

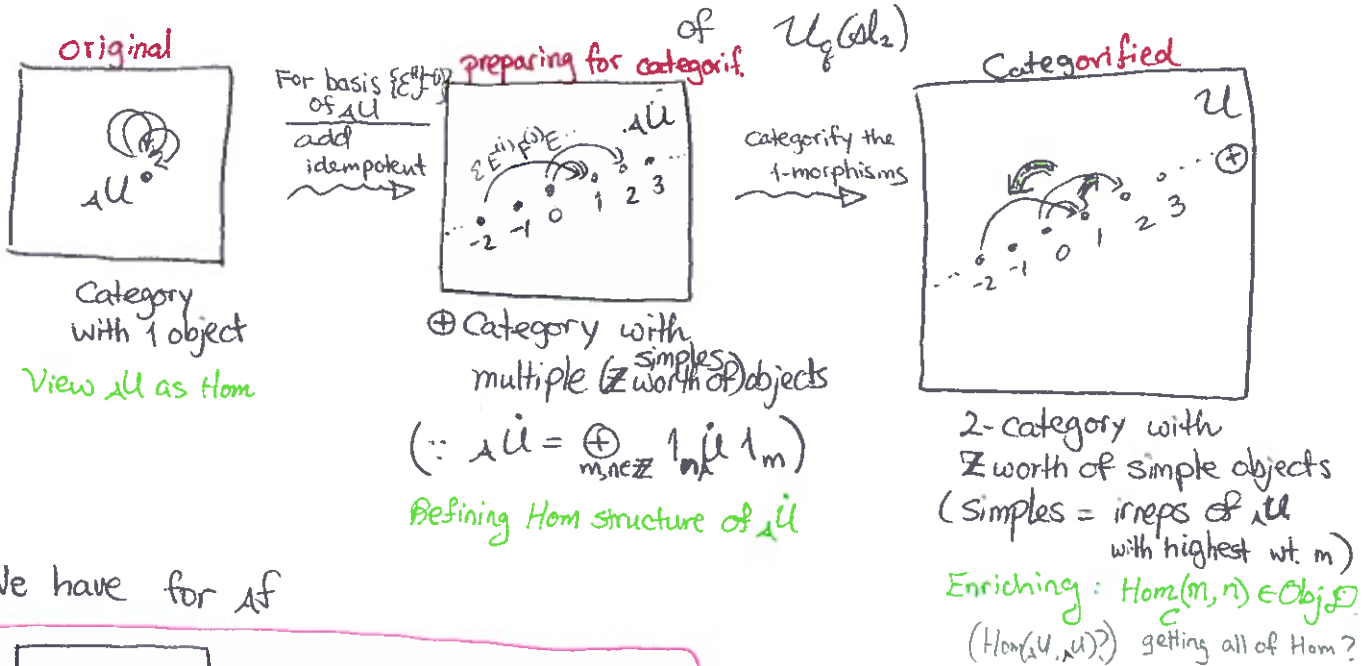
... on Section 3 of [LK1]

Goal: To categorify $\mathcal{A}\mathcal{U}$, the integral form of the quantum universal enveloping algebra of the negative half of the simply-laced Kac-Moody algebra $\sim \Gamma$.

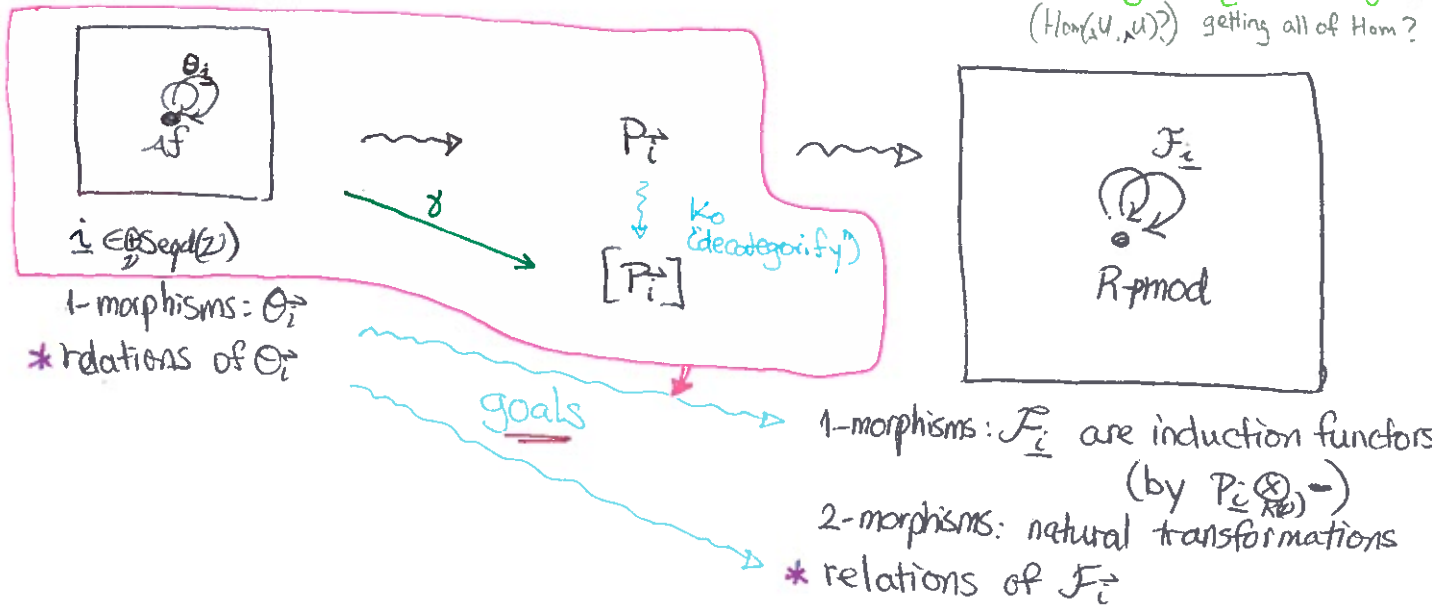
↳ works for $\hat{\mathfrak{sl}}_n$ as well.

Main Claim:

Parallel to the story of $\mathcal{A}\mathcal{U} \sim$ "idempotentized", integral form



We have for $\mathcal{A}\mathcal{F}$



Precise Claims: Global

Proposition 3.4 + Surjectivity:

$$\gamma: \mathcal{A}f \longrightarrow K_0(R)$$

$$\theta_i \longmapsto [P_i] \quad , \quad i \in \text{Seqd}(\mathcal{D}) \quad , \quad \text{any } \mathcal{D} \in \mathbb{N}_0[I]$$

$v(\Gamma)$

(i) γ is an isomorphism of $\mathbb{Z}[q, q^{-1}]$ -algebras

(ii) comultiplication of $\mathcal{A}f$ \longmapsto comultiplication of $K_0(R)$
 Lie algebra type γ [Res]

(iii) pairing on $\mathcal{A}f$ \longmapsto pairing on $K_0(R)$
 $(,) \quad (,) \quad \text{ie. } (x, y)_{\mathcal{A}f} = (\gamma(x), \gamma(y))_{K_0(R)}$

(iv) bar-involution of $\mathcal{A}f$ \longmapsto bar-involution of $K_0(R)$
 $\mathbb{Z}[q, q^{-1}]$ -antilinear $q \mapsto q^{-1}$
 $\theta_i \mapsto ?$
 $P_i \mapsto \overline{P_i} := \text{HOM}(P_i, R(\mathcal{D}))^\psi$
 $q \mapsto q^{-1}$ as a left module
 as a right module
 what if $i \in \text{Seqd}(\mathcal{D})$?

Thm 3.21

Any relation $\sum_k u_k \theta(k) = \sum_l v_l \theta(l)$ in $\mathcal{A}f$. $u_k, v_l \in \mathbb{N}[q, q^{-1}]$
 as t -morphisms

lifts to

relations $\bigoplus_k F_{\theta(k)}^{\oplus u_k} \cong \bigoplus_l F_{\theta(l)}^{\oplus v_l}$ in $R\text{-pmod}$
 (isomorphism of functors)

Definitions: (those appearing directly in Global claims)

③

$K_0(R)$

$$R := \bigoplus_{\mathcal{Z} \in \mathcal{N}[I]} R(\mathcal{Z})$$

no interactions
between different \mathcal{Z} 's

$$R\text{-mod} := \bigoplus_{\mathcal{Z} \in \mathcal{N}[I]} R(\mathcal{Z})\text{-mod}$$

graded
finitely generated

$$R\text{-fmod} := \bigoplus_{\mathcal{Z} \in \mathcal{N}[I]} R(\mathcal{Z})\text{-fmod}$$

$$\text{graded finitely-dim'l} \rightsquigarrow G_0(R) := \bigoplus_{\mathcal{Z} \in \mathcal{N}[I]} G_0(R(\mathcal{Z}))$$

$$R\text{-pmod} := \bigoplus_{\mathcal{Z} \in \mathcal{N}[I]} R(\mathcal{Z})\text{-pmod}$$

graded
projectives

take
Grothendieck
group

$$\rightsquigarrow K_0(R) := \bigoplus_{\mathcal{Z} \in \mathcal{N}[I]} K_0(R(\mathcal{Z}))$$

may be different than
what you might first expect

$$(\ , \) : K_0(R(\mathcal{Z})) \times K_0(R(\mathcal{Z})) \longrightarrow \mathbb{Z}[q^{-1}, q][(\mathcal{Z})_q]$$

$$([P], [Q]) \longmapsto \text{gdim}_k(P \otimes_{R(\mathcal{Z})} Q)$$

$$\text{Here, } (\mathcal{Z})_q = \text{gdim}_k(\text{Sym}(\mathcal{Z})) = \prod_{i \in I} \left(\prod_{a=1}^{\mathcal{Z}_i} \frac{1}{1 - q^{2a}} \right)$$