CHAPTER 4

THE TOPOLOGICAL WORLD

1. Gravity in a Quantum World

Classical concepts are adequate for the description and understanding of all observed features of gravitation. However, it is firmly established that the behaviour of matter in the real world can be fully understood only within the context of quantum theory. Material phenomena that can be effectively described with the use of classical (as opposed to quantum) variables arise as a consequence of the quantum behaviour of large systems, and are only manifested in macroscopic systems. Since general relativity, as it was formulated by Einstein and as I have presented it above, couples the space-time geometry to classical descriptors of matter, it is reasonable to conclude that GR is only valid when macroscopic (classical) systems are being investigated.

This limitation of GR has long been recognized, and many attempts have been made to remove it by constructing new theories that are valid in the quantum domain and which reduce to GR in the classical limit. The most common line of attack is to quantize the space-time metric much as one would any other field [23], however, quantized GR has been shown to be non-renormalizable [24] and it seems unlikely that renormalizability can be restored within a model that has the correct classical behaviour [25].

An alternative "semi-classical" approach was proposed by Møller [10]. He suggested that, rather than quantizing the gravitational field, it might be more appropriate to continue to think of it as a c-number field, with the expectation value of the quantum stress-energy tensor as its source:

$$G(\vec{U},\vec{V}) = 8\pi \langle T_{op}(\vec{U},\vec{V}) \rangle \quad . \tag{1.1}$$

These equations are to be solved self-consistently, with the quantum fields that contribute to the stress-energy being defined on the curved space-time that they determine. The major difficulty with the coupling (1.1) is that the simple normal ordering procedure used in special relativistic QFT to eliminate the zero-point energy from $\langle T_{op} \rangle$ has no obvious unique analogue on curved space-time [26]. The situation is not hopeless, however, and recent results obtained by imposing physical renormalization conditions at each order in perturbation theory [36] may well lead to a resolution of the problem. One of the most interesting features of this gravity modified quantum theory is that the linear superposition principle ceases to be valid because equation (1.1) is non-linear. This certainly represents a dramatic break from conventional quantum theory, but, as has been demonstrated by Everett in his "many worlds" interpretation of quantum mechanics [27], it is not unreasonable to assume that the entire universe is described by a single, smoothly evolving wave function, thereby eliminating the need for a superposition principle.

In addition to the two traditional approaches to the reconciliation of curved space-time and quantum theory, there have been several suggestions advocating the adoption of radically different world-views. Most notable amongst these are the <u>twistor theory</u> [28] being developed by Penrose and co-workers, in which the spin group $SL(2,\mathbb{C})$ plays a central role, and Finkelstein's <u>space-time code</u> [29] in which quantum processes are considered as fundamental and space secondary. Unfortunately, these theories are extremely complicated and they seem to be quite arbitrary. Unless they can be made more intuitive it is unlikely that they will ever gain popular acceptance.

Much of the recent interest in quantum gravity seems to have been stimulated by developments in elementary particle theory. The non-abelian gauge theories have provided a single formalism capable of handling all strong, weak, and electromagnetic interactions; and, as a bonus, the fibre bundle picture of gauge fields makes them look (at least superficially) similar to the gravitational field [13]. If the graviton could be added to the elementary particle zoo, using gauge theory techniques, then particle theory would, in a sense, be complete. From a different viewpoint, gravity appeared as the only remaining physical phenomenon that might be able to eliminate the singularities that occur throughout quantum field theory. In either case, it was (and is) suspected that gravity and particle physics are linked in some fundamental way, and that neither one can be fully understood without the other.

In the remainder of this chapter I shall pursue this connection between gravitation and particle physics, looking for answers, not in quantum theory, but in the structure of space-time. I take my guidance from Einstein, who showed most elegantly that the removal of unnecessarily restrictive assumptions can reveal beautiful and exciting physics.

2. Pregeometry is No Geometry

The key to special relativity was the revelation that time need not be absolute. Einstein quickly realized, though, that even the Minkowski space-time was too restrictive - its absolute geometric structure could not be justified. Freeing up the geometry led naturally to GR and an understanding of gravity never before dreamed of.

In early investigations of curved space-time it was just assumed that space and space-time have the topologies of $\mathbb{R}^3$ and $\mathbb{R}^4$, respectively, but it was soon realized that this assumption was also too restrictive. Observations of the universe extend out only a finite distance, so on a very large scale the topology is indeterminate. Other topologies (than $\mathbb{R}^4$) were investigated and gave interesting results, and GR quickly assumed a central role in the field of cosmology [30].

In the other direction, at small distances rather than large, the situation is similar. Experimentally, we have only been able to probe to 10^{-16}cm , which is a long way from the Planck length, $L_P \approx 10^{-33}$cm , at which gravitational effects are expected to significantly influence quantum processes. There is thus no compelling physical reason to suppose that the space-time topology remains trivial at lengths less than 10^{-16}cm (or 10^{-33}cm if you wish to be more conservative). Indeed, it has often been suggested that the space-time topology becomes extremely complex at short distances, with the degree of complexity increasing as the length scale decreases. As an extension of their already unified theory, Misner and Wheeler [9] showed that charge could be recovered from source-free electrodynamics by assuming a multiply connected space-time. Looking more towards quantum gravity, Wheeler [31] conjectured that space is a "foam-like" structure whose topology is constantly changing due to quantum fluctuations at lengths of the order of L_P . He envisaged particles as being macroscopic collective modes of the fluctuating topology/geometry.

With the completely arbitrary topology of Wheeler's quantum geometrodynamics, it would seem as though Einstein's programme of removing restrictive assumptions about the structure of space and time has been brought to a conclusion. But Wheeler is not yet satisfied. He argues that if one can obtain electromagnetism without electromagnetism and charge without charge, then one should also be able to obtain geometry without geometry; and he has coined the word <u>pregeometry</u> to symbolize the structure from which geometry arises.

The nature of pregeometry is very vague. Wheeler surmises that it might be topological or even "pretopological", but he has no specific model for it. By drawing a picture in which each topological configuration of space is endowed with a geometric structure, he seems to imply, however, that pregeometry is something conceptually distinct from (and in addition to) the topology of space or space-time. I believe that this picture is unnecessarily complicated, and that the pregeometry which Wheeler seeks is nothing more than the topology of space-time.

Consider a 4-dimensional topological manifold, $\mathcal{W}$, whose global topology is extremely complex. Although it is always possible to assign to $\mathcal{W}$ some particular geometry or field structure, I shall assume that all such fields are irrelevant - $\mathcal{W}$ is completely characterized by its topology. Moreover, the global topology of $\mathcal{W}$ is not subject to any restrictions beyond those that are necessary to preserve the manifold structure. Now imagine trying to describe some of the gross features of the global structure of $\mathcal{W}$ without knowing all the fine features. The usual topological descriptors become useless because they depend on a complete knowledge of the details, but perhaps there is an alternative mode of description. If we consider a topologically simple 4-manifold, $\mathcal{M}$, then perhaps we can replace (or symbolically represent) some of the topological complexities of $\mathcal{W}$ with the use of appropriately chosen fields on $\mathcal{M}$.

This is the basic picture of physics that I shall begin to develop below. I think of the objective world underlying all of our perceptions as an unimaginably complex, 4-dimensional manifold, $\mathcal{W}$. All of physics for all of time is coded into the topology of $\mathcal{W}$, but even this vast amount of information represents only a tiny fraction of the information contained in $\mathcal{W}$. If $\mathcal{W}$ is the (real) objective world, then the space-time-matter world of our perceptions is but a faint shadow. Matter and all its properties, life, and even intellect are contained in that shadow. In this world of perceptions, most information about the topological complexities of $\mathcal{W}$ is lost, and the remainder is represented by matter fields defined on a topologically simple, geometrical space-time manifold. Conventional space-time thus emerges as a replacement manifold for $\mathcal{W}$ - a simplified version of $\mathcal{W}$ which has no objective existence.

Wheeler's vision of transcending geometry is realized, not by appealing to some new mathematical or logical structure, but by recognizing the stupendous amount of information that can be coded into the topology of a 4-dimensional manifold. If we think of $\mathcal{W}$ as space-time viewed on a deeper level, then pregeometry is the space-time topology. When geometry is born, topological complexities must die; so geometric space-time is topologically simple at small distances (in contrast with Wheeler's geometrical space-time foam).

3. Breaking the Topological Code

My conjecture is that all of physics is encoded in the mathematics of 4-dimensional topological manifolds, and that the particular world we are a part of corresponds to a particular manifold, $\mathcal{W}$. The problem now is to break the code and extract physical laws from an (almost) unconstrained mathematical system. I cannot claim to have done this, but I do have suggestions for a scheme that I consider to be worth pursuing.

Again, just as in Chapter 2 , I shall begin with an investigation of 3-dimensional manifolds. My principal reference here is "3-manifolds" by John Hempel [32], which is quite a complete survey of progress, up until 1976, on the problem of classifying all 3-manifolds. This is a very difficult problem in topology and most of the techniques being used to solve it are beyond the grasp of a novice like myself. Nonetheless, there are some general results that are easily apprehended, and which seem particularly useful for the physics problem I have set myself.

Let M_1 and M_2 be connected 3-manifolds (possibly with boundaries) and let B_1 , B_2 be closed 3-cells in the interiors of M_1 , M_2 , respectively. Removing the interiors of these cells leaves the remainders $R_i = M_i - \text{Int } B_i$, $i = 1,2$. A third 3-manifold M is said to be a <u>connected</u> <u>sum</u> of M_1 and M_2 if there exist embedding maps $e_i : R_i \to M$ such that $e_1(R_1) \cap e_2(R_2) = e_1(\partial B_1) = e_2(\partial B_2)$ and $M = e_1(R_1) \cup e_2(R_2)$. This is denoted by $M = M_1 \# M_2$. If either M_1 or M_2 is non-orientable then

$M_1 \# M_2$ is unique up to equivalence; but if both are oriented then two distinct manifolds may arise, corresponding to the cases when $e_1^{-1} \circ e_2$ is an orientation preserving and orientation reversing homeomorphism of the 2-sphere boundaries. In situations I shall consider, this ambiguity will never occur.

"Connected sum" is a well defined associative and commutative operation, so for any finite k the notation $M_1 \# M_2 \# \ldots \# M_k$ is unambiguous.

For any 3-manifold M it is obvious that $M \# S^3 = M$, so the 3-sphere behaves as an identity element for connected sums. M is said to be prime if $M = M_1 \# M_2$ implies that one of M_1 , M_2 is a 3-sphere.

3.1 Theorem Each compact 3-manifold can be expressed as a connected sum of a finite number of prime factors [32].

Prime decomposition is not unique. Hempel shows that if $M = M_1 \# (S^2 \times S^1)$ where M_1 is non-orientable, then $M = M_1 \# P$. Here P is the non-orientable S^2 bundle over S^1 (the 3-dimensional analogue of the Klein bottle) and both $S^2 \times S^1$ and P are prime. To get around this problem he defines a normal prime factorization of a 3-manifold M to be a prime factorization $M = M_1 \# \ldots \# M_k$ such that some M_i is $S^2 \times S^1$ only if M is orientable. This leads to the central result:

3.2 Theorem Let $M = M_1 \# \ldots \# M_k = M^*_1 \# \ldots \# M^*_{k^*}$ be two normal, prime factorizations of a compact 3-manifold M. Then $k = k^*$ and (after reordering) M_i is homeomorphic to M^*_i [32]. That is, there is a unique normal, prime factorization for each compact 3-manifold (with boundary).

With this last result, the problem of classifying all compact 3-manifolds is reduced to the problem of finding and classifying all prime 3-manifolds. However this is still a very difficult task which is far from complete. Even though an infinite number of prime 3-manifolds have already been identified there are many more yet to be found.

Now, what I want to do is to build up the topological space-time manifold, W, by stacking together 3-dimensional submanifolds. This will create a picture concordant with the perceived special status of space-like hypersurfaces in geometrical space-time. It will also introduce the concept of time on a fundamental level. Although it is not clear whether this assumption is necessary or not, I shall assume, for illustrative purposes, that W is endowed with a differential structure of class C^∞. The Whitney embedding theorem [33] then allows me to consider W as a smooth submanifold of $\mathbb{R}^8$; inducing on W a (non-physical) Riemannian metric, g_8.

Let B be a closed 8-cell in $\mathbb{R}^8$ such that $W' = W \cap B$ is a connected, compact (yet still extremely complex), 4-dimensional submanifold-with-boundary of W and $\partial W' = W \cap \partial B$ is a

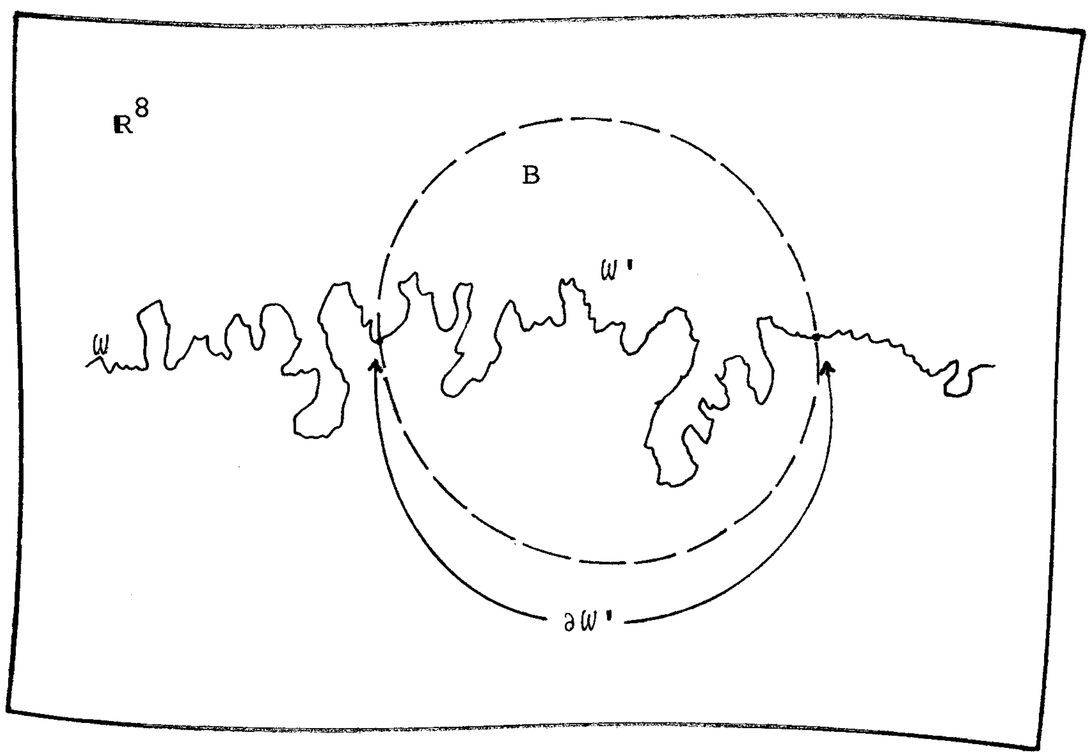


Figure 3.1 W may always be considered as a submanifold of $\mathbb{R}^8$. The closed 8-cell, B , is chosen such that $W' = W \cap B$ is connected and $\partial W' = W \cap \partial B$ is closed.

closed 3-manifold (Figure 3.1). W' will correspond to a closed 4-cell in geometric space-time, whereas the entire geometric space-time (corresponding to all of W) may have a non-trivial topology. Choose, for the "initial" 3-dimensional submanifold of W', a submanifold S_0 such that $\partial S_0 = S_0 \cap \partial W'$ is a 2-sphere which divides $\partial W'$ into two pieces, one to the "past" and one to the "future" of ∂S_0. With this condition on the boundary, the prime decomposition of S_0 must take the form

$$S_0 = M_1 \# \ldots \# M_k \# B^3 \quad , \tag{3.3}$$

where B^3 is the closed ball in 3-dimensions and the manifolds M_i have no boundaries [34].

Turn now to the geometry induced on S_0 by the embedding of W in $\mathbb{R}^8$. Since its geometry has no physical significance, we may deform W (and B) in $\mathbb{R}^8$ to make it assume whatever geometric configuration we wish, subject of course to the constraints imposed by topology. In particular, we can assume that the embedding has been chosen so that S_0 takes the form of a Euclidean space onto which a large number, k, of small, widely spaced, prime 3-manifolds, M_i, have been fastened (Figure 3.2). For comparison, the 2-dimensional analogue of S_0 is a Euclidean disc onto which have been fastened (by cutting and pasting) a large number of very small and widely separated handles (or crosscaps). The main difference is that 3 dimensions provides an infinite variety of distinct objects, rather than just

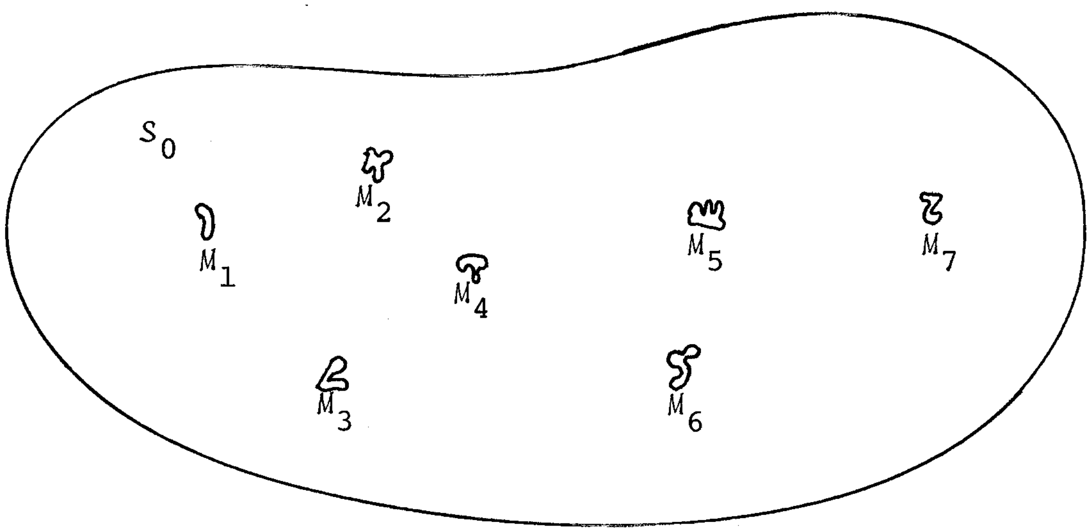


Figure 3.2 The geometry of S_0 is chosen to make it appear like a Euclidean space onto which small, widely separated, prime 3-manifolds, M_i , have been fastened.

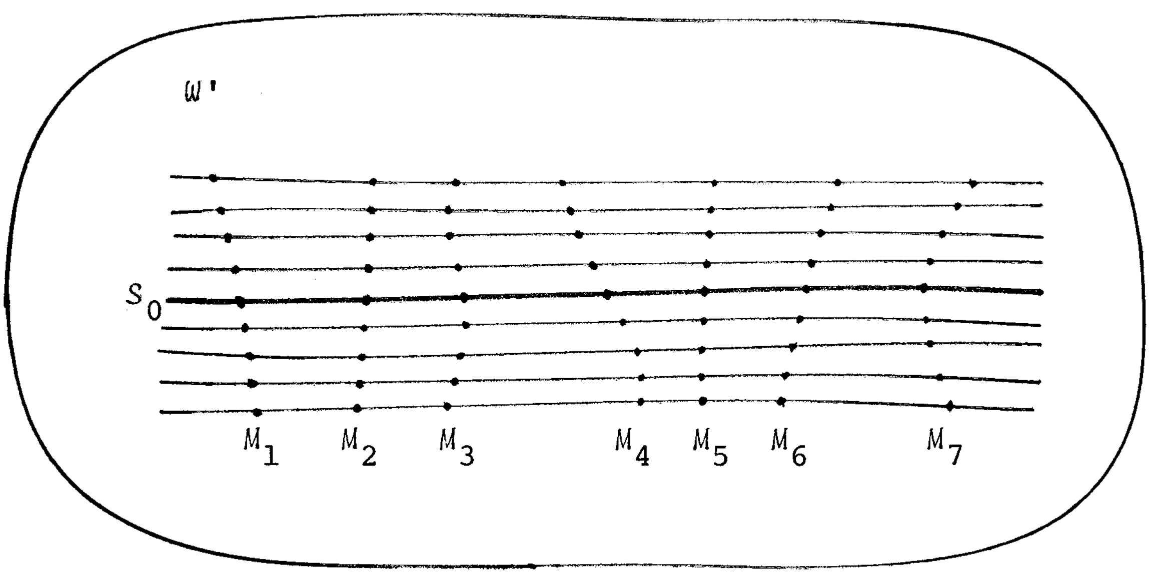


Figure 3.3 Hypersurfaces near S_0 are assigned similar geometries so that the topological anomalies appear to travel on smooth paths through W' .

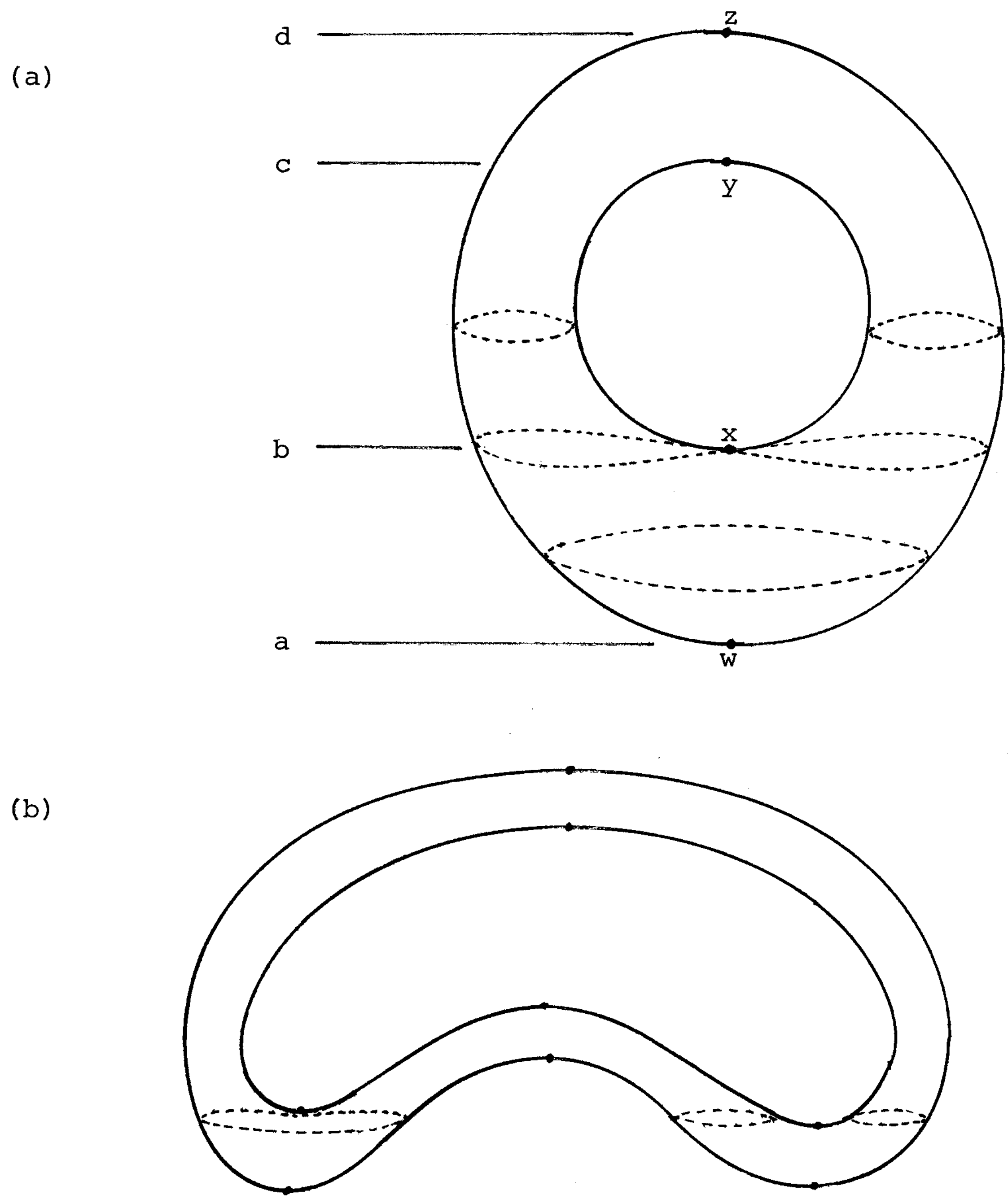


Figure 3.4 (a) The topology of 2-dimensional slices through the solid torus changes at the critical points w,x,y,z . (b) A different choice of geometry can produce additional critical points.

handles or crosscaps.

Sufficiently small deformations of S_0 in W' yield new hypersurfaces with the same topology as S_0 . Let $\{S_t: t \in I_\delta\}$ be a continuous family of mutually disjoint hypersurfaces in W' such that, for $t > s$, S_s may be continuously deformed through W' into S_t , and $\partial S_t = S_t \cap \partial W'$ is a 2-sphere in the piece of $\partial W'$ lying to the future of ∂S_s . Each of the 3-manifolds S_t , $t \in I_\delta$, is homeomorphic to S_0 and has the same prime factorization: $M_1 \# \ldots \# M_k \# B^3$. By choosing the induced geometries to be similar to that already chosen for S_0 , we arrive at a simple geometric picture of an open region of W' containing S_0 which portrays space-time as an (almost) Euclidean space that is being traversed by k very small and widely separated topological anomalies (Figure 3.3).

If we try to deform S_0 too far through W' , however, we will run into topological obstructions because W' does not have the product topology $S_0 \times I$, with I a closed interval. This is best illustrated in three dimensions, rather than four, by looking at 2-dimensional slices through the solid torus (Figure 3.4(a)). Between a and b all of the slices have the topology of a disc, but at b the 2-dimensional section ceases to be a manifold (due to the singular point x), and between b and c each section is the disjoint sum of two discs. The point x , and also w,y,z are called critical points of the torus [35]. Although a different choice of geometry, such as in Figure 3.4(b), could have produced additional critical points (which need be neither isolated nor non-degenerate), only the four isolated and non-degenerate critical points of Figure 3.4(a) are demanded by the topology of the torus.

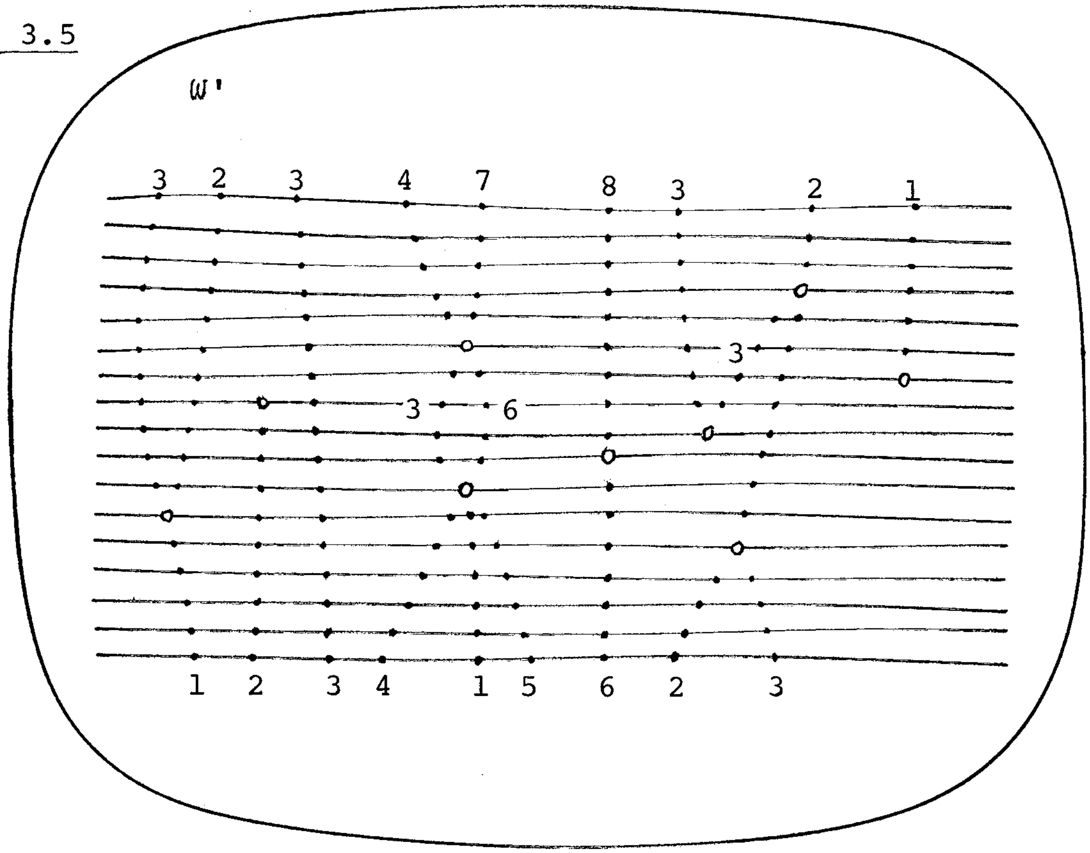
Returning to the topological space-time manifold, $\mathcal{W}'$, we see that we can continue constructing surfaces S_t , for ever increasing t , until at some $t_c > \delta$ a critical point of $\mathcal{W}'$ is reached. Beyond t_c the topology of the hypersurfaces differs from that of S_0 . However, the change in topology that takes place at an isolated critical point is small compared to the tremendous complexity of S_0 ; indicating that most of the prime factors, M_i , that appear in the factorization (3.3) of S_0 do not participate in the topological changes and continue to appear in the prime factorization of S_t for $t > t_c$. Since the topological change takes place at an isolated critical point in $\mathcal{W}'$, those prime factors of $S_{t_c-\varepsilon}$ and $S_{t_c+\varepsilon}$ that <u>do</u> participate must "meet" at the critical point. Adapting to this situation the specialized geometry introduced above leads to the geometric representation of $\mathcal{W}'$ shown in Figure 3.5(a), in which distinct prime 3-manifolds are labelled by distinct integers.

It should be clear, from the above analysis, that all of the topological complexities of $\mathcal{W}'$ are now represented by the (labelled) graph-like structure shown in Figure 3.5(b). Each line corresponds to a prime, compact 3-manifold without boundary;

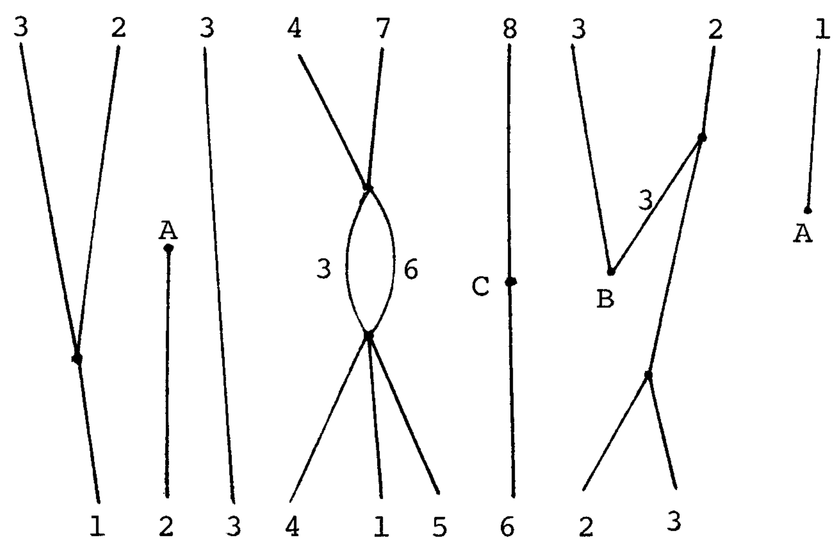
Figure 3.5 (a) Different hypersurfaces of $\mathcal{W}$ may have different topologies. Changes in the hypersurface topology take place at isolated critical points (small circles), with only a small number of prime 3-manifolds, M_i , meeting at each critical point. (b) Stripping the inessential details from the geometric picture, (a), leaves a simple graphical representation of $\mathcal{W}'$.

Figure 3.5

(a)



(b)



and the integer(s) associated with it identify (through some as yet unknown classification scheme) which element of the infinite collection of distinct prime 3-manifolds is being considered. The vertices, which correspond to critical points of W' , need no labels because they are completely characterized by the labelled lines that emanate from them. Critical points such as B are non-essential and can be eliminated by a more careful choice of the hypersurfaces, S_t . However, the remaining critical points (or vertices) are demanded by the topology of W' , just as the four critical points of Figure 3.4(a) are demanded by the topology of the torus. The minimal graph, obtained by eliminating all non-essential vertices (such as B), is thus unique up to the operation of flipping external lines from the past to future and vice versa (which corresponds to choosing a different 2-sphere in $\partial W'$ to bound S_0).

The vertices labelled A and C in Figure 3.5(b) are included only because I cannot prove that such vertices do not exist. The possibility of having vertices such as A seems remote, however, and I shall assume from now on that they never occur. If vertices such as C exist, which I also doubt, then they can be eliminated by assigning the same label to all lines that can be joined to each other by vertices like C . Thus, the lines labelled 6 and 8 in Figure 3.5(b) would be assigned the same label, say 6 . The new graph so obtained would be uniquely determined by W' , but it would no longer provide a faithful representation of this region of topological space-time.

By noting the remarkable similarity between the (not necessarily faithful) graphical representation of $\mathcal{W}'$ and the Feynman diagrams of quantum field theory, we can now initiate the transition from topological space-time to geometrical space-time. Think of $\mathcal{W}'$ as not just something that bears a resemblance to a Feynman diagram, but rather think of it as being a Feynman diagram. Think of each line segment in Figure 3.5(b) as a distinct virtual particle; and think of each vertex as an unrenormalized particle interaction vertex.

The integer labels on the lines in Figure 3.5(b) identify which prime 3-manifold is to be associated with each line, but they serve equally well to identify the elementary particles. There is thus a one-to-one correspondence between the distinct, prime, compact 3-manifolds without boundary (excluding $S^2 \times S^1$) and the elementary particles of quantum physics. It follows immediately that there is a countable infinity of distinct elementary particles, and not just a (small) finite number as is most often supposed.

Allowed particle interactions - that is, allowed vertices - are determined in conventional quantum theory by the phenomenological field theory which the Feynman diagrams represent. In the topological space-time, however, the rules which determine what lines may meet at a vertex (critical point) are purely topological in nature. They are imposed by the simple requirement that $\mathcal{W}$ be a 4-dimensional manifold, and they may, in principle, be derived. Unfortunately, this "selection rule" problem seems just as difficult as, and is clearly dependent upon, the classification of prime 3-manifolds.

Assume, nonetheless, that the classification and selection rule problems are solved, yielding a complete array of particles and interactions. The particles are naturally divided, by the orientability of their corresponding prime 3-manifolds, into two classes. One orientability class will yield bosons in geometrical space-time and the other class will yield fermions. Which is which must be decided with the aid of the selection rules. Ultimately, the selection rules must also be called on to identify the particular prime 3-manifold that corresponds to each known elementary particle (electron, photon, etc.).

Turn, at last, to field theory. Abandon $\mathcal{W}'$ and replace it by the topologically trivial manifold, $\mathcal{M}' \simeq B^4$ (with B^4 the closed ball in $\mathbb{R}^4$). Assume, for the time being, that $\mathcal{M}'$ has a globally Minkowskian metric, η ; and construct on $\mathcal{M}'$ a quantum field theory, $\mathcal{Q}$, with fields ψ_i , $i \in \omega$, such that a one-to-one correspondence between the ψ_i's and the prime 3-manifolds (without boundaries) may be found, which places the interaction vertices of the Feynman graphs of $\mathcal{Q}$ in one-to-one correspondence with the allowed vertices of $\mathcal{W}'$. The parameters (masses and coupling constants) of $\mathcal{Q}$ will, of course, be undetermined, but even when this freedom is ignored there may still be several theories, $\mathcal{Q}_1$, $\mathcal{Q}_2$, $\mathcal{Q}_3$,...., which satisfy the above requirements. From this collection of candidate theories choose the one theory, $\mathcal{Q}$, that is completely determined by its Feynman graphs.

Propagators in $\mathcal{Q}$ carry the virtual particles through M' with constant velocities; and interactions of the fields take the simplest possible form that is consistent with the required vertices. Whenever two or more different particles (fields) have exactly equivalent, yet distinct, allowed interactions the associated fields have identical masses and coupling constants in $\mathcal{Q}$ (even though these parameters are not yet known). "Internal" symmetries, such as colour SU(3) , thus arise out of the topology of $\mathcal{W}$ in a natural way.

In order to fix the masses and coupling constants, go back now and reconsider the geometry of M' . It was necessary to assume a c-number metric in the first instance because without it the whole quantum theory, $\mathcal{Q}$, would collapse. However, a physical metric, g , cannot be arbitrarily imposed, as η was. Instead, g must arise out of a logical analysis of the topology of $\mathcal{W}'$ and, in particular, the graphical representation of $\mathcal{W}'$ obtained above. Since all of this topological information has already been exploited in the construction of $\mathcal{Q}$, our only option is to have $\mathcal{Q}$ determine g in some self-consistent way. The correct coupling will give g the simplest possible form; and Møller's proposal,

$$G(\vec{U},\vec{V}) = 8\pi\langle T_{op}(\vec{U},\vec{V})\rangle \quad , \tag{1.1}$$

seems ideally suited for this purpose. All partial derivatives in $\mathcal{Q}$ are now converted to covariant derivatives (minimal coupling) and the state $\rangle$ is determined by the particular topology of $\mathcal{W}$. It is to be assumed, as well, that a unique procedure has been found for eliminating the zero-point energy from T_{op} (cf. Section 1); and that $\mathcal{Q}$ has been renormalized on the background g, leaving only "physical" masses and coupling constants.

The specific geometry obtained from (1.1) depends not only on the state, $\rangle$, but also on the values, m_i, c_α, that are chosen for the masses and coupling constants. To obtain a unique space-time geometry require that

$$\frac{\delta g}{\delta m_i} = 0 \quad \text{and} \quad \frac{\delta g}{\delta c_\alpha} = 0 \quad . \tag{3.4}$$

Solve these equations to find the unique set of physical masses and coupling constants and hence the unique geometrical space-time, $\{\mathcal{M}',g,\mathcal{Q}\}$, corresponding to $\mathcal{W}'$.

I have moved quickly through this formal construction of geometrical field theory, and in doing so I have passed over many very real problems, both technical and philosophical. Most of these are due to the non-linearity of the semi-classical field theory. As mentioned above, some progress has been made on the factor ordering (which must be solved to make $\langle T_{op}\rangle$ finite); but the problem of renormalizing interacting quantum fields on a self-consistently determined background geometry remains untouched. Also required by the non-linearity of (1.1) are significant changes, of both an interpretive and a mathematical nature, in the foundations of quantum theory.

A new problem, more directly related to my topological picture of the world, arises from equations (3.4). These equations may be thought of as a generalized bootstrap, with the mutual interactions of the elementary particles determining all the masses and coupling constants. However, because the metric is a tensor rather than a scalar field it may be impossible to satisfy all of the equations (3.4); and even if g had only one degree of freedom (such as the Newtonian gravitational potential) the solutions, m_i , c_α , would, in general, not be constants but rather functions of the space-time coordinates. In this latter case, one could reasonably expect counter-terms from the renormalization to suppress fluctuations of the parameters, with any remanent variations having a length scale much larger than the radius, $g/\nabla g$, characteristic of changes in the local geometry. No great problems will arise in the quantum theory as long as the geometric radius remains large when compared with the Compton wavelengths of the particles being considered; but if the masses and coupling constants change too rapidly, or if it becomes necessary to consider more than one component of g , then the entire field theory will collapse. It is worth noting that in our local region of the universe the metric is, in fact, adequately specified by the one component, g_{00}, and that fluctuations of g about η have characteristic lengths of about 10^{15}m (at the surface of the sun) or more. Thus, only in the neighbourhood of a gravitational shock wave or some other equally catastrophic gravitational event should the masses and coupling constants be expected to change noticeably and, perhaps, become ill defined.

4. Summary

Although the technical hurdles still to be cleared are immense, the rough outline presented above shows that the idea of extracting field physics from the topology of a 4-dimensional world is not so crazy as to be impossible. By constructing the graphical representation of $\mathcal{W}'$ (and, by extension, $\mathcal{W}$) we are actually led directly to quantum field theory. The intricate web of virtual particles and interactions is reduced systematically, through the renormalization procedure, to leave a "physical" graph that represents physical particles propagating and interacting in a geometry of their own creation. This geometry depends on the particular topology of $\mathcal{W}$ and is a c-number field - gravity is <u>not</u> quantized. Uniqueness of the fields is assured by choosing the masses and coupling constants such that infinitesimal variations of these parameters leave the geometry unchanged.

The splitting of space and time is essential in the construction of a semi-local (neither global nor local) representation of the topology of W . So also is the dimension, three, of space, because a non-trivial connected sum decomposition is possible only in three-dimensions [37]. (A similar decomposition is possible in 2-dimensions, but all factors are identical.)

In the end, though, the most remarkable and compelling feature of this topological world-view is its simplicity. Providing the theory is born out by further analysis, all perceived physical phenomena, including gravity, quantum effects, and the detailed behaviour of the elementary particles, will be understood as characteristics of an unconstrained 4-dimensional topological manifold.