CHAPTER 5

CONCLUSION

I have investigated the structure of space-time from two very different points of view. Adopting a traditional world-view in Chapters 2 and 3, and restricting attention to the classical domain, I have developed a general formalism for describing, in a coordinate-free way, the evolution of the metrical geometry of the universe. This evolution can be described in terms of tensor fields intrinsic to the space-like hypersurface only if the tensor field, g , from which the hypersurface metric, ${}^3g = \Pi g$, is constructed, is a (pseudo-)Riemannian metric on space-time. The invariantly defined, normalized time derivatives of ${}^3g$ are then the extrinsic curvature, K ; the hypersurface projection of the space-time Ricci tensor, ΠS ; and the hypersurface projections of the invariant derivatives of S , to all orders, along the unit normals to the hypersurface: $\Pi(\nabla_{\vec{n}}S)$, $\Pi(\nabla_{\vec{n}}\nabla_{\vec{n}}S - \nabla_{\nabla_{\vec{n}}\vec{n}}S)$, etc..

I have shown how to construct dynamical theories of the evolution of ${}^3g$ by supposing that ${}^3g$ and its time derivatives up to some finite order, m , are essential initial data fields, but that the $(m+1)$th time derivative of ${}^3g$ is some explicit functional of the lower derivatives and of additional initial data fields that characterize the distribution of matter in space-time. The integrability conditions are constraints on the initial data which allow one to consistently and unambiguously construct all time derivatives of ^3g , of order greater than $(m+1)$, in terms of the initial data. In the simplest case, $m = 1$, it is only possible to construct one consistent set of dynamical and constraint equations for the metric, and these are the Einstein gravitational field equations. Higher order gravitational theories (with $m>1$) cannot be ruled out, however, nor can any restrictions be placed on the functional form of the stress-energy tensor (when $m = 1$), aside from the obvious condition that it must be a symmetric tensor with vanishing covariant divergence.

With the hope of obtaining a unified theory of gravitation and quantum phenomena, I have proposed, in Chapter 4, that the objective world underlying all of our perceptions is a 4-dimensional topological manifold, $\mathcal{W}$, with no physically significant field structure, but instead an unconstrained and extremely complex global topology. Aided by the connected sum decomposition theorem for 3-manifolds, I have demonstrated that $\mathcal{W}$ may be uniquely represented by a labelled graph which has properties remarkably similar to those of the Feynman graphs of QFT. The lines of this graph correspond to prime 3-manifolds (without boundary) and the vertices correspond to isolated, non-degenerate critical points of $\mathcal{W}$. By exploiting this similarity with Feynman graphs, I have been able to show how the space-time of our perceptions, with its geometry and quantum fields, might arise as a replacement manifold for $\mathcal{W}$, and how the phenomenology and laws of field physics might emerge from the unconstrained topological structure of $\mathcal{W}$. Neither geometry nor quantum fields are fundamental - instead each arises to give meaning to the other, with geometry providing a substrate for the quantum fields and the stress-energy of the quantum fields (corresponding to the unique world, $\mathcal{W}$) determining the space-time geometry.

In both of these world-views - the geometrical and the topological - the splitting of space and time has played a central role. With each passing instant a new universe, a new face of the world, is revealed to us. To a very great extent the changes that take place in the appearance of the universe are predictable, and the rules used to make predictions are the laws of physics. Thus, in the geometrical world-view, the evolution of the geometry of space is predicted with the use of the Einstein field equations; while, in the topological world-view, changes in the topology of the 3-dimensional slices of $\mathcal{W}$, although random, are subject to the restrictions imposed by the topological selection rules. The geometrical replacement manifold for $\mathcal{W}$ bridges the gap between pure geometry and pure topology, capturing the topological selection rules in the quantum field theory, $\mathcal{Q}$, and resurrecting GR in the form of Møller's semi-classical theory of gravity.

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