Home Problems: Set I

1. CN 1.1 (Course notes)

2. (a) Show that if X^i_{jk} and Y^i_{jk} are (1,2) tensors the there sum is also a (1,2) tensor.
   (b) Show that if $X_{ij} = X_{ji}$ holds in one coordinate system it holds in all coordinate systems.
   (c) Show that $(Z_{ijk} + Z_{jki} + Z_{kij})X^i X^j X^k = 3Z_{ijk}X^i X^j X^k$.
   (d) Let $S_{ij} = S_{ji}$ be a symmetric (0,2) tensor and $A^{ij} = -A^{ji}$ a anti-symmetric (2,0) tensor. Show that $S_{ij}A^{ij} = 0$.

3. Assume that δ^i_j is a (1,1) tensor that in a particular coordinate system x^i has components $\delta^i_j = \begin{cases} 1 & i = j \\ 0 & i \neq j \end{cases}$. Show that δ^i_j is the same in all coordinate systems, i.e it is a numerical tensor.

4. (a) An object with an index need not always be a tensor. Let x^i be some curvilinear coordinate system on $\mathbb{R}^N$. Is the object x^i a (1,0) tensor? Argue for why it is or why it is not.
   (b) An object without an index is not necessarily a scalar, i.e a (0,0) tensor. Show that the determinant of the metric $det(g)$ is not a scalar by deriving the transformation law of $det(g)$ under general coordinate transformations.