Problem Set #2

Quantum Error Correction
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Due Tues., Jan. 23, 2007

Problem #1. Distance 2 Stabilizer Codes

a) Find the generators of the stabilizer for a $[[4, 2, 2]]$ QECC. Give a basis of state vectors for it.

b) Find generators for $[[2k, 2k - 2, 2]]$ and $[[2k + 1, 2k - 2, 2]]$ stabilizer codes for all $k \geq 2$.

c) Prove that there is no $[[3, 1, 2]]$ stabilizer code.

Problem #2. The 9-Qubit Code as a CSS Code

a) The 9-qubit code is a CSS code formed from two classical linear codes C_1 and C_2. Write down the generator matrices for C_1 and C_2.

b) Find the distances d_1 and d_2 of C_1 and C_2. Note that $\min(d_1, d_2) = 2$ but the 9-qubit code corrects one error nevertheless. How is this possible?

Problem #3. Stabilizer Entangled States

A quantum error-correcting code with 0 entangled qubits can still be interesting. When a stabilizer S has n generators on n qubits, the subspace $T(S)$ is just a single state, called a *stabilizer state*.

a) Show that the four Bell states $|00\rangle \pm |11\rangle$, $|01\rangle \pm |10\rangle$ are stabilizer states and give their stabilizers. Show that the state $|010\rangle - |101\rangle$ is a stabilizer state and give its stabilizer. (In all cases, it is enough to specify a set of generators for the stabilizer.)

b) For a general stabilizer entangled state, suppose we partition the qubits into two disjoint sets A and B, and let S_A be the set

$$S_A = \{P_A \,|\, P = P_A \otimes I_B \in S,\ P_A \text{ acts on } A,\, I_B \text{ acts on } B\} \tag{1}$$

That is, S_A is the set of elements of S that act only on the qubits in A.

Show that S_A meets the conditions to be a stabilizer.

c) Show that if we discard the qubits in B, the remaining state is a uniform mixture of states in $T(S_A)$.