Entanglement of Dirac field modes for uniformly accelerated observers

Gelo Noel M. Tabia[1,*]

[1] *Perimeter Institute for Theoretical Physics, 31 Caroline Street North, Waterloo, Ontario N2L 2Y5, Canada*
Department of Physics and Astronomy and Institute for Quantum Computing, University of Waterloo, 200 University Avenue West, Waterloo, ON, N2L 3G1, Canada

The main purpose of this note is to briefly review the analysis of the entanglement between modes of a free Dirac field for observers that are in uniform acceleration relative to each other. As a slight generalization of existing results, we consider quantum states diagonal in the generalized Bell basis and consider mixed states with varying amounts of entanglement.

I. INTRODUCTION

In quantum information, entanglement serves as an important resource for many computational tasks. Recently, properties of entanglement in a relativistic setting have been studied by several authors. The earlier analyses used a single-mode approximation to show how the entanglement between bosonic [1] and fermionic [3] field modes is degraded from the point of view of a uniformly accelerated observer. The canonical scenario considers two parties, an inertial observer Alice and a uniformly accelerated observer Rob, each assumed to possess a detector sensitive to only one of two modes that are maximally entangled from an inertial perspective. In this setting, we assume a continuum of Minkowski wavepackets that are sufficiently peaked around a particular value of Minkowski momentum for Alice or Unruh frequency for Rob.

Ref. [2] shows that the single-mode approximation actually works only for certain family of states. They demonstrate how the usual maximally entangled state examined in the literature corresponds to a Minkowski mode with frequency ω entangled with a specific type of Unruh mode with Rindler frequency Ω, which can be achieved with Minkowski wave packets with smearing functions peaked according to some Fourier transform constraints.

Of particular interest is the infinite acceleration limit, which can be seen as describing a situation where Alice falls into a black hole while Rob barely escapes through eternal uniform acceleration. The results show that in the bosonic case, the entanglement goes to zero in the limit of infinite acceleration, and the rate of decay is independent of the choice of Unruh mode. In the fermionic case, there is always some residual entanglement and this minimum value depends on the Unruh mode used. The analogy with the black hole scenario tells us that the degradation is a consequence of a communication horizon that causes Rob to lose information about the state shared with Alice.

In this note, as an attempt to better familiarize ourselves with the specific details of these results, we explore a small generalization by considering states with different amounts of initial entanglement in the inertial setting. We will consider states of the form

$$\rho = \sum_i w_i \left|\phi_i\right\rangle \left\langle\phi_i\right| \tag{1}$$

where the basis states are given by

$$\begin{aligned} \left|\phi_1\right\rangle &= \cos\alpha \left|00\right\rangle + \sin\alpha \left|11\right\rangle , \\ \left|\phi_2\right\rangle &= \sin\alpha \left|00\right\rangle - \cos\alpha \left|11\right\rangle , \\ \left|\phi_3\right\rangle &= \cos\alpha \left|01\right\rangle + \sin\alpha \left|10\right\rangle , \\ \left|\phi_4\right\rangle &= \sin\alpha \left|01\right\rangle - \cos\alpha \left|10\right\rangle . \end{aligned} \tag{2}$$

When $\alpha = \frac{\pi}{4}$, this is the Bell basis with $\left|\phi_1\right\rangle$ corresponding to the Bell state used in the canonical scenario in noninertial frames, and ρ represents a Bell-diagonal state. Thus, we consider mixtures of states with the same amount of arbitrary entanglement defined by $\alpha \in [0, \frac{\pi}{4}]$. In particular, we explore entangled states for Dirac field modes [3], which allows us to use techniques in quantum information that involve finite-dimensional density matrices.

II. PRELIMINARIES

Suppose that Rob is a uniformly accelerated observer in the (t, x) plane of Minkowski spacetime. It is appropriate to use Rindler coordinates to describe his frame. Let $\bar{u} = t - x$ and $\bar{v} = t + x$. Using the coordinate transformation

$$t = \frac{e^{a\xi}}{a} \sinh a\tau, \quad x = \frac{e^{a\xi}}{a} \cosh a\tau \tag{3}$$

for $a > 0$ and $-\infty < \tau, \xi < \infty$, we get

$$t \pm x = \pm \frac{e^{a(\xi \pm \tau)}}{a}. \tag{4}$$

For the metric, we have

$$ds^2 = dt^2 - dx^2 = d\bar{u}\ d\bar{v}. \tag{5}$$

If we let $u = \tau - \xi$ and $v = \tau + \xi$ then

$$ds^2 = e^{2a\xi} du\ dv = e^{2a\xi}(d\tau^2 - d\xi^2). \tag{6}$$

* Electronic address: gtabia@perimeterinstitute.ca

Thus, for the Rindler coordinates (τ, ξ), lines of constant ξ are hyperbolae corresponding to world lines of uniformly accelerated observers with proper acceleration a.

In Rindler coordinates there are two regions called wedges which correspond to $x > |t|$ and $x < |t|$, referred to as region I and II, respectively. Any uniformly accelerated observer remains in either region I or II since these represent causally disconnected regions in spacetime, with the lines $\bar{u} = 0$ and $\bar{v} = 0$ acting as event horizons.

Consider a free Dirac field ψ in Minkowski space time, which satisfies

$$i\gamma^\mu \partial_\mu \psi - m\psi = 0 \tag{7}$$

where m is the mass, γ^μ are the Dirac matrices, and ψ is a Dirac spinor. In an inertial frame, Minkowski coordinates $x^\mu = (t, x)$ are typically used to describe such fields. We can write ψ in terms of the positive and negative energy solutions of the Dirac equation,

$$\psi = \int dk \ \left(a_k \psi_k^+ + b_k^\dagger \psi_k^- \right) \tag{8}$$

where k serves as a shorthand label for the modes, including energy and spin, that is,

$$\psi_k^\pm = \frac{1}{\sqrt{2\pi\omega}} \phi_s^\pm e^{\pm i(\vec{k}\cdot\vec{x} - wt)} \tag{9}$$

where $\omega^2 = m^2 + \vec{k}^2$ and ϕ_s is a constant spinor with $s = \{\uparrow, \downarrow\}$. The mode functions satisfy the orthonormality relations

$$\begin{aligned} &\left\langle \psi_k^+, \psi_j^+ \right\rangle = \delta(k-j) = -\left\langle \psi_k^-, \psi_j^- \right\rangle, \\ &\left\langle \psi_k^\pm, \psi_j^\mp \right\rangle = 0, \end{aligned} \tag{10}$$

where the inner product is given by

$$\langle \psi_k, \psi_j \rangle = \int dx \ \psi_k^\dagger \psi_j. \tag{11}$$

The operators a_k and b_k are particle and antiparticle annihilation operators, respectively, with anticommutation relations

$$\{a_k, a_j^\dagger\} = \{b_k, b_j^\dagger\} = \delta_{kj} \tag{12}$$

and other anticommutators vanishing. The Minkowski vacuum state $|0\rangle$ is defined by

$$a_k |0\rangle = b_k |0\rangle = 0. \tag{13}$$

In Rindler coordinates, region I and II admit separate quantizations, which leads to positive and negative energy solutions $\psi_{k,I}^\pm$ and $\psi_{k,II}^\pm$, respectively. Since the Rindler metric is independent of τ, the solutions are of the form $e^{i\omega\tau}\phi_s(\xi)$.

With $\omega > 0$, the mode solutions with time dependence $e^{\mp i\omega\tau}$ represent positive-frequency solutions for region I and II respectively since

$$\partial_\tau \psi_{k,I}^+ = -i\omega\psi_{k,I}^+, \qquad \partial_{-\tau}\psi_{k,II}^+ = -i\omega\psi_{k,II}^+. \tag{14}$$

The Rindler modes obey the orthonormality relations

$$\begin{aligned} &\left\langle \psi_{k,\mu}^\pm, \psi_{j,\nu}^\mp \right\rangle = 0, \\ &\left\langle \psi_{k,\mu}^\pm, \psi_{j,\nu}^\pm \right\rangle = \delta_{\mu\nu}\delta(k-j), \end{aligned} \tag{15}$$

where $\mu, \nu = I, II$. Note that we distinguish Rindler states and operators from Minkowski ones through the labels identifying the Rindler wedge they belong to.

The Dirac field written in terms of Rindler modes is given by

$$\psi = \int dk \ \left(a_{k,I}\psi_{k,I}^+ + b_{k,I}^\dagger \psi_{k,I}^- + a_{k,II}\psi_{k,II}^+ + b_{k,II}^\dagger \psi_{k,II}^- \right) \tag{16}$$

where $a_{k,\mu}, b_{k,\mu}$ with $\mu = I, II$ denote Rindler particle and antiparticle operators. These operators satisfy the usual Dirac anticommutation rules, i.e.,

$$\{a_{k,\mu} a_{j,\nu}^\dagger\} = \{b_{k,\mu} b_{j,\nu}^\dagger\} = \delta_{\mu\nu}\delta(k-j), \tag{17}$$

and all other anticommutators vanish, including those between different regions. The Rindler vacuum in regions $\mu = I, II$ are given by

$$a_{k,\mu} |0\rangle_\mu = b_{k,\mu} |0\rangle_\mu = 0. \tag{18}$$

The transformation between Minkowski and Rindler modes is given by

$$\psi_j^+ = \sum_{\mu=I,II} \int dk \ \left(\alpha_{jk,\mu}\psi_{k,\mu}^+ + \beta_{jk,\mu}^* \psi_{k,\mu}^- \right). \tag{19}$$

where the Bogoliubov coefficients are obtained by taking the inner product of the Rindler mode functions with the Minkowski mode ones, which with $\tan r = e^{-\pi\Omega}$ yields

$$\begin{aligned} &\alpha_{jk,I} = e^{i\pi/4} \frac{e^{i\theta\Omega}}{\sqrt{2\pi\omega}} \cos r \delta_{ss'}, \qquad \alpha_{jk,II} = \alpha_{jk,I}^*, \\ &\beta_{jk,I} = -e^{i\pi/4} \frac{e^{i\theta\Omega}}{\sqrt{2\pi\omega}} \sin r \delta_{ss'}, \quad \beta_{jk,II} = \beta_{jk,I}^*, \end{aligned} \tag{20}$$

where ω is the energy of the Minkowski mode, Ω is the energy of the Rindler mode, and θ is defined so that $\Omega = m\cosh\theta$ and $\Omega^2 = m^2 + \kappa^2$.

With $a_k = \left\langle \psi_k^+, \psi \right\rangle$, the Minkowski particle annihilation operator in terms of the Rindler ones is

$$a_j = \sum_{\mu=I,II} \int dk \ \left(\alpha_{jk,\mu}^* a_{k,\mu} + \beta_{jk,\mu} b_{k,\mu}^\dagger \right). \tag{21}$$

To consider transformations between states, define the Fock basis for each Dirac field mode to be

$$|k\rangle = |k\rangle_R |k\rangle_L \tag{22}$$

with

$$|k\rangle_R = |n\rangle_I^+ |m\rangle_{II}^- ,$$
$$|k\rangle_L = |n'\rangle_I^- |m'\rangle_{II}^+ , \tag{23}$$

where $\pm$ denotes particle and antiparticle states. With these basis states, we can write

$$a_j = \frac{1}{\sqrt{2\pi\omega}} \int dk \ \ (\eta^* a_{k,R} + \eta a_{k,L}) \tag{24}$$

where $\eta = e^{i\pi/4} e^{i\theta\Omega}$ and $a_{k,R}$ and $a_{k,L}$ are the Unruh operators

$$a_{k,R} = a_{k,I} \cos r - b_{k,II}^\dagger \sin r,$$
$$a_{k,L} = a_{k,II} \cos r - b_{k,I}^\dagger \sin r. \tag{25}$$

To find the Minkowski vacuum in this basis, consider the ansatz

$$|0_\Omega\rangle_R = \sum_{n,s} f(n,\Omega,s) \, |n_\Omega, s\rangle_I^+ \, |n_\Omega, -s\rangle_{II}^- ,$$
$$|0_\Omega\rangle_L = \sum_{n,s} g(n,\Omega,s) \, |n_\Omega, s\rangle_I^- \, |n_\Omega, -s\rangle_{II}^+ , \tag{26}$$

where $\pm$ labels particle and antiparticle modes, s labels the spin, and $|0\rangle = \bigotimes_\Omega |0_\Omega\rangle$ is the Minkowski vacuum expressed as a vacuum product state with

$$|0_\Omega\rangle = |0_\Omega\rangle_R \, |0_\Omega\rangle_L . \tag{27}$$

For Dirac fields, the Pauli exclusion principle reduces the sum in Eq. (26) to just two terms each. Using the anticommutation relations and the conditions for the Unruh mode vacuum,

$$a_{\Omega,R} \, |0_\omega\rangle_R = 0, \qquad a_{\Omega,L} \, |0_\omega\rangle_L = 0, \tag{28}$$

we can show that the Minkowski mode vacuum state $|0_\Omega\rangle$ is given by

$$|0_\Omega\rangle = \Big(\cos r \, |0_\Omega\rangle_I^+ \, |0_\Omega\rangle_{II}^- + \sin r \, |1_\Omega\rangle_I^+ \, |1_\Omega\rangle_{II}^- \Big)$$
$$\otimes \Big(\cos r \, |0_\Omega\rangle_I^- \, |0_\Omega\rangle_{II}^+ - \sin r \, |1_\Omega\rangle_I^- \, |1_\Omega\rangle_{II}^+ \Big) . \tag{29}$$

To simplify the notation, let

$$|nmpq\rangle = |n_\Omega\rangle_I^+ \, |m_\Omega\rangle_{II}^- \, |p_\Omega\rangle_I^- \, |q_\Omega\rangle_{II}^+ . \tag{30}$$

Then,

$$|0_\Omega\rangle = \cos^2 r \, |0000\rangle - \sin r \cos r \, |0011\rangle$$
$$+ \sin r \cos r \, |1100\rangle - \sin^2 r \, |1111\rangle . \tag{31}$$

We apply the Unruh mode particle creation operator

$$a_{k,U}^\dagger = q_R a_{\Omega,R}^\dagger + q_L a_{\Omega,L}^\dagger \tag{32}$$

with $|q_R|^2 + |q_L|^2 = 1$ to create a Minkowski one-particle state, which in shorthand notation reads

$$|1_\Omega\rangle_U^+ = q_R \left(\cos r \, |1000\rangle - \sin r \, |1011\rangle \right) \tag{33}$$
$$+ q_L \left(\sin r \, |1101\rangle + \cos r \, |0001\rangle \right) \tag{34}$$

and can be obtained in a straightforward manner by expressing the Unruh operators in terms of the Rindler operators.

III. MIXED-STATE ENTANGLEMENT OF FERMIONIC MODES

We are interested in entangled states of the form

$$\rho = \sum_i w_i \rho_i \tag{35}$$

where $\sum_i w_i = 1$ and $\rho_i = |\phi_i\rangle \langle\phi_i|$ are basis states corresponding to

$$|\phi_1\rangle = \cos\alpha \, |0_\omega\rangle \, |0_\Omega\rangle_U + \sin\alpha \, |1_\omega\rangle \, |1_\Omega\rangle_U ,$$
$$|\phi_2\rangle = \sin\alpha \, |0_\omega\rangle \, |0_\Omega\rangle_U - \cos\alpha \, |1_\omega\rangle \, |1_\Omega\rangle_U ,$$
$$|\phi_3\rangle = \cos\alpha \, |0_\omega\rangle \, |1_\Omega\rangle_U + \sin\alpha \, |1_\omega\rangle \, |0_\Omega\rangle_U ,$$
$$|\phi_4\rangle = \sin\alpha \, |0_\omega\rangle \, |1_\Omega\rangle_U - \cos\alpha \, |1_\omega\rangle \, |0_\Omega\rangle_U . \tag{36}$$

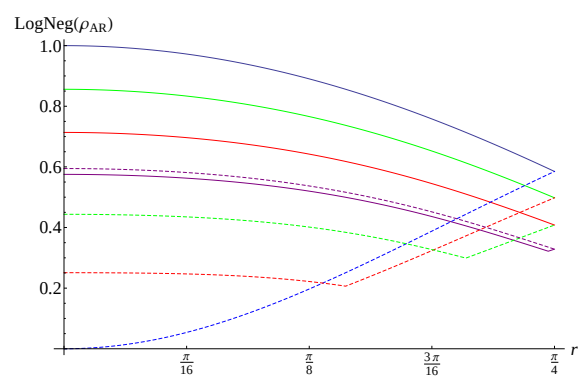

FIG. 1. Log-neg plot for $w_1 = 1, a = \pi/4$.

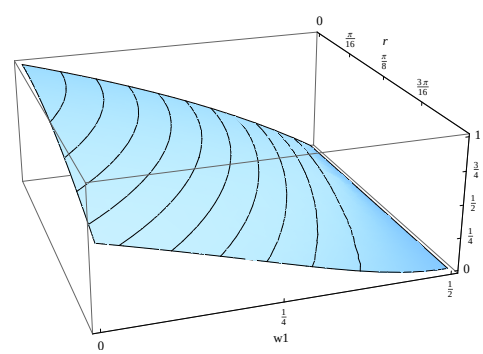

FIG. 2. Plot of log-neg vs. r and w_1, for $q_R = 1$ and $\alpha = \pi/4$.

Tracing over region II, we get

$$\begin{aligned}\rho_{A,I} = &\left(q_L w_3 q_L^* c^2 C^2 + w_1 c^2 C^4 + q_L w_4 q_L^* C^2 s^2 + w_2 C^4 s^2\right) |000\rangle \langle 000| + \left(w_1 c^2 C^2 S^2 + w_2 C^2 s^2 S^2\right) |001\rangle \langle 001| \\ &+ \left(q_R w_3 q_R^* c^2 C^2 + q_R w_4 q_R^* C^2 s^2 + q_L w_3 q_L^* c^2 S^2 + w_1 c^2 C^2 S^2 + q_L w_4 q_L^* s^2 S^2 + w_2 C^2 s^2 S^2\right) |010\rangle \langle 010| \\ &+ \left(q_R w_3 q_R^* c^2 S^2 + q_R w_4 q_R^* s^2 S^2 + w_1 c^2 S^4 + w_2 s^2 S^4\right) |011\rangle \langle 011| \\ &+ \left(q_L w_2 q_L^* c^2 C^2 + w_4 c^2 C^4 + q_L w_1 q_L^* C^2 s^2 + w_3 C^4 s^2\right) |100\rangle \langle 100| + \left(w_4 c^2 C^2 S^2 + w_3 C^2 s^2 S^2\right) |101\rangle \langle 101| \\ &+ \left(q_R w_2 q_R^* c^2 C^2 + q_R w_1 q_R^* C^2 s^2 + q_L w_2 q_L^* c^2 S^2 + w_4 c^2 C^2 S^2 + q_L w_1 q_L^* s^2 S^2 + w_3 C^2 s^2 S^2\right) |110\rangle \langle 110| \\ &+ \left(q_R w_2 q_R^* c^2 S^2 + q_R w_1 q_R^* s^2 S^2 + w_4 c^2 S^4 + w_3 s^2 S^4\right) |111\rangle \langle 111| + \sigma_{\mathrm{A,I}} + \sigma_{\mathrm{A,I}}^\dagger \end{aligned} \tag{37}$$

where $c = \cos\alpha, s = \sin\alpha, C = \cos r, S = \sin r$, and σ_{nd} is defined by the non-diagonal entries

$$\begin{aligned}\sigma_{\mathrm{A,I}} = &\left(-q_L w_3 q_R^* c^2 CS - q_L w_4 q_R^* C s^2 S\right) |000\rangle \langle 011| + \left(-q_L w_3 c C^2 s S + q_L w_4 c C^2 s S\right) |000\rangle \langle 101| \\ &+ \left(w_1 q_R^* c C^3 s - w_2 q_R^* c C^3 s\right) |000\rangle \langle 110| + \left(-w_1 q_L^* c C^2 s S + w_2 q_L^* c C^2 s S\right) |001\rangle \langle 100| \\ &+ \left(w_1 q_R^* c C s S^2 - w_2 q_R^* c C s S^2\right) |001\rangle \langle 111| + \left(q_R w_3 c C^3 s - q_R w_4 c C^3 s\right) |010\rangle \langle 100| \\ &\left(q_L w_3 c s S^3 + q_L w_4 c s S^3\right) |010\rangle \langle 111| + \left(q_R w_3 c C s S^2 - q_R w_4 c C s S^2\right) |011\rangle \langle 101| \\ &+ \left(-w_1 q_L^* c s S^3 + w_2 q_L^* c s S^3\right) |011\rangle \langle 110| + \left(-q_L w_2 q_R^* c^2 CS - q_L w_1 q_R^* C s^2 S\right) |100\rangle \langle 111| . \end{aligned} \tag{38}$$

If we trace over region I, we get the reduced state of Alice and anti-Rob, which is given by

$$\begin{aligned}\rho_{A,II} = &\left(q_R w_3 q_R^* c^2 C^2 + w_1 c^2 C^4 + q_R w_4 q_R^* C^2 s^2 + w_2 C^4 s^2\right) |000\rangle \langle 000| \\ &+ \left(q_L w_3 q_L^* c^2 C^2 + q_L w_4 q_L^* C^2 s^2 + q_R w_3 q_R^* c^2 S^2 + w_1 c^2 C^2 S^2 + q_R w_4 q_R^* s^2 S^2 + w_2 C^2 s^2 S^2\right) |001\rangle \langle 001| \\ &+ \left(w_1 c^2 C^2 S^2 + w_2 C^2 s^2 S^2\right) |010\rangle \langle 010| + \left(q_L w_3 q_L^* c^2 S^2 + q_L w_4 q_L^* s^2 S^2 + w_1 c^2 S^4 + w_2 s^2 S^4\right) |011\rangle \langle 011| \\ &+ \left(q_R w_2 q_R^* c^2 C^2 + w_4 c^2 C^4 + q_R w_1 q_R^* C^2 s^2 + w_3 C^4 s^2\right) |100\rangle \langle 100| \\ &+ \left(q_L w_2 q_L^* c^2 C^2 + q_L w_1 q_L^* C^2 s^2 + q_R w_2 q_R^* c^2 S^2 + w_4 c^2 C^2 S^2 + q_R w_1 q_R^* s^2 S^2 + w_3 C^2 s^2 S^2\right) |101\rangle \langle 101| \\ &+ \left(w_4 c^2 C^2 S^2 + w_3 C^2 s^2 S^2\right) |110\rangle \langle 110| + q_L w_2 q_L^* c^2 S^2 + q_L w_1 q_L^* s^2 S^2 + w_4 c^2 S^4 + w_3 s^2 S^4 |111\rangle \langle 111| \\ &+ \tau_{A,II} + \tau_{A,II}^\dagger \end{aligned} \tag{39}$$

where

$$\begin{aligned}\tau_{A,II} = &\left(q_R w_3 q_L^* c^2 CS + q_R w_4 q_L^* C s^2 S\right) |000\rangle \langle 011| + \left(w_1 q_L^* c C^3 s - w_2 q_L^* c C^3 s\right) |000\rangle \langle 101| \\ &+ \left(q_R w_3 c C^2 s S - q_R w_4 c C^2 s S\right) |000\rangle \langle 110| + \left(q_L w_3 c C^3 s - q_L w_4 c C^3 s\right) |001\rangle \langle 100| \\ &+ \left(q_R w_3 c s S^3 - q_R w_4 c s S^3\right) |001\rangle \langle 111| + \left(w_1 q_R^* c C^2 s S - w_2 q_R^* c C^2 s S\right) |010\rangle \langle 100| \\ &+ \left(w_1 q_L^* c C s S^2 - w_2 q_L^* c C s S^2\right) |010\rangle \langle 111| + \left(w_1 q_R^* c s S^3 - w_2 q_R^* c s S^3\right) |011\rangle \langle 101| \\ &+ \left(q_L w_3 c C s S^2 - q_L w_4 c C s S^2\right) |011\rangle \langle 110| + \left(q_R w_2 q_L^* c^2 CS + q_R w_1 q_L^* C s^2 S\right) |100\rangle \langle 111| \end{aligned} \tag{40}$$

contains the upper triangular matrix entries of $\rho_{A,II}$, in the basis $|lmn\rangle = |l\rangle |m\rangle_{II}^- |n\rangle_{II}^+$.

To quantify the entanglement between Alice and Rob in region I, we compute the logarithmic negativity of the partially transposed matrix ρ_{AR}^{PT}. The logarithmic negativity E_N is an easy-to-compute entanglement monotone that gives an upper bound to distillable entanglement is defined as

$$E_N(\rho) = \log_2 (2\mathcal{N} + 1) \tag{41}$$

where $\mathcal{N}$ is the sum of the negative eigenvalues of ρ, i.e.,

$$\mathcal{N}(\rho) = \frac{1}{2} \sum_i \left(|\lambda_i| - \lambda_i\right) . \tag{42}$$

Same as what was found before, the eigenvalues of ρ_{AR}^{PT} depend only on $|q_R|$ and not on any relative phase between q_R and q_L.

In the following graphs, we plot the logarithmic negativity for the case $w_3 = w_4 = 0, w_2 = 1 - w_1$ for different values of w_1 and α. For Figs. 1 to 6, the blue, green, red, and purple lines correspond to $q_R = 1.0, 0.9, 0.8, 0.7$, respectively. The dashed lines of the same color correspond to the entanglement between Alice and Rob's region II mode. One only needs to consider the case $\frac{1}{\sqrt{2}} \leq |q_R| \leq 1$ since when $|q_R|^2 < |q_L|$, the roles of the region I and II modes are just reversed.

Fig. 1 reproduces the result reported in Ref. [2], where the fermionic modes of Alice and Rob are maximally entangled in an inertial frame. Fig. 2 shows a plot of the entanglement with w_1 and r both varied, for $q_R = 1$, with $w1 = 0$ corresponding to the canonical case explored in the literature. The contour lines shown are lines of constant logarithmic negativity.

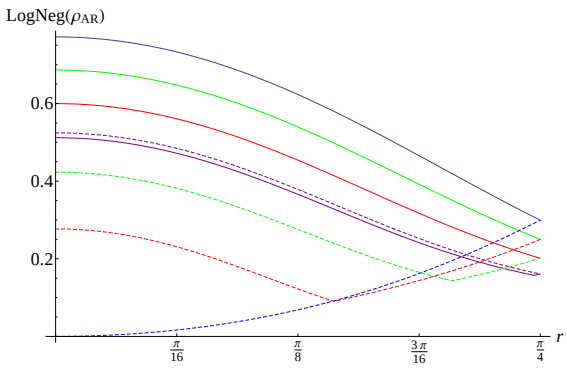

FIG. 3. Log-neg plot for $w_1 = 1, a = \pi/8$.

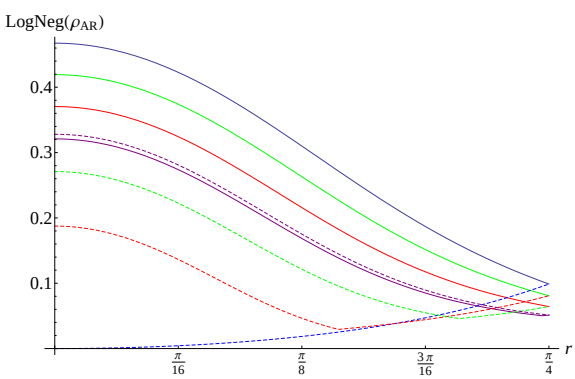

FIG. 4. Log-neg plot for $w_1 = 1, a = \pi/16$.

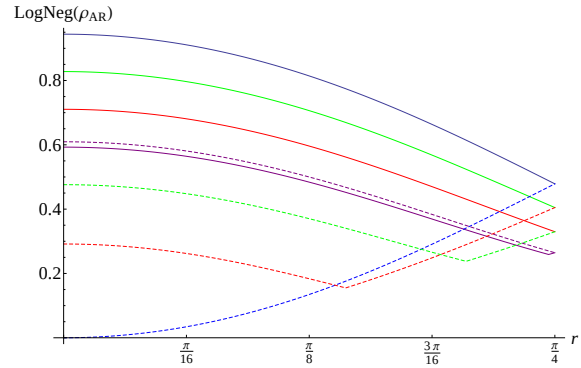

FIG. 5. Log-neg plot for $w_1 = 2/3, a = 3\pi/16$.

Qualitatively, we find the same behavior at different levels of entanglement for the inertial bipartite state for Alice and Rob: the entanglement gets degraded at

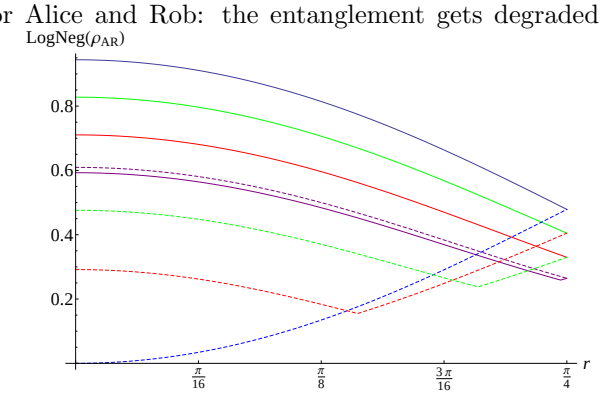

FIG. 6. Log-neg plot for $w_1 = 3/4, a = \pi/8$.

greater accelerations due to the Unruh effect at a rate that depends on the choice of $|q_R|$, with the most resilient state obtained for $|q_R| = 1$.

We also observe the conservation of total logarithmic negativity for the state between Alice and Rob's region I mode, and Alice and Rob's region II mode for $|q_R| = 1$: the sum of the two is always equal to the entanglement in the inertial bipartite state. However, this compensating levels of entanglement does not hold for $|q_R| < 1$.

Quantitatively, how the amount of entanglement decreases with acceleration varies with intermediate levels of entanglement in the inertial state, as we observe when we vary α and w_1. Nevertheless, for $|q_R| > 1/\sqrt{2}$, the logarithmic negativity essentially decreases monotonically for the Alice-Rob region I modes, for any α or w_1, while we observe the characteristic dip and rise in the logarithmic negativity of Alice-Rob region II modes.

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[2] D. E. Brushi, et al., *Phys. Rev. A* **82** (2010) 042332.
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