Problem Set #4

Quantum Error Correction
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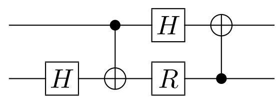
Due Thursday, Feb. 1, 2018

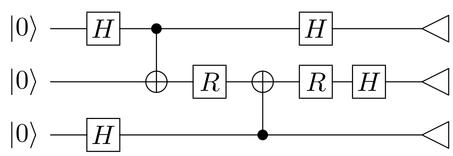
Problem #1. Analyzing Clifford group circuits

In the following diagrams, $R = R_{\pi/4}$ is the matrix $\mathrm{diag}(1, i)$ and H is the Hadamard transform.

a) For the following Clifford group circuit, compute the overall action on Paulis and use that to write down the 4×4 unitary matrix performed by the circuit:



b) For the following Clifford group circuit, use Clifford simulation techniques to compute the full probability distribution of the 8 possible classical outputs after measuring all qubits in the computational basis:



Problem #2. Twirling

Let $S(\rho)$ be a quantum operation (a completely positive trace-preserving map) taking n qubits to n qubits. **Hint:** (For both parts) Any $2^n \times 2^n$ matrix can be expanded in the basis of Pauli operators.

a) Consider the following quantum operation: Choose a uniformly random $P \in \mathcal{P}_n/\{\pm I, \pm iI\}$ (i.e., a Pauli ignoring global phase). Apply $P^\dagger$, then S, then P (for the same P). Show that, averaging over P, the resulting quantum operation is a Pauli channel.

b) Now instead of choosing a random Pauli, choose a random Clifford and do the same thing, i.e., uniformly random $C \in \mathcal{C}_n/\{e^{i\phi} I\}$, apply $C^\dagger$, then S, then C. Show that, averaging over C, the resulting quantum channel is a depolarizing channel.

Problem #3. Transversal gates for qutrit code

For this problem, consider the following $[[4, 2, 2]]_3$ stabilizer code on qutrits:

	X	X	X^{-1}	X^{-1}
	Z	Z	Z	Z
$\overline{X}_1$	X	X^{-1}	I	I
$\overline{Z}_1$	Z	I	Z	I
$\overline{X}_2$	I	X	I	X^{-1}
$\overline{Z}_2$	I	I	Z	Z^{-1}

For each of the physical gates below, determine if it is valid gadget (meaning it preserves the codespace) and, if so, what logical gate it performs. (You can specify the logical gate via a circuit or as a transformation on the logical Paulis.)

a) $\mathcal{F}^{\otimes 4}$, where $\mathcal{F}$ is the qutrit Fourier transform, $\mathcal{F}|a\rangle = \sum_b \omega^{ab}|b\rangle$.

b) $S_2^{\otimes 4}$, where S_2 is multiplication by 2, $S_2|a\rangle = |2a\rangle$, with arithmetic mod 3.

c) $R^{\otimes 4}$, where R is the quadratic phase gate, $R|a\rangle = \omega^{a(a-1)/2}|a\rangle$.

d) $SUM^{\otimes 4}$, where SUM is the two-qutrit sum gate, $SUM|a\rangle|b\rangle = |a\rangle|a+b\rangle$, with arithmetic mod 3.

e) Find a valid gadget of the form $\mathcal{F}^{a_1} \otimes \mathcal{F}^{a_2} \otimes \mathcal{F}^{a_3} \otimes \mathcal{F}^{a_4}$, with $\mathcal{F}$ the Fourier transform as in part a and not all of the a_i's are the same. Give the logical gate performed by this transversal gate.