Space-time Structure
and the Origin of Physical Law

by

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ABSTRACT

The laws of physics are viewed as mathematical statements which should follow from some set of fundamental principles. Included amongst these principles are basic notions of space, time and, since the development of relativity theory, space-time. In the first part of the thesis a traditional world-view is adopted, with space-time a topologically simple geometrical manifold, matter being represented by smooth classical fields, and space a Riemannian submanifold of space-time. Using a completely coordinate-free notation, it is shown how to characterize the space-time geometry in terms of fields defined on 3-dimensional space. Accepting only a finite number of the fields induced on space as independent initial data, a procedure is then given for constructing dynamical and constraint equations which will consistently and unambiguously propagate these fields forward in time. When the geometrical initial data is restricted to include only the hypersurface metric, ${}^{3}g$, and the extrinsic curvature, K , the resulting dynamical and constraint equations combine to form the Einstein gravitational field equations (with the cosmological term).

This is a new and very direct approach to general relativity, which shows quite clearly that the raison d'être of the Einstein field equations is to propagate the spatial metric forward in time in a consistent fashion. Higher order gravitational equations cannot be ruled out, however, nor does this investigation of the

space-time geometry provide the basis for a theory of matter. In an attempt to remove some of this arbitrariness, it is conjectured that matter fields are not observed directly, but only indirectly through their influence on the space-time geometry. This would imply the existence of a "super" already unified theory, modelled after the Misner - Wheeler already unified theory of gravity and electromagnetism [9], and it would provide an intuitive physical argument for the correctness of the Einstein equations.

The problem of synthesizing gravitational and quantum physics is approached by adopting a new and radically different world-view. It is proposed that the objective world underlying all our perceptions is a 4-dimensional topological manifold, $\mathcal{W}$, with no physically significant field structure, but instead an unconstrained and extremely complex global topology. Conventional space-time, with its geometry and quantum fields, is then a topologically simple replacement manifold for $\mathcal{W}$, with the fields on space-time replacing the topological complexities of $\mathcal{W}$. A preliminary outline of the correspondence is presented, using as its basis a remarkable similarity between a natural graphical representation of $\mathcal{W}$ and the Feynman graphs of quantum field theory. The technical problems are formidable, but if they can be overcome then this theory may be able to explain the origin of quantum phenomena and the detailed phenomenology of the elementary particles.

... It is part of the martyrdom which I endure for the cause of the Truth that there are seasons of mental weakness, when Cubes and Spheres flit away into the background of scarce-possible existences; when the Land of Three Dimensions seems almost as visionary as the Land of One or None; nay, when even this hard wall that bars me from my freedom, these very tablets on which I am writing, and all the substantial realities of Flatland itself, appear no better than the offspring of a diseased imagination, or the baseless fabric of a dream.

Edwin A. Abbott
From Flatland

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