the notion that it may have affected the evolution of the universe at early times. Thus, the primordial generation of perturbations during a possible inflationary phase; production of light elements in Big Bang Nucleosynthesis; acoustic oscillations in the background radiation; and the evolution of structure at more recent times all may yield clues and tests of the idea of an everpresent Λ.

If the idea of an everpresent Lambda is correct, it will manifest a pleasing symmetry between the very small and the very large. For then, not only will gravity, which is largely absent in the micro-world, become once again important on Planckian scales under the influence of quantum mechanics, but quantum mechanics, which is largely absent in the macro-world, will become once again important on cosmic scales under the influence of gravity.

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[1] Well before supernovae observations pointed to an accelerating universe, there was ample evidence that the best fit universe was one with a cosmological constant. See e.g. L. Krauss and M.S. Turner, *Gen. Rel. Grav.* **27**, 1137 (1995); J.P. Ostriker and P.J. Steinhardt, *Nature* **377**, 600 (1995).

[2] The two independent sets of observations of Type Ia supernovae are described in A. Riess et al, Astronom. J. 116, 1009 (1998) and S. Perlmutter et al, Astrophys. J., 517, 565 (1999).

[3] Fits to all observations are given in e.g. X. Wang, M. Tegmark, and M. Zaldarriaga, Phys.Rev. D65 123001 (2002)

[4] A. Einstein, Sitz. Preuss. Akad. d. Wiss. Phys.-Math **142** (1917).

[5] Two examples of this are: (i) the historical tension between the predicted age of an Einstein-de Sitter universe and observed ages of globular clusters (e.g. L. M. Krauss and B. Chaboyer, "New Globular Cluster Age Estimates and Constraints on the Cosmic Equation of State and The Matter Density of the Universe," eprint: `astro-ph/0111597`) and (ii) the need to

lower the matter density (but still keep the universe flat) in order to fit the observed clustering of matter (e.g. S. Dodelson, E.I. Gates, & M.S.Turner, Science, **274**, 69 (1996); A.R. Liddle et al, *Mon. Not. R. astron. Soc.* **282**, 281 (1996)).

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[8] R.D. Sorkin, "Forks in the Road, on the Way to Quantum Gravity", talk given at the conference entitled "Directions in General Relativity", held at College Park, Maryland, May, 1993, *Int. J. Th. Phys.* **36**: 2759–2781 (1997), eprint: `gr-qc/9706002`;
R.D. Sorkin, Two Talks given at the 1997 Santa Fe workshop: "A Review of the Causal Set Approach to Quantum Gravity" and "A Growth Dynamics for Causal Sets", presented at: "New Directions in Simplicial Quantum Gravity" July 28 - August 8, 1997. The scanned in transparencies may be viewed at http://t8web.lanl.gov/people/emil/Slides/sf97talks.html

[9] For a classical dynamics meant as a stepping stone to the quantum one see David P. Rideout and Rafael D. Sorkin, "A Classical Sequential Growth Dynamics for Causal Sets", *Phys. Rev. D* **61**, 024002 (2000) eprint:`gr-qc/9904062`

[10] L. Bombelli, J. Lee, D. Meyer and R.D. Sorkin, "Spacetime as a Causal Set", *Phys. Rev. Lett.* **59**:521-524 (1987);

[11] See reference [9] and, for example, Xavier Martin, Denjoe O'Connor, David Rideout and Rafael D. Sorkin, "On the "renormalization" transformations induced by cycles of expansion and contraction in causal set cosmology", *Phys. Rev. D* **63**, 084026 (2001) eprint: `gr-qc/0009063`

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[15] See for example G. Steigman, eprint: `astro-ph/0208186` and S. Burles, K. M. Nollett, and M. S. Turner, Astrophys. J. **552**, L1 (2001).

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