CHAPTER 1

INTRODUCTION

This thesis is a theoretical investigation of the fundamental nature and form of the physical world. Holding fast to the belief that the phenomena we perceive are manifestations of some exceptionally coherent and conceptually simple mathematical structure, I have placed the emphasis throughout on philosophical ideas rather than on phenomenological details. Elementary notions of time and evolution are shown to place significant constraints on the geometric structure of space-time, yielding, in the simplest case, a new derivation of general relativity. Geometrical field theory need not be fundamental, however; and preliminary arguments are given to support the conjecture that all of field theory, including quantum phenomena and particle physics phenomenology, is subordinate to and derivable from the unconstrained topological structure of space-time.

1. First Principles

Tightly bound into our modern world-view are several intuitive notions regarding time, space, space-time, and evolution. These constitute a (still evolving) set of guiding principles that we feel should be embodied in any reasonable physical theory. Recognizing that they are subject to change and personal differences, I shall place particular emphasis, in my statement of these principles, on our perception of the world rather than on the world itself.

1.1 Uniqueness - All of the physical phenomena that we perceive are manifestations of a unique and definite structure, which I call the world.

1.2 Time - There is a natural partial ordering for the phenomena that we are capable of perceiving (henceforth called events). If two events are ordered, then one lies either to the future or to the past of the other, and they are said to be time-ordered. A future directed time-like path from an event a to a future event b is a time-ordered set of events, $T(a,b) = \{a,\ldots,b\}$, which is maximal in the sense that the addition of any event, x , that lies neither to the past of a nor the future of b would necessarily destroy the time-ordering of $T(a,b)$. When given the natural topology induced by the time-ordering, each such path is homeomorphic with the closed interval $[0,1]$ in $\mathbb{R}$.†

1.3 The Universe - Two events, x_1 and x_2 , are said to be space-like if and only if there exist events x_3 to the past and x_4 to the future of x_2 such that neither x_3 nor x_4 lies to the past or future of x_1 . If $\mathcal{U}$ is a set of

† A summary of much of the mathematics and the notation used in this thesis is provided in Appendix I.

mutually space-like events that is maximal with respect to the space-like condition, then all events in $\mathcal{U}$ may be simultaneously perceived, and $\mathcal{U}$ is called the universe at some time.

1.4 History - If $\mathcal{U}_1$ and $\mathcal{U}_2$ are two non-intersecting universes such that $x_1 \in \mathcal{U}_1$ lies to the future (past) of some $x_2 \in \mathcal{U}_2$, then $\mathcal{U}_1$ is said to lie to the future (past) of $\mathcal{U}_2$. A history is a time-ordered one-parameter family, $\mathcal{H}$, of non-intersecting universes, $\mathcal{U}(t)$, $t \in [a,b]$, which is maximal in the sense that every event x that lies to the future of some event in $\mathcal{U}(a)$ and to the past of some event in $\mathcal{U}(b)$ is contained in $\mathcal{U}(t)$ for some $t \in (a,b)$. The sequence of universes of events of which we progressively become aware as time passes is a history.

1.5 Space and Matter - The universe appears to us to be a synthesis of two very different kinds of structures: space and matter. Serving as a stable repository for matter, space is a three-dimensional differential manifold, $\mathcal{S}$, of indeterminate global topology. Matter, the stuff that "resides in space" and gives events their distinguishing features, is characterized by a countable (and perhaps finite) number of tensor fields, ${}^3F_i$, $i \in \omega$, of class C^∞ on $\mathcal{S}$.

1.6 Geometry - Defined on $\mathcal{S}$ is a Riemannian (ie. positive definite) metric tensor field, ${}^3g$, of class C^∞. This is manifested in the sizes and shapes that we perceive to be characteristic features of all matter distributions.

1.7 Continuity - As time passes, the geometry and the matter distributions that are defined on space undergo continual change, but these changes are never discontinuous. Denote by ${}^3F_i(t)$ and ${}^3g(t)$ the matter and metric tensor fields on S that correspond to the universe $U(t) \in H$. Then for each $x \in S$ the tensors ${}^3F_i(t;x)$ and ${}^3g(t;x)$ are continuous functions of t . Moreover, there always exists a parameterization, $t = t(t')$, such that ${}^3F_i(t';x)$ and ${}^3g(t';x)$ are differentiable functions of t' of class C^∞ .

1.8 Space-time - There is a unique, smooth, 4-dimensional manifold, M , which has defined on it a set of tensor fields, F_j , $j \in \omega$, such that any universe $U(t)$ may be realized as a submanifold, $S(t) \subset M$, with the fields ${}^3F_i(t)$, ${}^3g(t)$ induced by the embedding $e_t : S \to M$; $e_t(S) = S(t)$. Each history is a foliation of some region $M' \subset M$, M' having the topology of $\mathbb{R}\times S$. The manifold M , together with whatever geometric structure it may have, is called <u>space-time</u>.

1.9 Evolution Rules - Let $\{S(t); t \in (a,b)\}$ be a foliation of $M' \subset M$ by space-like hypersurfaces such that for each $t_0 \in (a,b)$ and all $x \in S(t_0)$, every past directed time-like path through x intersects every one of the hypersurfaces $S(t)$, for $t \in (a,t_0)$. Then, for each $t_0 \in (a,b)$, a knowledge of the fields ${}^3F_i(t_0)$ and ${}^3g(t_0)$, and a finite number of their derivatives with respect to t , on $S(t_0)$ is sufficient to completely determine these fields on any future hypersurface $S(t)$,

$t \in (t_0, b)$, and also to determine the fields F_j at all points of M' to the future of $S(t_0)$. The rules that govern this "evolution" do not depend in any way on the field configurations.

In brief form, the first of these principles, (1.1), asserts my belief in the existence of an objective reality; items (1.2) to (1.8) establish the conventional space-time-matter picture (without clocks) as the appropriate model for the space of our perceptions; and (1.9) stipulates that the universe must evolve in a predictable fashion.

2. Classical Physics

It should be clear that all of the principles (1.1) to (1.9) originate in classical (as opposed to quantum) physics, and so I shall explore their implications only in the classical context. The main results that I shall obtain in Chapter 2 are:

2.1 The space-time appropriate for modelling perceived (classical) phenomena has defined on it a locally Minkowskian metric tensor field, g , whose projection onto any space-like hypersurface provides the metric, ^3g , for that universe; and

2.2 In the absence of additional fields, the metric g satisfies a covariant set of local partial differential equations in which the highest derivative of g enters linearly. When restricted to second order, these are Einstein's vacuum gravitational field equations (with the cosmological term).

These are certainly not new results, but I believe that the novel approach that I have used in their derivation provides some new and useful insight into why general relativity (GR), or a related higher order theory, should be considered as fundamentally correct.

Many different approaches have been used to derive GR in the past. Einstein made use, primarily, of the classical correspondence principle, requiring that Newtonian gravity emerge as the weak field, small velocity, limit of a generally covariant theory [1]. A Lagrangian formulation was found by Hilbert [2], and recently Hojman, Kuchař, and Teitelboim (HKT) have derived general relativity by investigating the integrability conditions of the Hamiltonian equations of motion [3]. Several authors have started from a quantum picture and used (Lorentz) gauge invariance arguments to obtain the Einstein equations [4], while Boulware and Deser have used aspects of quantum <u>particle</u> physics as their starting point [5]. A good summary of the earlier derivations is provided by Misner, Thorne, and Wheeler [6].

The present derivation is closest, in spirit, to that of HKT. Rather than assuming, at the outset, that the space-time metric, g , satisfies some set of covariant field equations on M , they focussed attention on the space-like hypersurfaces of the space-time manifold. They sought a set of evolution rules that would propagate data, defined on some initial hypersurface, forward in time onto a future space-like slice, and they stipulated that the result should be independent of the choice of intermediate space-like hypersurfaces used in the time integration. This much of the HKT formalism I have kept, although I have abandoned their notation in favour of a more intuitive, coordinate-free notation.

I deviate from HKT in the assumptions that actually fix the dynamics. They assume that (in the vacuum case) the spatial metric, ${}^3g$, and a conjugate super-momentum provide all the necessary initial data, and they postulate that the equations governing the evolution of these fields take the Hamiltonian form [7], with the Poisson bracket algebra of the super-Hamiltonian and super-momentum closing exactly as the commutator algebra of the generators of hypersurface deformations. The philosophical motivation for these assumptions is not at all clear, and in the first part of my derivation I eliminate them completely. Without any new assumptions to replace them, I obtain (2.1) and the first half of (2.2). In order to pick the Einstein equations from the myriad of possibilities, a new assumption is necessary, however, and I have chosen the more conventional route of restricting the initial data to be ${}^3g$ and its "velocity", which is denoted by K . The obvious advantage in choosing this data, as opposed to that used by HKT, is that this choice obviates the need to assume anything about a Hamiltonian structure.

Space-times with matter fields, that is, fields F_j in addition to g , are considered in Chapter 3. It is shown, using the formalism developed in Chapter 2, that matter fields are quite able to influence the evolution of the spatial geometry, and that the space-time geometry, in turn, influences the evolution of the matter fields. When the geometrical initial data is once again restricted to include only ${}^3g$ and K , the standard Einstein coupling of matter and geometry is recovered - a result that is no surprise.

The most troublesome aspect of matter, at this stage, is that we have no philosophically appealing (non-empirical) criteria for deciding what kinds of matter fields should be considered, nor what equations they should satisfy (aside from the geometric constraints mentioned above). Many attempts have been made to construct unified field theories using alternative (non-Riemannian) geometries, and so obtain a geometric picture of electromagnetism or other kinds of matter fields. But these theories can always be reformulated in terms of Riemannian geometry plus tensor fields, making the physical significance of the alternative geometries uncertain at best.

Rainich [8], and later Misner and Wheeler [9], showed that the electromagnetic field need not be considered as something in addition to the metric, however, since it could actually be extracted from a suitably constrained space-time metric. A short discussion of this "already unified" theory of gravitation and electromagnetism is presented in Chapter 3. A somewhat different interpretation of the Misner - Wheeler results is then proposed and it is argued that, if all primary physical measurements are measurements of the metric geometry of space-time, then all matter fields, not just the electromagnetic field, must leave distinctive imprints in the space-time geometry, from which all of the observable characteristics of the various fields may be recovered. Fields which do not leave such distinct imprints could be eliminated from consideration, reducing the arbitrariness of the theory eventually adopted.

3. Quantum and Particle Physics

While classical field theory, and general relativity in particular, provides a very elegant and simple formalism for describing some of the features that we perceive to be characteristic of the world, it is unable to give us any insight into the nature or origin of quantum phenomena. It cannot tell us what kinds of (quantum) particles the world is made of or how they interact with each other. Normally, it is just assumed that quantum field theory (QFT) is the correct formalism to use when investigating quantum effects; and the particular particles that are known to exist are described with the use of phenomenological models, which are constantly being updated.

I find this situation highly unsatisfactory on two counts: (1) there is no compelling logical rationale (aside from its empirical successes) to indicate that QFT (or even ordinary QM) is a reasonable formalism to adopt; and

(2) the particle physics phenomenology we adopt has no philosophical basis to guide us in our search for a fundamental theory. At the same time, though, I cannot help but be impressed by the remarkable achievements of quantum theory - achievements that must indicate to even the strongest opponents of quantum mechanics that there is something correct about this inscrutable formalism.

Why does quantum theory work? What are the guiding principles from which it follows? How can we replace particle physics phenomenology by fundamental theory? In Chapter 4, a radically new and strikingly simple view of the world is presented, which I believe may eventually provide satisfying answers to all of these questions. Carrying to its logical conclusion Einstein's lead in removing restrictive assumptions about the structure of space-time, it is proposed that *the objective world underlying all of our perceptions is a 4-dimensional topological manifold*, $\mathcal{W}$, *which has no physically significant geometry or field structure*, *but instead an unconstrained* (*and extremely complex*) *global topology*. Direct perception of the detailed structure of $\mathcal{W}$ would be analogous to observing all of the virtual particles in the quantum mechanical vacuum, which is clearly impossible. But many characteristic features of the topology of $\mathcal{W}$ are perceptible. These we interpret as fields (metric and quantum fields) on a topologically simple 4-dimensional manifold, that we call space-time. Our conventional space-time thus emerges, in this picture, as a *replacement manifold* for the objective world, $\mathcal{W}$, with the fields on space-time capturing as much information about the topology of $\mathcal{W}$ as possible (but not nearly all of that information).

The only restriction placed on the form of $\mathcal{W}$ is that it must be a 4-dimensional topological manifold. No additional physical laws are necessary, or even allowed, and therein lies the great beauty of this new world-view. Although this work is still very preliminary, it is argued that (an improved version of) quantum field theory and all of the phenomenology of the elementary particles should follow directly from a detailed study of the topology of 4-manifolds. The space-time geometry emerges in a natural way, yielding an intuitive understanding of the semi-classical coupling of gravity and matter first proposed by Møller [10], and indicating most emphatically that gravity should not be considered as another quantum field.

Unfortunately, due to the extreme difficulty of the mathematics involved and the attendant paucity of knowledge about three- and four-dimensional manifolds, it is not possible, at the present, to make testable predictions based on this new world-view. The potential for a great wealth of predictions is there, however, and I can only hope that the prospect of applications in fundamental physics will stimulate mathematicians to develop the relevant tools more rapidly.