gr-qc/0003043
SU–GP–00/02–1

Indications of causal set cosmology *

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Abstract

Within the context of a recently proposed family of stochastic dynamical laws for causal sets, one can ask whether the universe might have emerged from the quantum-gravity era with a large enough size and with sufficient homogeneity to explain its present-day large-scale structure. In general, such a scenario would be expected to require the introduction of very large or very small fundamental parameters into the theory. However, there are indications that such "fine tuning" is not necessary, and a large homogeneous and isotropic cosmos can emerge naturally, thanks to the action of a kind of renormalization group associated with cosmic cycles of expansion and re-contraction.

Until as recently as a year ago, it could have been said that we had no proven method by which to arrive at a dynamical law for causal sets. That is, the theory remained essentially in a kinematical stage, aside from some considerations of a very general nature about how a sum-over-histories might be formulated for causal sets. What has changed the situation is the discovery of a family of dynamical laws in which the "time-evolution" of the causal set appears as a process of stochastic growth [1]. At a technical level, such a dynamics may be defined in terms of a Markov process with a time-varying state-space — a process that might be described as the law of motion of a "stochastic spacetime". It turns out that relatively little freedom remains, once one postulates a dynamics of this kind: the picture of sequential growth leads almost uniquely to the dynamical family of [1] provided that one agrees to honor the discrete analogs of general covariance and (classical) causality. I will not try to summarize these developments in any detail here, or even to introduce

* Published in *Int. J. Theor. Ph.* **39 (7):** 1731-1736 (2000) as part of the proceedings of the Peyresq IV workshop on Quantum and Stochastic Gravity, String Cosmology and Inflation held June 28-July 3, 1999 in Peyresq, France.

the causal set idea itself. For that, the reader is referred to [2] and [1]. Rather, I wish to consider briefly the possible implications of some of these developments for cosmology.

It is true that the "sequential growth dynamics" found in [1] are classical (non-quantum), and it is true also that one does not know at present whether any of them leads to something like the Einstein equations, or even to anything resembling a spacetime at all. On the other hand, directions in which one might seek their quantum generalization are not hard to discern, and — still at the classical level — there is available at least one plausible guess at a choice of growth parameters which might reproduce something like classical spacetime. In these circumstances, and given also the accumulation of mathematical knowledge concerning at least one special case of these dynamics, it does not seem out of place to look for indications of how the theory taking shape might offer its own solutions to some of the recognized puzzles of cosmology. Specifically, I am thinking of the unexplained "large numbers" in cosmology related to the large size of the universe and its high degree of homogeneity and isotropy. (Lurking behind these issues is the question of why the cosmological constant Λ is so small. Causal sets so far have provided at best vague hints of why this should be so, but they *have* led to a prediction [3] of fluctuations about $\Lambda = 0$, and indeed, fluctuations of a time-dependent magnitude whose predicted value for the current universe is just that which seems to be indicated by the most recent observations.)

If we suppose that the cosmic microwave radiation we see today is descended directly from radiation which was present at the conclusion of the quantum-gravity era, * then we can straightforwardly evolve present conditions back to describe the universe (as much as we can see of it) as it was just after the "Planck time", by which I mean the time when the Hubble parameter $H = \dot{a}/a$ was near 1 in natural units. One finds (using the $1/a^4$ dependence of the energy density of radiation, and barring any conspiracies involving a time-varying cosmological constant) that the temperature at that epoch was also near to unity but the radius of curvature was some 28 orders of magnitude or more above the Planckian value. This "large number" (which corresponds to the large ratio of the present-day Hubble radius $1/H$ to the present-day wavelength of the microwave background) is one for which current theory has no convincing explanation.

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* This assumption is denied in "inflationary" scenarios according to which all matter visible today was created much later, in a process of "reheating".

Only two ways of obtaining such a large number have seemed appealing: either derive it from some other large number of the underlying theory (which then has to be explained in its turn) * or relate it to some conjunctural (i.e. historical) number of cosmology whose large size is not in need of explanation, such as the age of the universe or the number of cycles of contraction and re-expansion it has undergone to date. This second way of proceeding is the one to which some of the recent causal set results lend themselves.

To understand why, one must know that, despite being representable formally as a Markov process, a sequential growth dynamics exhibits a long memory, such that the present effective laws of motion are influenced by past behavior. (Indeed the process is formally Markovian only because one includes the entire past in the stochastically evolving "state".) The passage of time, according to this dynamics consists in a sequence of "births" of new elements of the causal set, each of which comes into being with a definite set of pre-existing "ancestor elements". The dynamical law is specified by giving the relative probability of each possible choice of ancestor-set (called "the precursor" in [1]), and this in turn, turns out to be given by a relatively simple expression depending only on the total size ϖ of the precursor and the size m of its maximal layer, ** namely

$$\lambda(\varpi, m) = \sum_k \binom{\varpi - m}{k - m} t_k \,, \tag{1}$$

where $t_0,\ t_1,\ t_2 \cdots$ is a sequence of non-negative "coupling constants" that completely characterizes the dynamics (and where $t_0 \equiv 1$). Notice in this formula how the behavior of the n^{th} element is influenced not only by the "contemporaneous coupling constant" t_n, but by the entire history of t's up to that "time".

Now among the possible choices of the t_n, two may be singled out for special consideration. The first choice,

$$t_n = t^n \tag{2}$$

for some fixed t $(0 < t < \infty)$, is known as *transitive percolation* and describes a simplistic, time-reversal invariant dynamics in which the future of each element is independent of its

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* for example, the ratio of the Planck mass to the Higgs mass

** In other language, ϖ is the number of all ancestors and m is the number of "immediate ancestors" or "parents".

past and of relatively "spacelike" regions. (See [1] and [4] for a more complete definition of transitive percolation dynamics.) The second choice,

$$t_n = \frac{t^n}{n!}\,, \tag{3}$$

has been suggested as a candidate which might yield spacetimes with genuine local degrees of freedom and a more realistic effective law of motion [1].

Let us consider transitive percolation first, since its properties are much better understood. One knows in particular that, with probability 1, the universe it describes undergoes an infinite succession of cycles of expansion, stasis and contraction punctuated by so called *posts* [5], each of which serves as the progenitor of all the elements born in the next cycle. The region issuing from any such post is independent of what preceded it, and has for its effective dynamics that of *originary percolation*, which is the same as plain percolation, except that no element can be born without having the post among its ancestors [4]. The size to which the region following a post re-expands is governed by the parameter t, or equivalently the probability $p = t/(1+t)$. For $t \ll 1$, the universe stops expanding at a "spatial volume" of not much more than $1/t$, whose value therefore would have to exceed (say) $(10^{28})^3 \sim 10^{84}$ in order to do justice to conditions at the time of the "big bang", assuming, of course, that the dynamics of transitive percolation is at all relevant to the very early universe.* The "fine tuning" or "large number" problem is then why t should have such a small magnitude, rather than a value near unity.

It is here that the memory effects embodied in (1) enter. Let us suppose for definiteness that the true dynamics is given by $t_n = t^n/n!$, and let us also suppose, for the sake of argument, that an infinite number of posts will occur for this dynamics as well. What then will be the effective dynamics for the portion of the causal set following some given post? (I'll call this portion the "current era".) Let e_0 be the post and let it have N_0 elements to its past (N_0 ancestors). Then, by definition, an element x born in the current era with ϖ *current* ancestors (including e_0) will have in reality $\varpi + N_0$ ancestors in the full causal set. On the other hand, its number of parents (maximal elements of past(x)) will

* We will see in a moment why this might be the case. The number 10^{84} assumes that a spacelike hypersurface in the continuum corresponds to a maximal antichain in the causal set, meaning a maximal set of causally unrelated elements. It assumes also that the spatial volume of such a hypersurface is equal, up to a factor of order unity, to the cardinality of the corresponding antichain.

be unaffected by the region preceding e_0, since the presence of e_0 prevents any element in that region from being an immediate ancestor of x. For the region, future(e_0), we thus acquire an *effective dynamics* described by weights $\widehat{\lambda}(\varpi, m)$ related to the fundamental weights $\lambda(\varpi, m)$ by the simple equation

$$\widehat{\lambda}(\varpi, m) = \lambda(\varpi + N_0, m) \,. \tag{4}$$

Each cosmic cycle thus acts to renormalize the coupling constants for the next cycle, and the dynamics in any given cycle differs from the original or "bare" dynamics by the action of this cosmological "renormalization group". It turns out that, when expressed as a transformation of the elementary coupling constants t_n, this action is very simple. For $N_0 = 1$ we have

$$\widehat{t}_n = t_n + t_{n+1} \tag{5}$$

and for $N_0 = 2, 3, 4, \cdots$ we just iterate this transformation N_0 times. (For defining the dynamics, only the ratios of the t_n matter. Hence, the t_n lie in a projective space, and (5), though it appears linear, is really a projective mapping). Equation (5) seems so simple that one could hope to analyze it fully, finding in particular all the attractors and their "basins of attraction". Potentially such an analysis could pick out as favored dynamical laws those to which the universe tends to evolve under the action of the "cosmic renormalization group". For now, we can note [6] that the only fixed points of (5) are those of the percolation family, $t_n = t^n$. (proof: In order that ratios $t_n : t_m$ not be altered by (5), it is necessary and sufficient that $\widehat{t}_n = c t_n$ for some constant c. But this holds iff $t_{n+1} = t_n t$ with $t = c - 1$.)

In [6], Djamel Dou has studied the action of this cosmic renormalization group on (3), as well as on some other choices of the t_n which can be regarded as simple "deformations" of (2), like $t_n = t^n \, p! \, n!/(n+p)!$. For the latter cases he finds that the "renormalization group flow" defined by (5) leads back to the fixed point set (2), indicating that percolation is to some degree an "attractor" in the space of all dynamics. For the former case, the story is more interesting. In the limit of large N_0, and for $m^2 \ll N_0 t$, $\varpi \ll N_0$, one finds that $\widehat{\lambda}(\varpi, m)$ corresponds to percolation (2) *with an* N_0*-dependent parameter* t given by

$$\widehat{t} = \sqrt{t/N_0} \tag{6}$$

The effective dynamics is thus once again transitive percolation, but only for a limited time,[†] and with an effective parameter t that diminishes from one cosmic cycle to the next.

Now, the germ of a resolution to our cosmological puzzles is contained in these results. Let us adopt the cosmology of (3) with its single free parameter taken to be a number of order unity (i.e. no "fine-tuning"), and let us assume that repeated posts occur. After each post, the ensuing cosmological cycle will begin with a stage governed by the dynamics (2) with a parameter $t = \widehat{t}$ which diminishes rapidly from cycle to cycle. During each such stage, the causal set will expand to a spatial volume of at least $O(\widehat{t}^{-1})$, a magnitude which increases rapidly from cycle to cycle. Moreover, it is not difficult to see that the earliest portion of this percolation stage (that for which $\widehat{n} \ll \widehat{t}^{-1}$) will be a phase of exponential tree-like growth (a tree being a poset in which every element but the first has precisely one parent.) [*] At the conclusion of each tree-like phase, we will have a homogeneous [**] universe with a "spatial volume" that grows larger with each successive cycle. In other words, by waiting long enough, we will automatically obtain conditions very like those needed for the "big bang" in whose aftermath we live. The "unnaturally" large size with which spacetime began in our particular phase of expansion would then reflect nothing more than the fact that a sufficiently great number of causal set elements had accumulated in previous cosmic cycles.

Before concluding, I would like to thank Chris Stephens and Alan Daughton for numerous early conversations about the cosmology of percolation dynamics. This research was partly supported by NSF grant PHY-9600620 and by a grant from the Office of Research and Computing of Syracuse University.

[†] The initial phase of effective percolation could not last forever. If it did, we could prove that another post would occur, whereafter, by (6), we'd have to have percolation with a *smaller* t, contradicting our original assumption.

[*] Computer simulations confirm this tree-like character.

[**] and also isotropic, to the extent that the causal set is sufficiently like a manifold that this term has meaning.

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