Title: Entanglement and the Foundations of Statistical Mechanics

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Abstract:

ENTANGLEMENT AND THE FOUNDATIONS OF STATISTICAL MECHANICS

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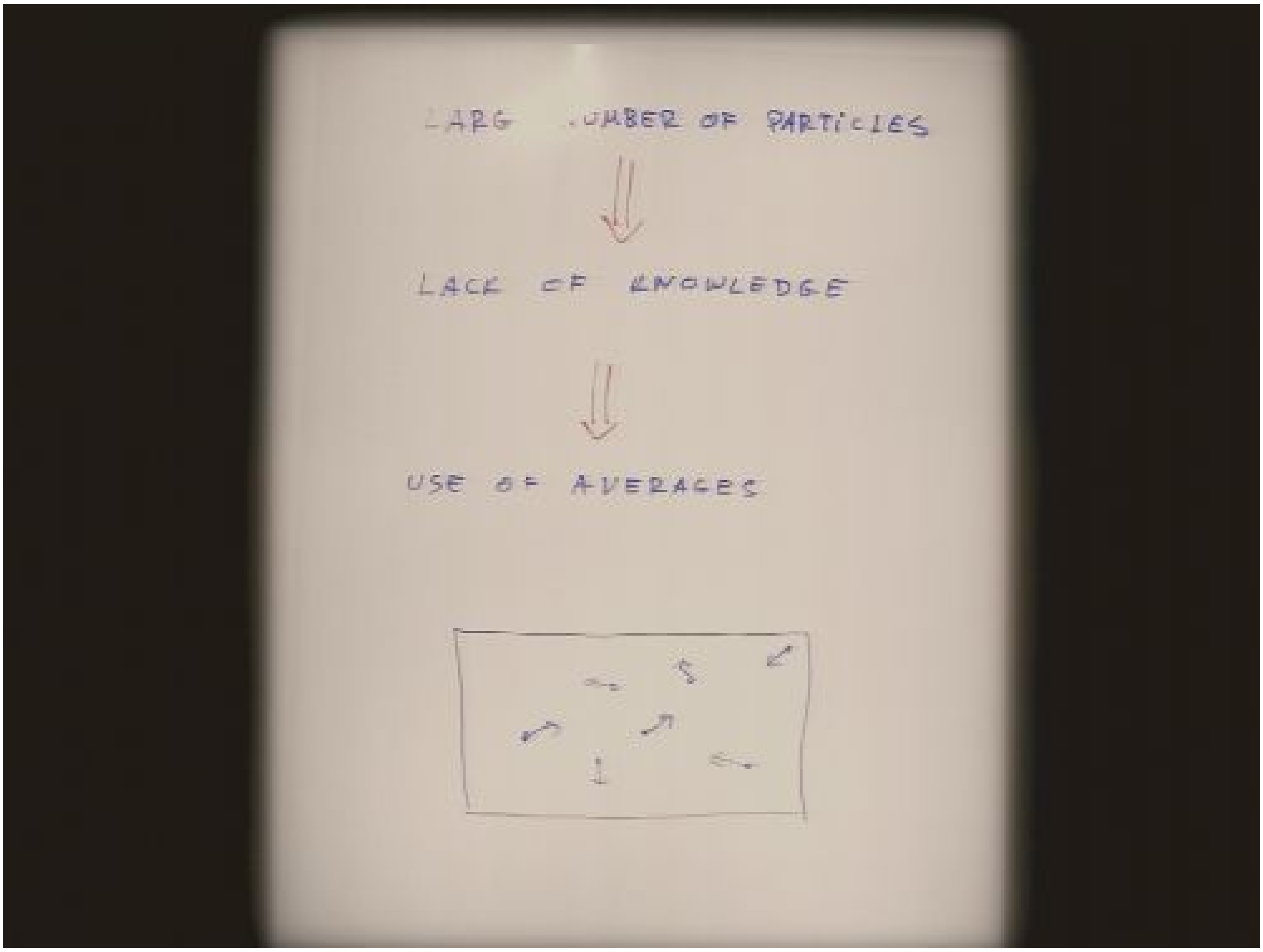
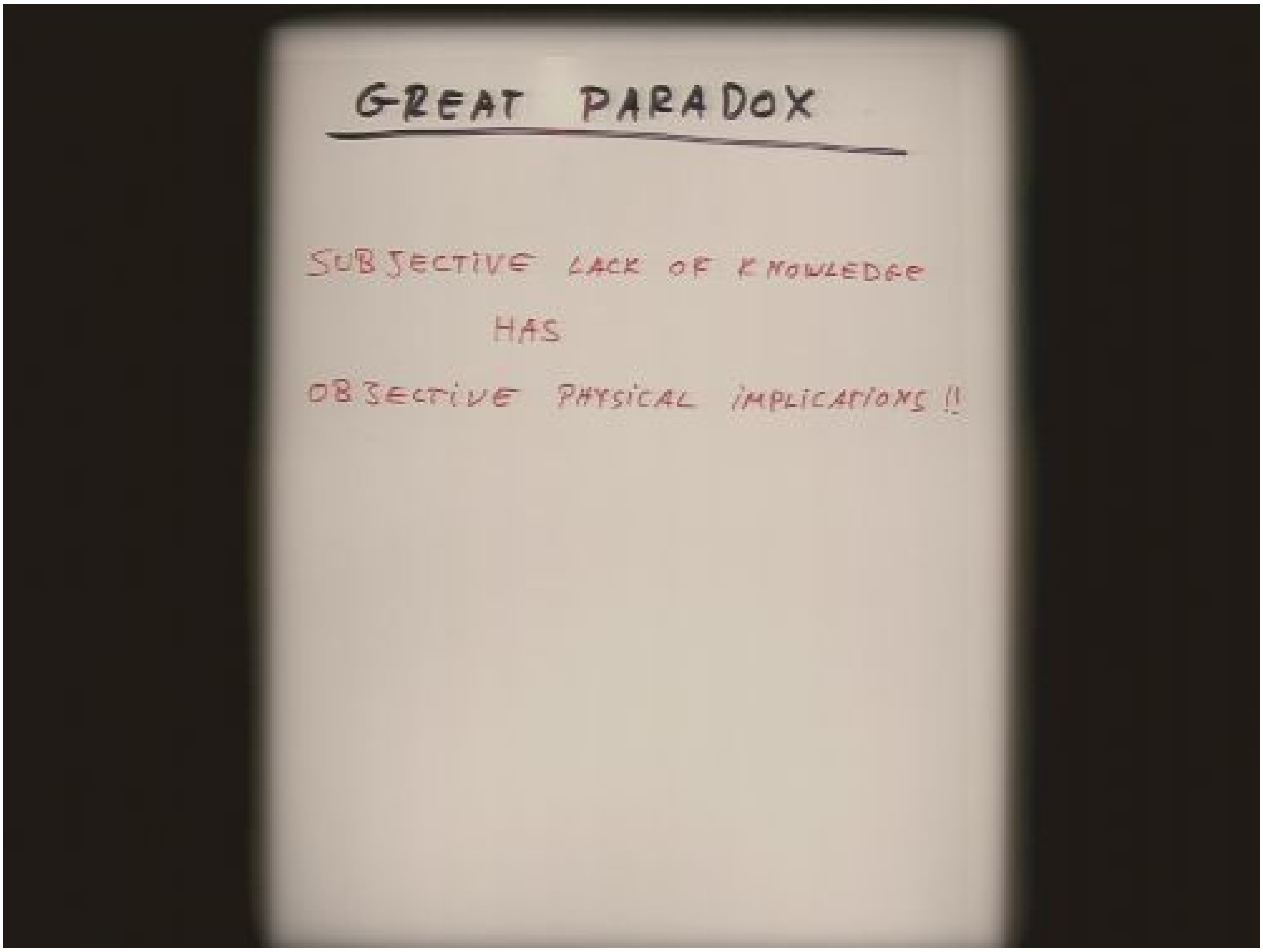
- YAKIR AHARONOV, NOAH LINDEN


LARG UMBER OF PARTICLES
LACK OF KNOWLEDGE
USE OF AVERAGES


GREAT PARADOX
SUBJECTIVE LACK OF KNOWLEDGE
HAS
OBJECTIVE PHYSICAL IMPLICATIONS !!

MACRO STATE (DETERMINED BY MACROSCOPIC PARAMETERS P, V, E ...)

$\Downarrow$

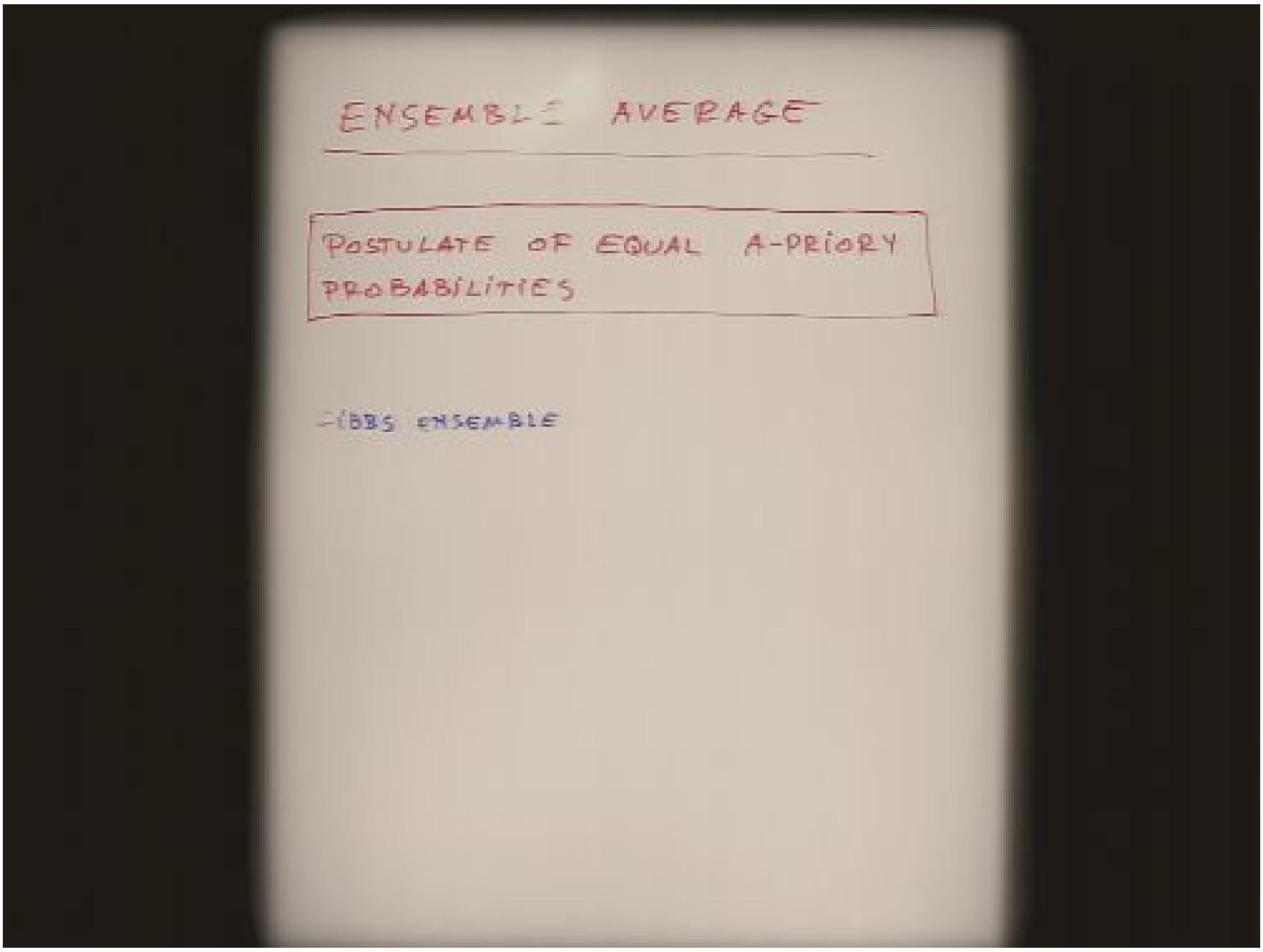
NUMBER OF (MICRO) STATES COMPATIBLE WITH THE MACRO STATE N

$\Downarrow$

$S = \ln N$ ENTROPY

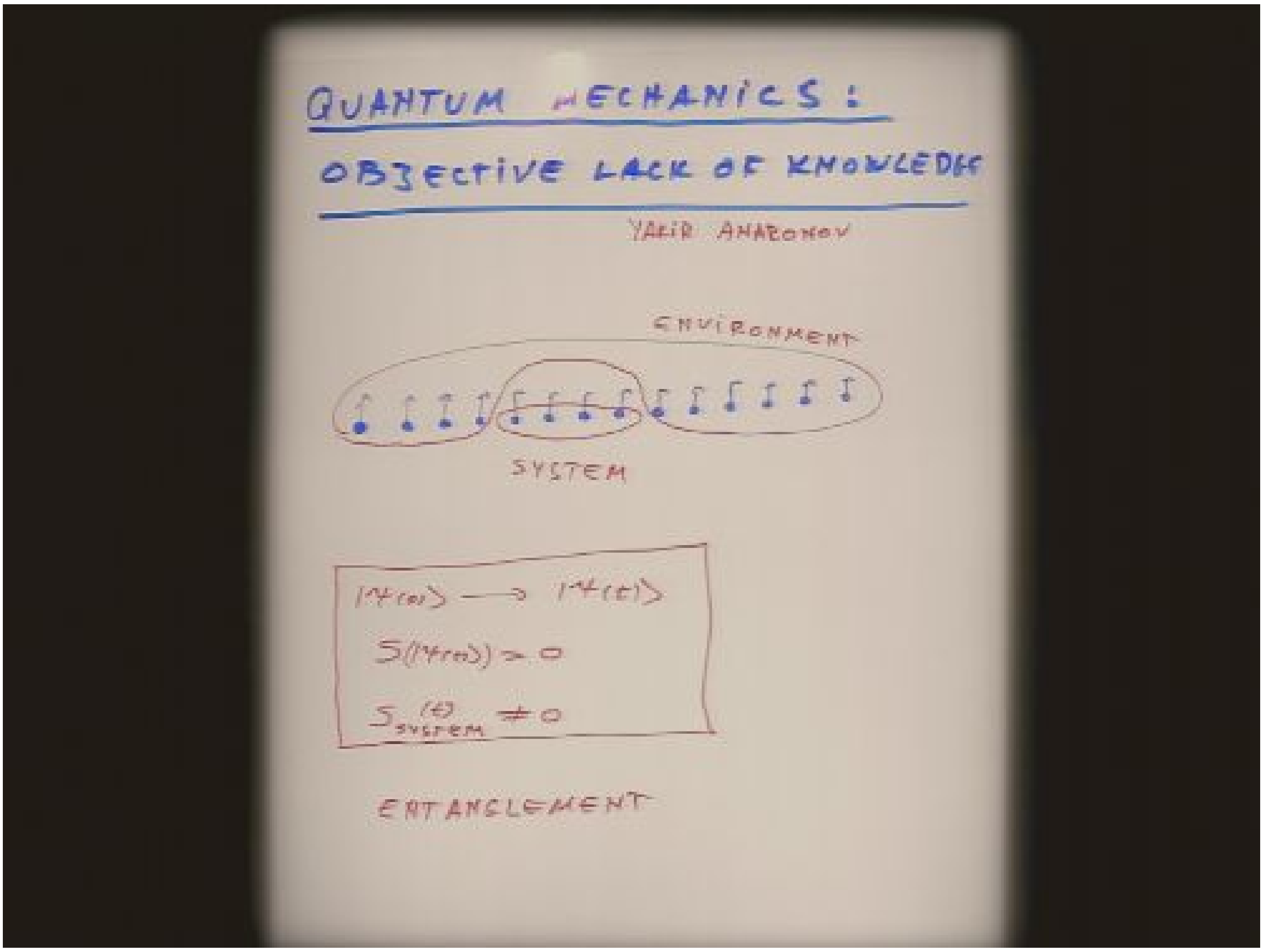
$\Downarrow$

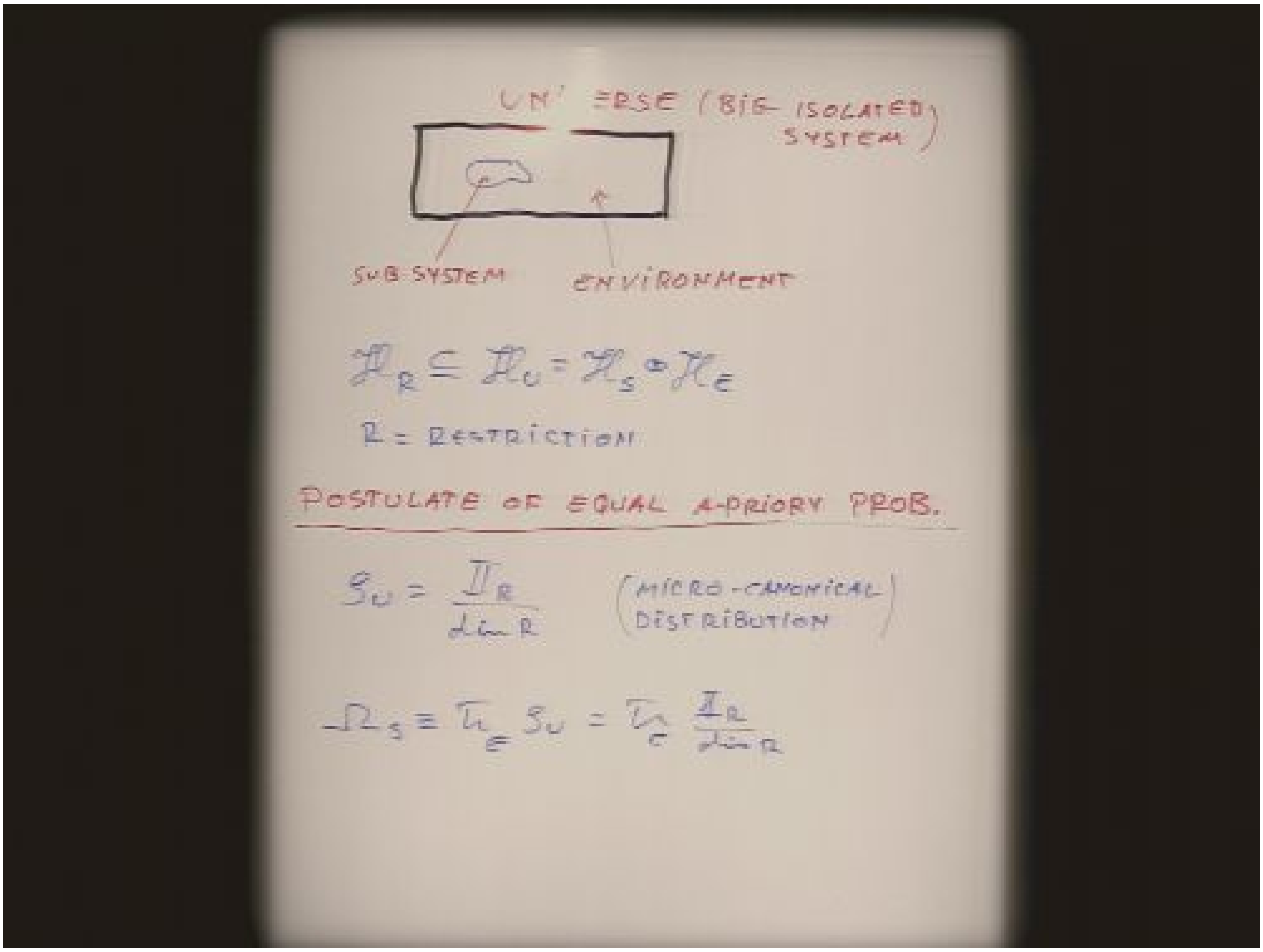
$$\frac{dS}{dE} = \frac{1}{T}$$


ENSEMBLE AVERAGE
POSTULATE OF EQUAL A-PRIORY PROBABILITIES
GIBBS ENSEMBLE

TIME AVERAGE

ERGODICITY


QUANTUM MECHANICS:
OBJECTIVE LACK OF KNOWLEDGE
YAKIR AHARONOV
ENVIRONMENT
SYSTEM
$|\Psi(0)\rangle \longrightarrow |\Psi(t)\rangle$
$S(|\Psi(0)\rangle) = 0$
$S^{(t)}_{SYSTEM} \neq 0$
ENTANGLEMENT


UN' ERSE (BIG ISOLATED SYSTEM)
SUB SYSTEM
ENVIRONMENT
$\mathcal{H}_R \subseteq \mathcal{H}_U = \mathcal{H}_S \otimes \mathcal{H}_E$
R = RESTRICTION
POSTULATE OF EQUAL A-PRIORY PROB.
$\rho_U = \frac{\mathbb{I}_R}{\dim R}$
(MICRO-CANONICAL DISTRIBUTION)
$\rho_S = \mathrm{Tr}_E\, \rho_U = \mathrm{Tr}_E \frac{\mathbb{I}_R}{\dim R}$

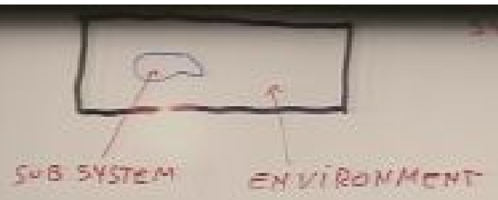
$$\mathcal{H}_R \subseteq \mathcal{H}_U = \mathcal{H}_S \otimes \mathcal{H}_E$$

R = RESTRICTION

POSTULATE OF EQUAL A-PRIORY PROB.

$$\rho_U = \frac{\Pi_R}{\dim R}$$ (MICRO-CANONICAL DISTRIBUTION)

$$\Omega_S \equiv \mathrm{Tr}_E\, \rho_U = \mathrm{Tr}_E \frac{\Pi_R}{\dim R}$$



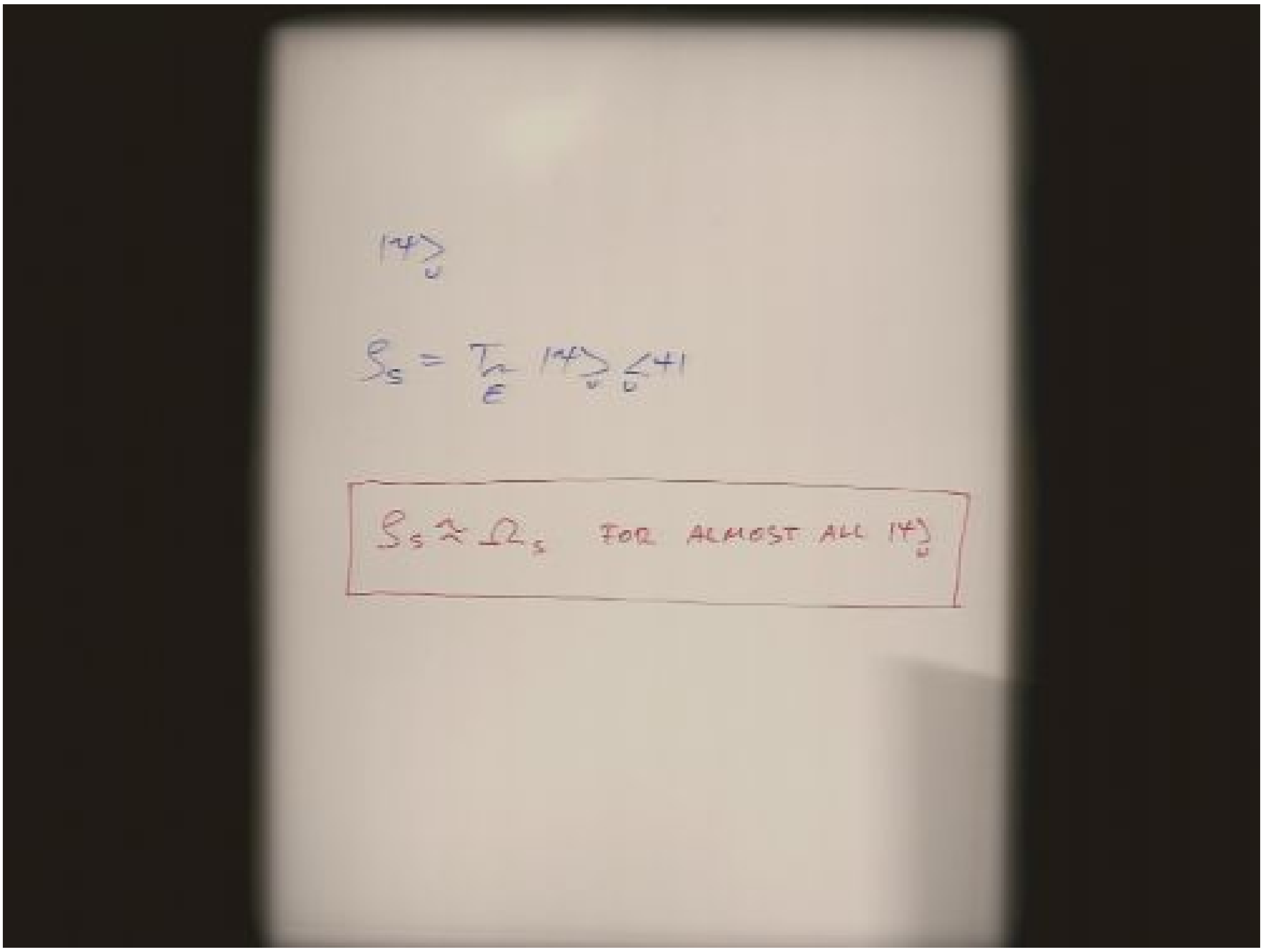
$$\mathcal{H}_R \subseteq \mathcal{H}_U = \mathcal{H}_S \otimes \mathcal{H}_E$$

R = RESTRICTION

POSTULATE OF EQUAL A-PRIORY PROB.

$$\rho_U = \frac{\mathbb{I}_R}{\dim R}$$ (MICRO-CANONICAL DISTRIBUTION)

$$\Omega_S \equiv \mathrm{Tr}_E\, \rho_U = \mathrm{Tr}_E \frac{\mathbb{I}_R}{\dim R}$$


$|\psi\rangle_U$
$\rho_S = \mathrm{Tr}_E\, |\psi\rangle_U \langle\psi|_U$
$\boxed{\rho_S \approx \Omega_S \text{ FOR ALMOST ALL } |\psi\rangle_U}$

$|\psi\rangle_U$

$$\rho_S = \mathrm{Tr}_E \, |\psi\rangle_U \langle\psi|$$

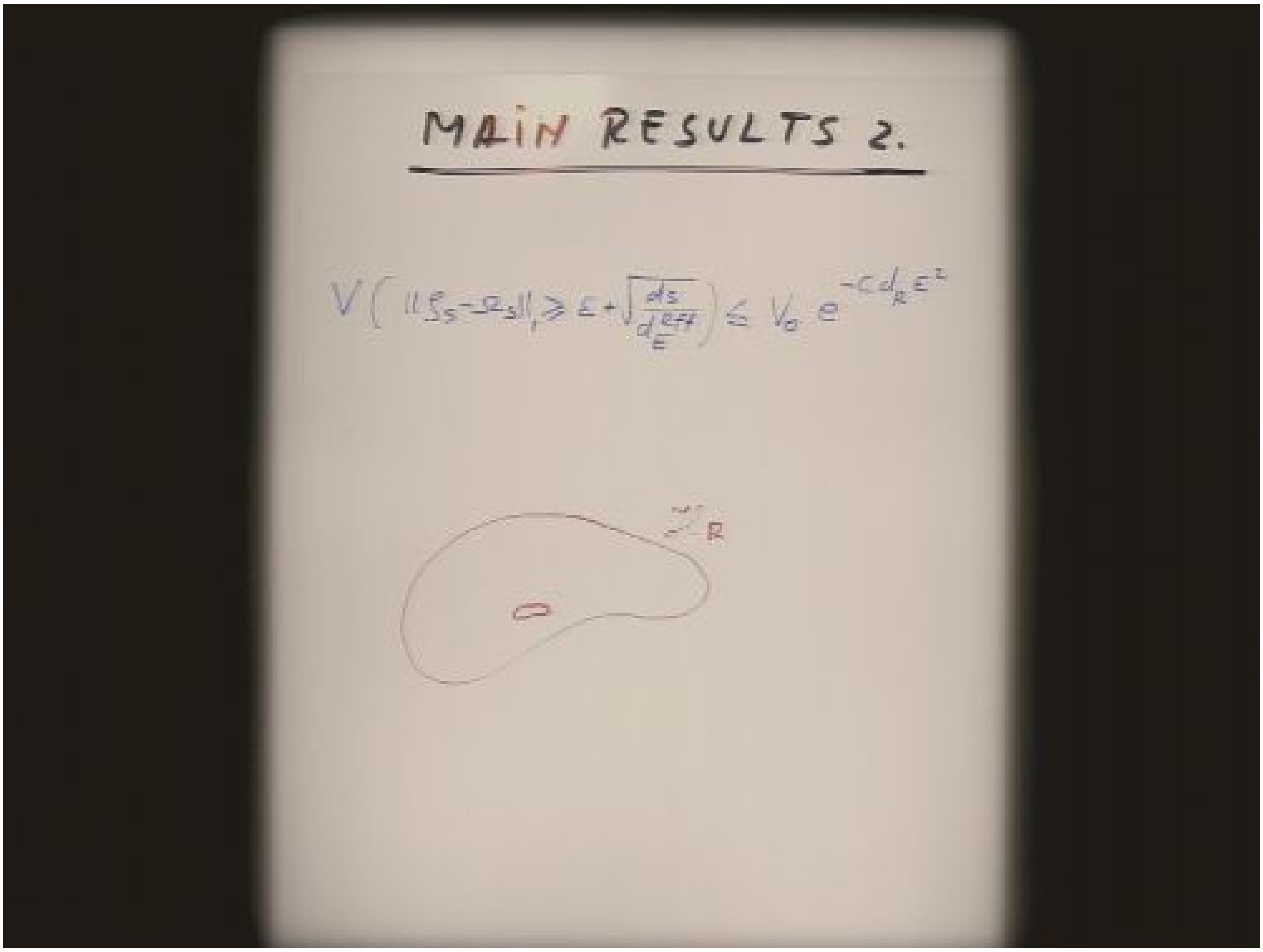
$$\boxed{\rho_S \approx \Omega_S \quad \text{FOR ALMOST ALL } |\psi\rangle_U}$$

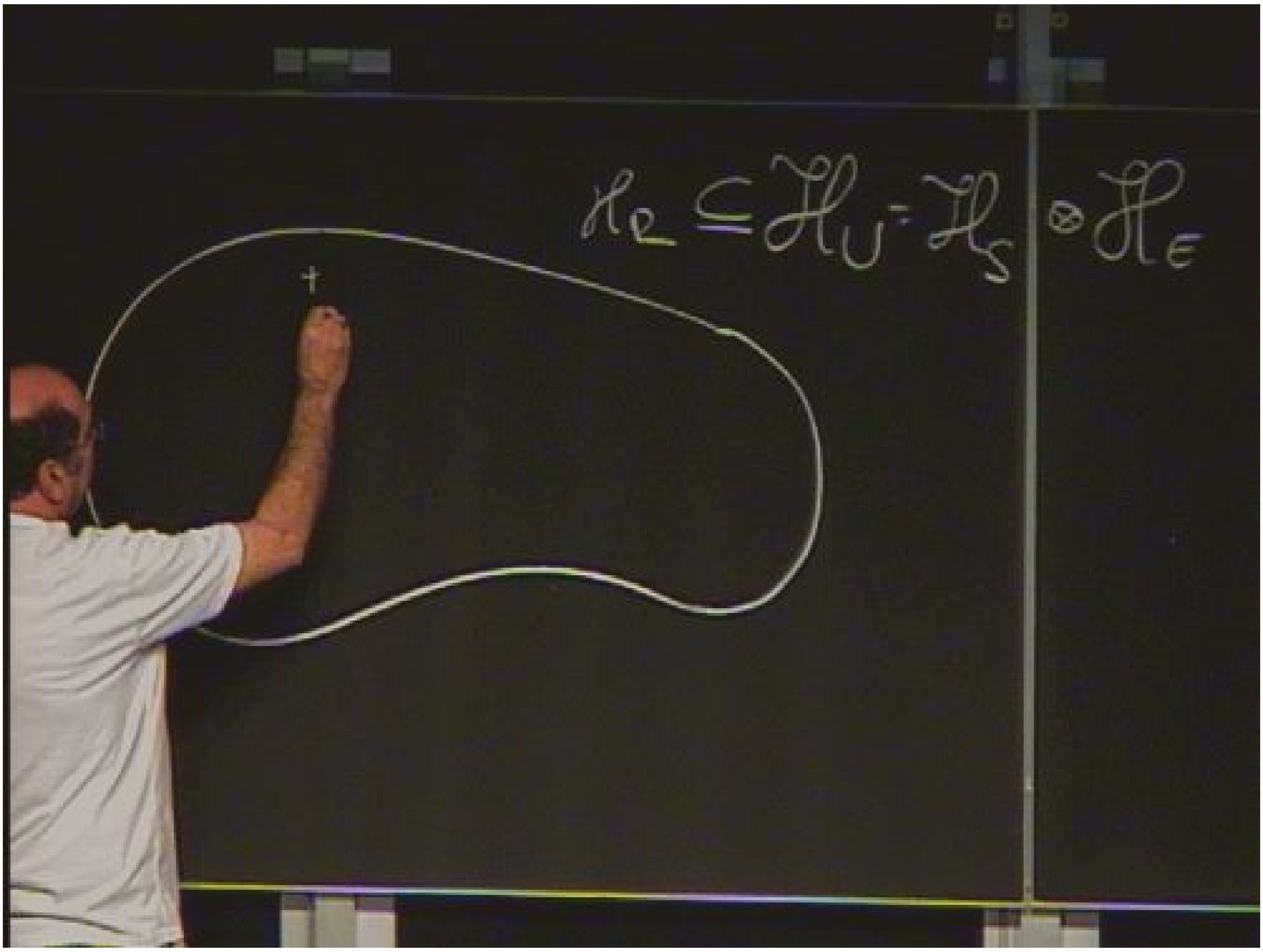
MAIN RESULTS 1.

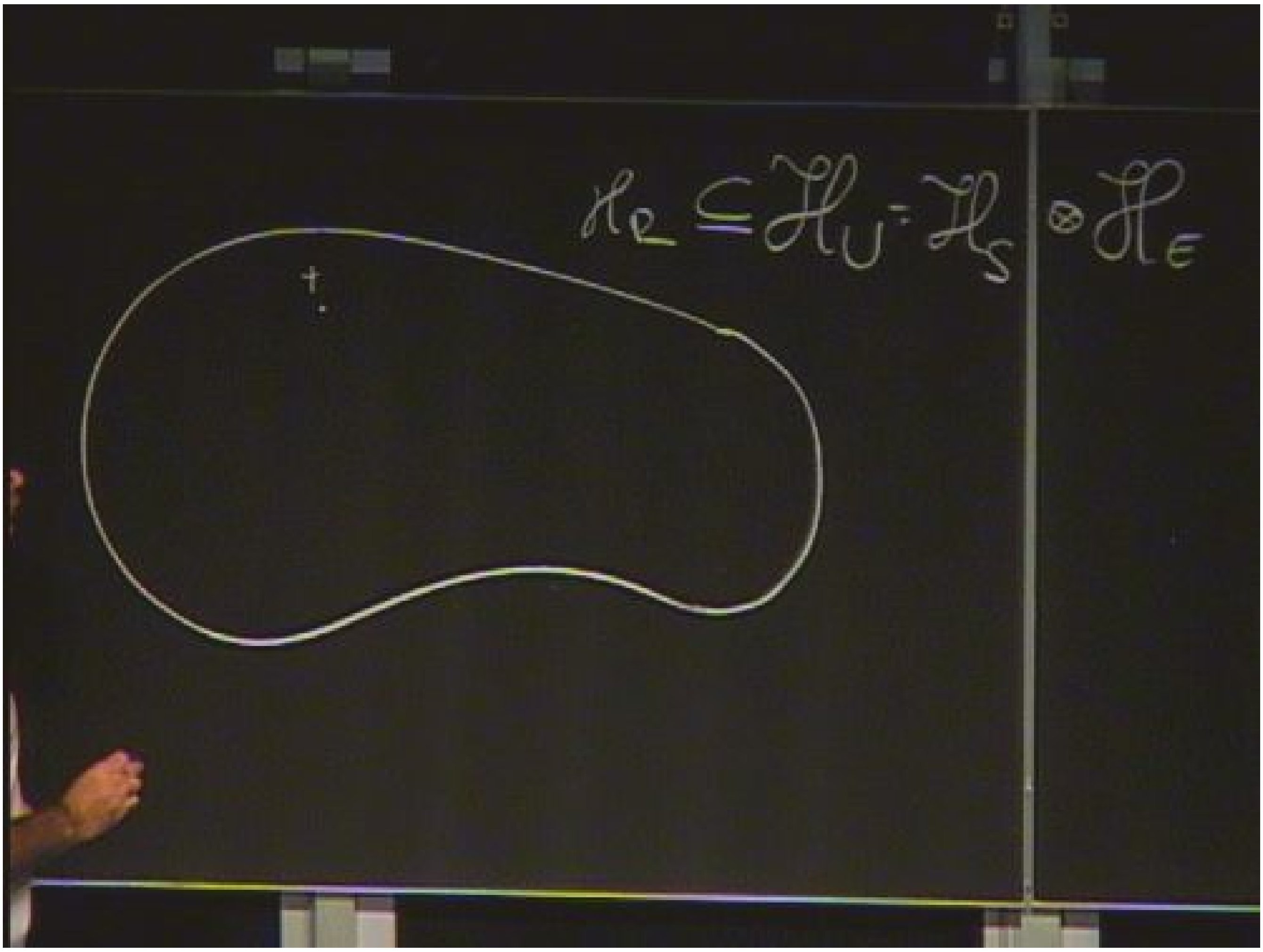
$$\langle \| \rho_S - \Omega_S \|_1 \rangle \leq \sqrt{\frac{d_S}{d_E^{eff}}}$$

$d_c \quad 2^{n-k}$

$d_s = 2^k$


MAIN RESULTS 2.
$V\left(\|\rho_S-\Omega_S\|_1 \geq \epsilon+\sqrt{\frac{d_S}{d_E^{eff}}}\right) \leq V_0\, e^{-C d_R \epsilon^2}$


$\mathcal{H}_R \subseteq \mathcal{H}_U = \mathcal{H}_S \otimes \mathcal{H}_E$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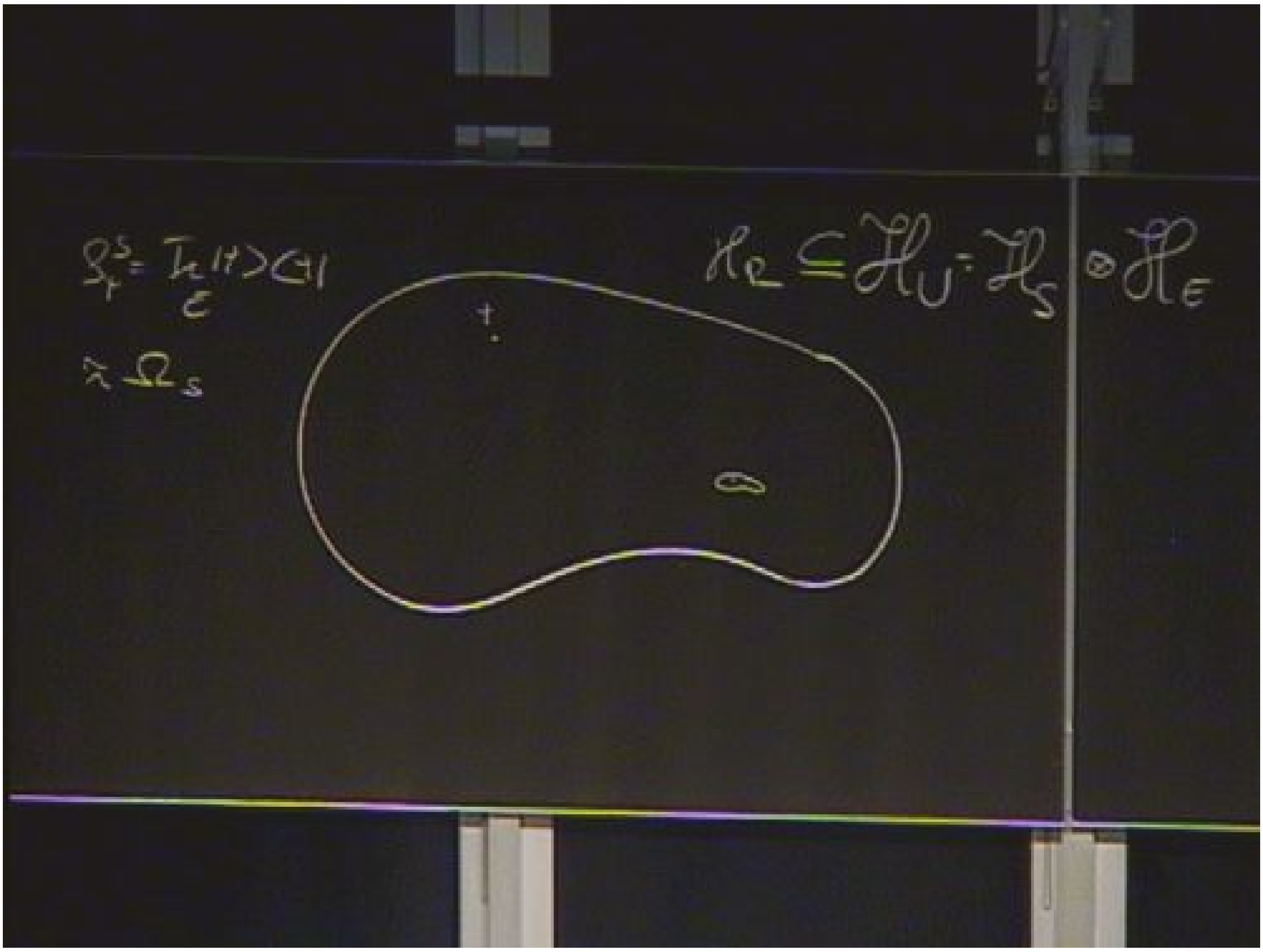
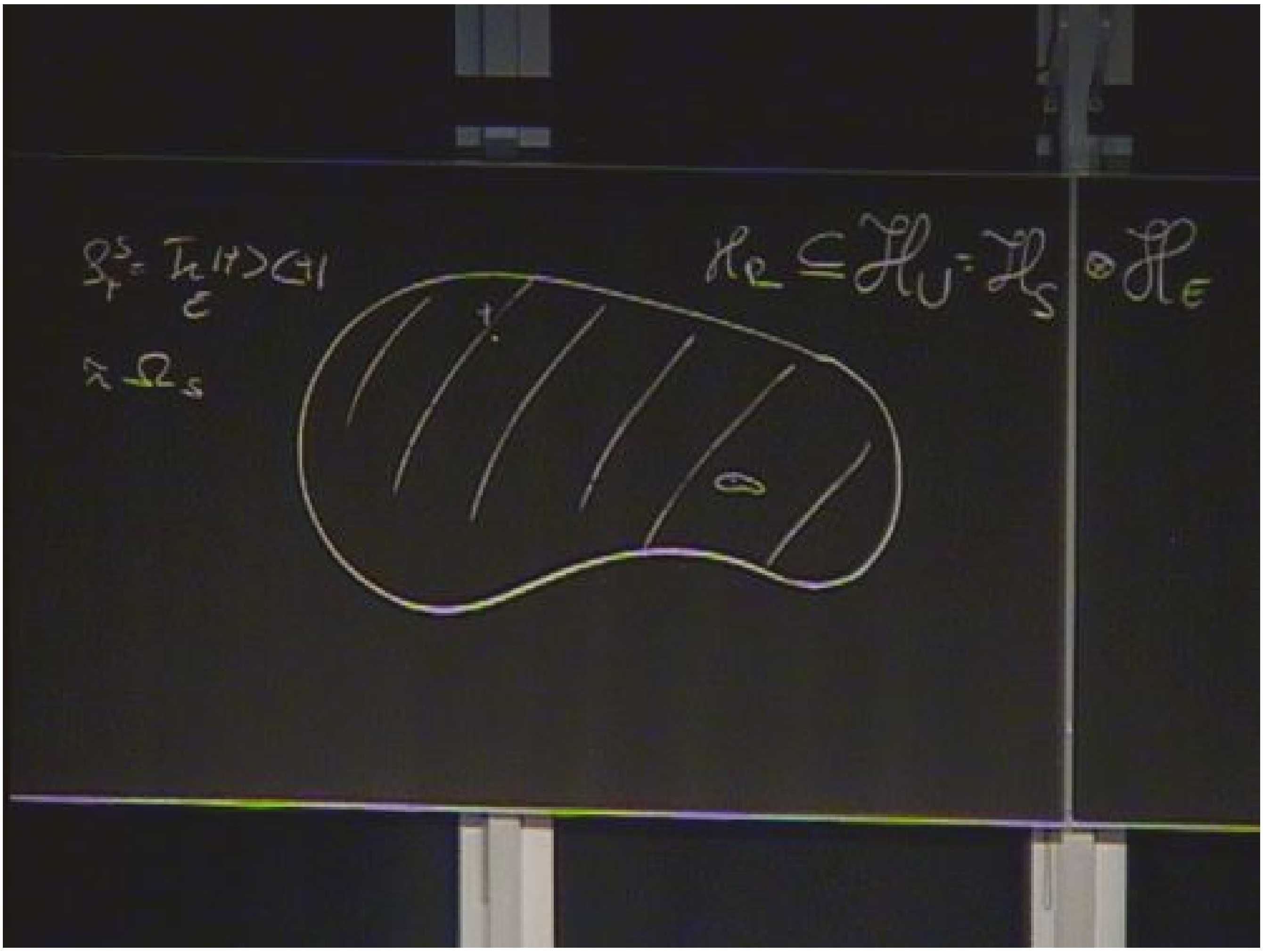
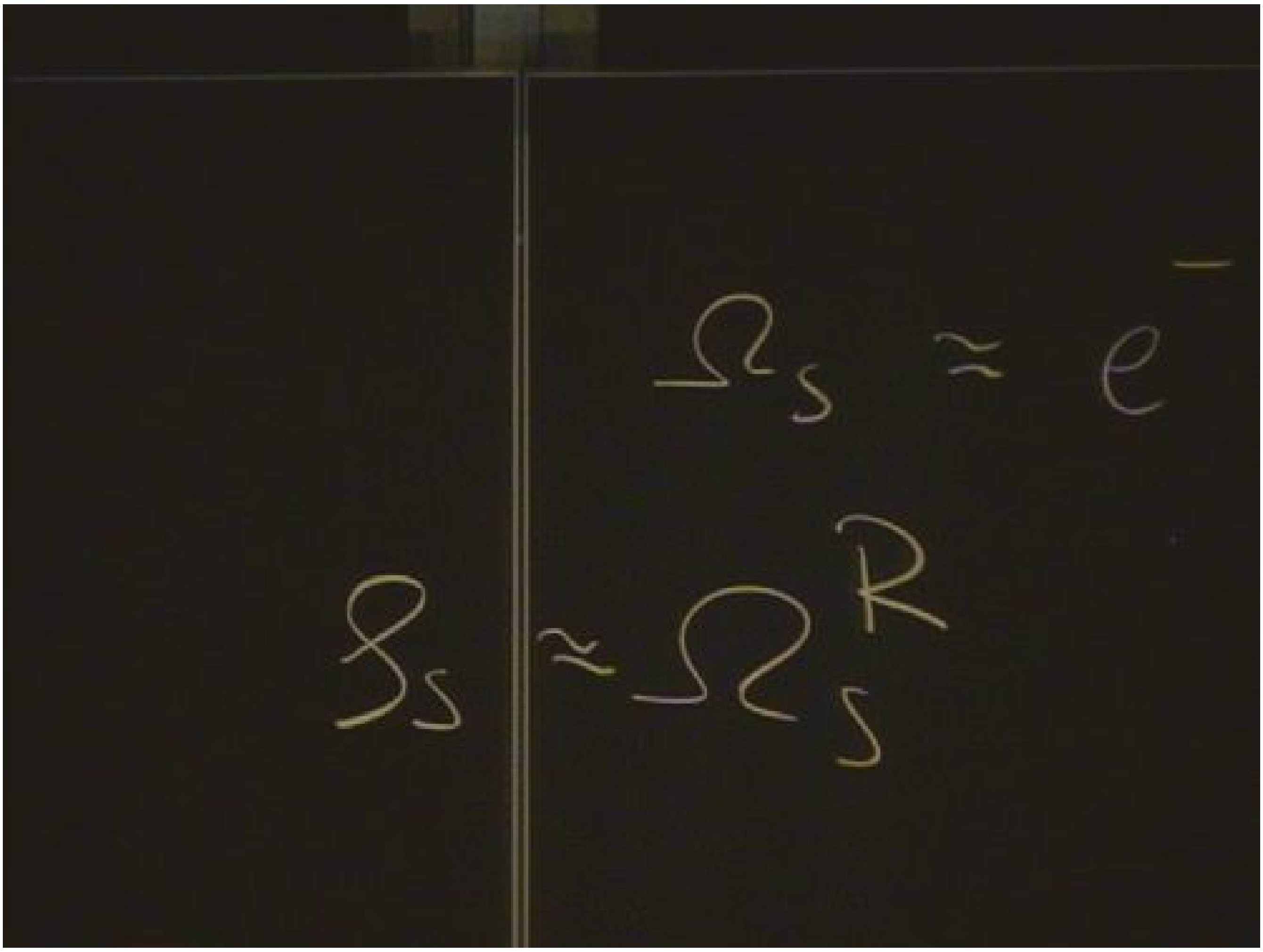

$\rho_t^S = \mathrm{Tr}_E |\psi\rangle\langle\psi|$
$\tilde{\lambda}\ \Omega_S$
$\mathcal{H}_R \subseteq \mathcal{H}_U = \mathcal{H}_S \otimes \mathcal{H}_E$


$\rho_t^S = \mathrm{Tr}_E |t\rangle\langle t|$
$\approx \Omega_S$
$\mathcal{H}_R \subseteq \mathcal{H}_U = \mathcal{H}_S \otimes \mathcal{H}_E$

~~POSTULATE OF EQUAL A-PRIORY PROBABILITIES~~

PRINCIPLE OF APPARENTLY EQUAL A-PRIORY PROBABILITIES


$\Omega_s \approx e^{-}$
$\rho_s \approx \Omega_s^R$

MAIN TOOL: LEVY'S LEMMA

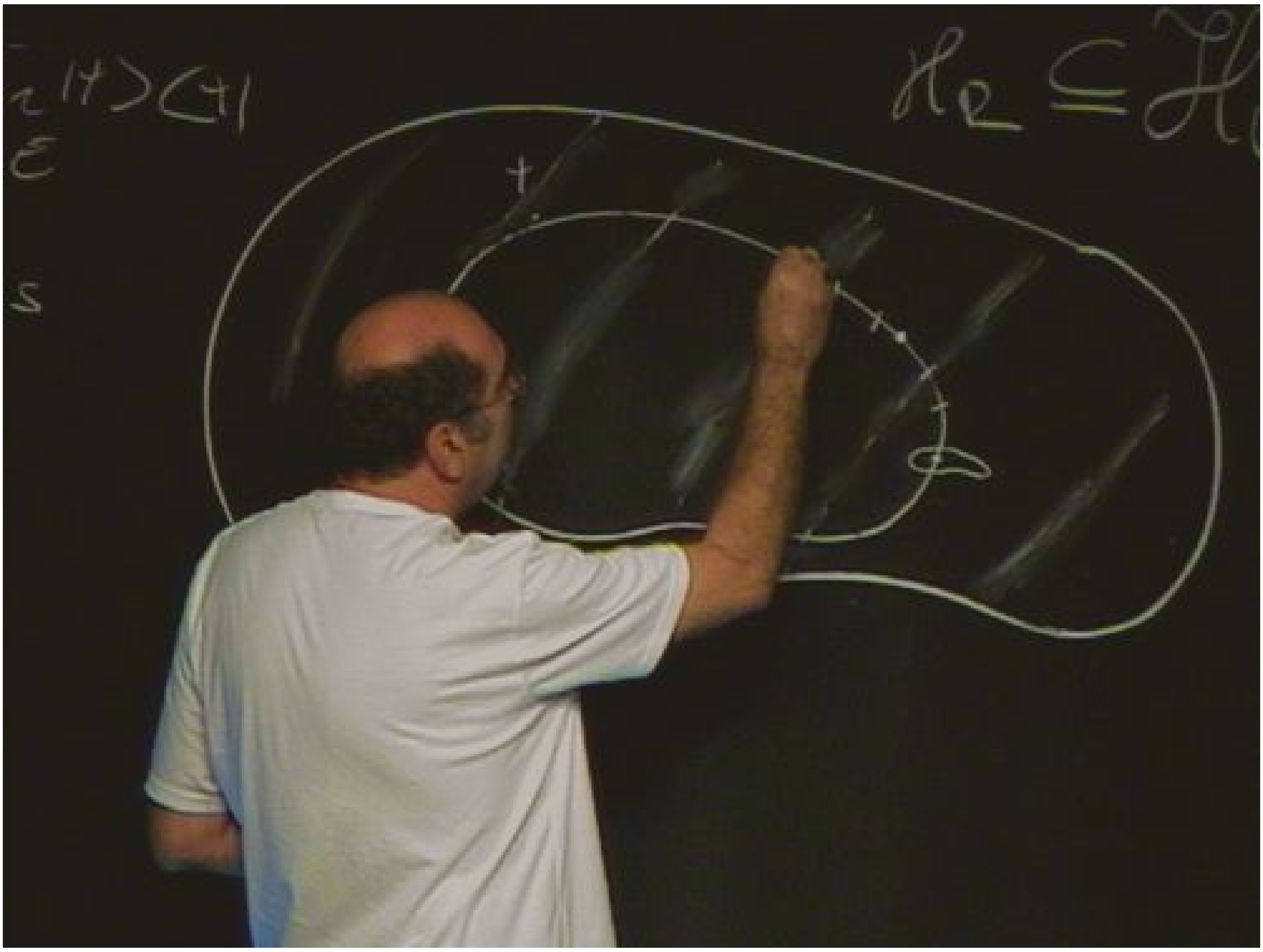
$f : S^d \to R$

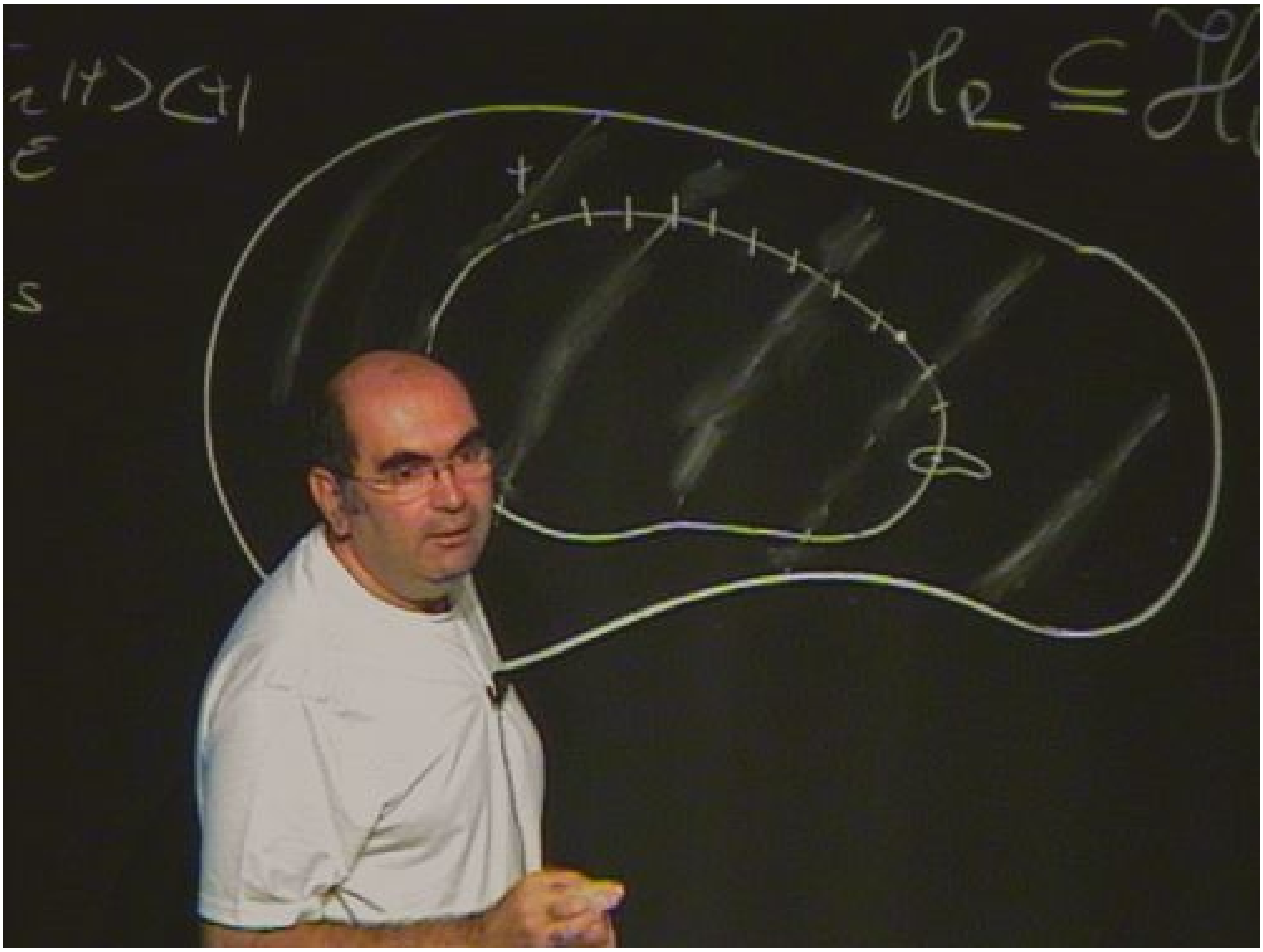
$$\mathrm{Prob}\left[\, |f(p) - \langle f \rangle| \geq \epsilon \,\right] \leq 2e^{-\frac{C(d+1)\epsilon^2}{\eta^2}}$$

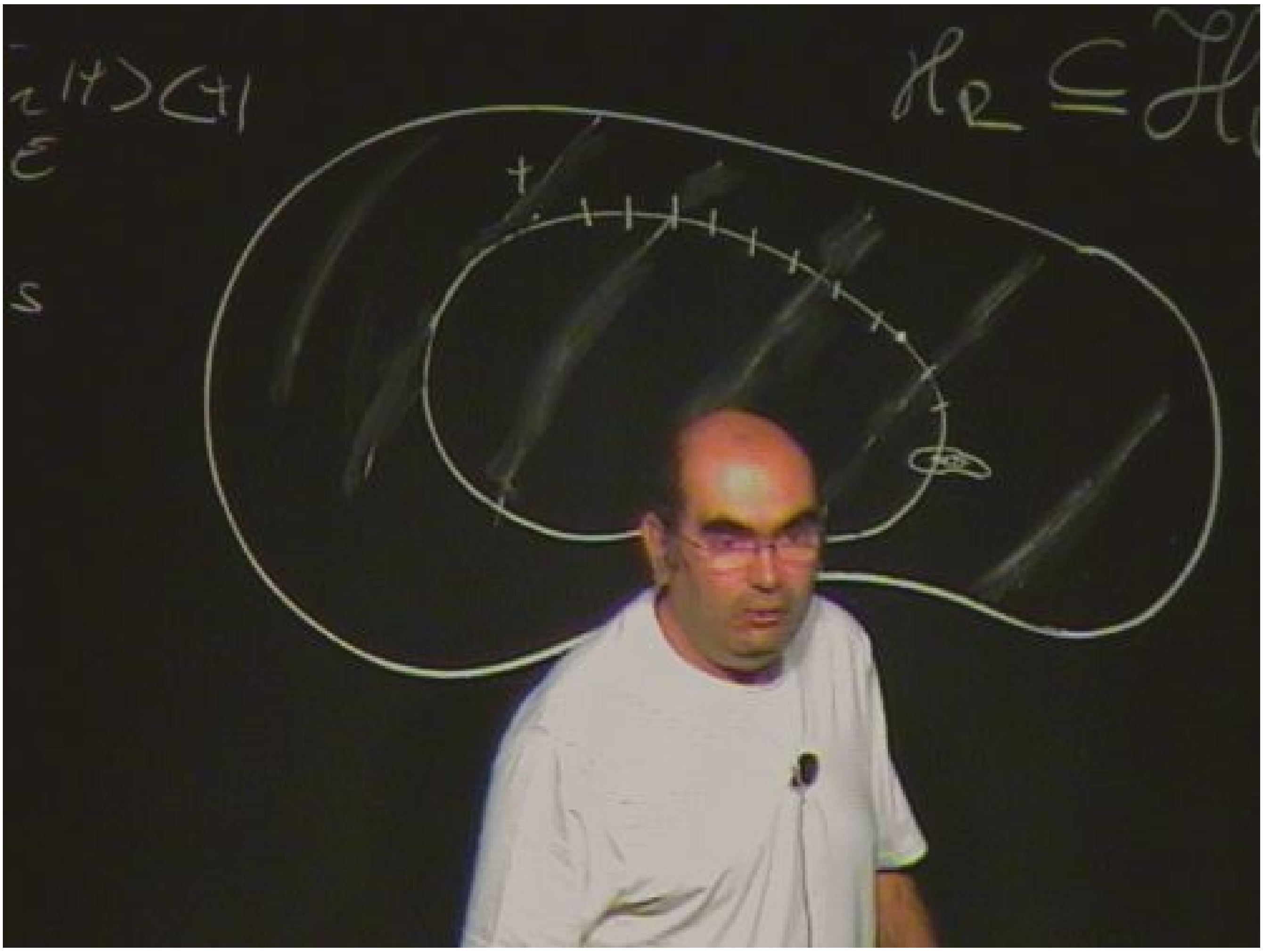
$\eta = \max |\nabla f|$

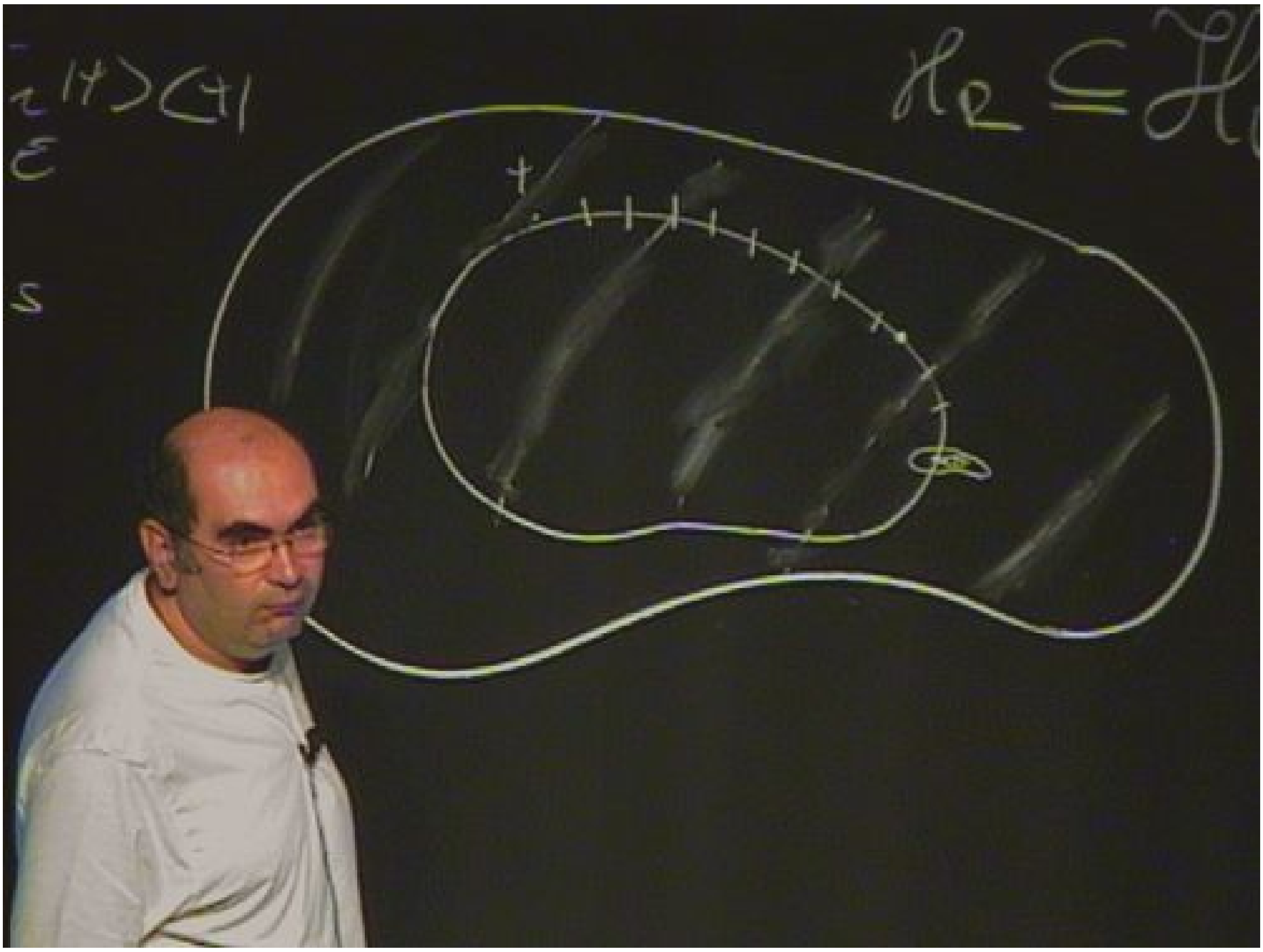
$f(\psi) = \|\rho_S(\psi) - \Omega_S\|_1$

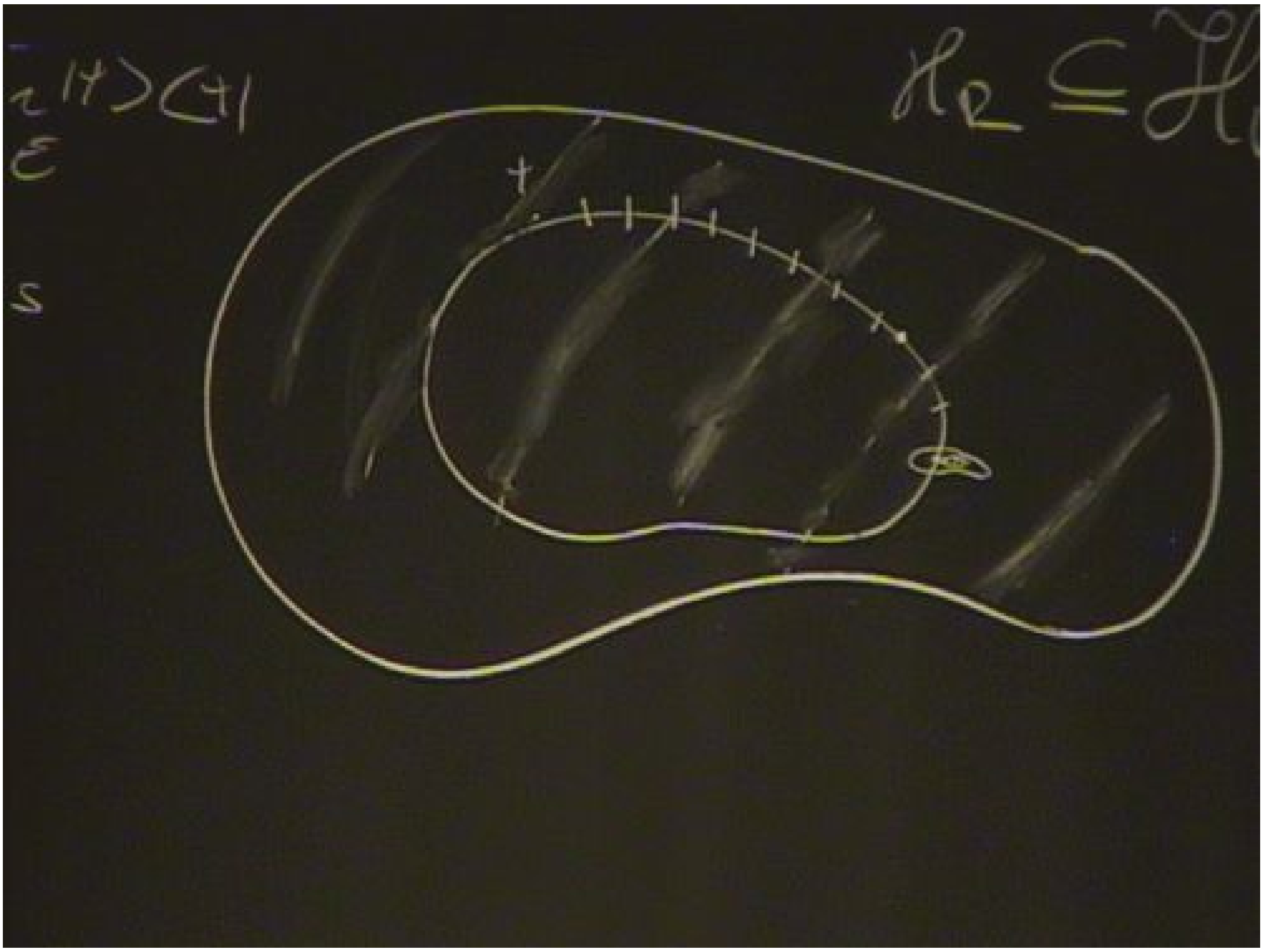
$\|M\|_1 \equiv \mathrm{Tr}\sqrt{MM^\dagger}$



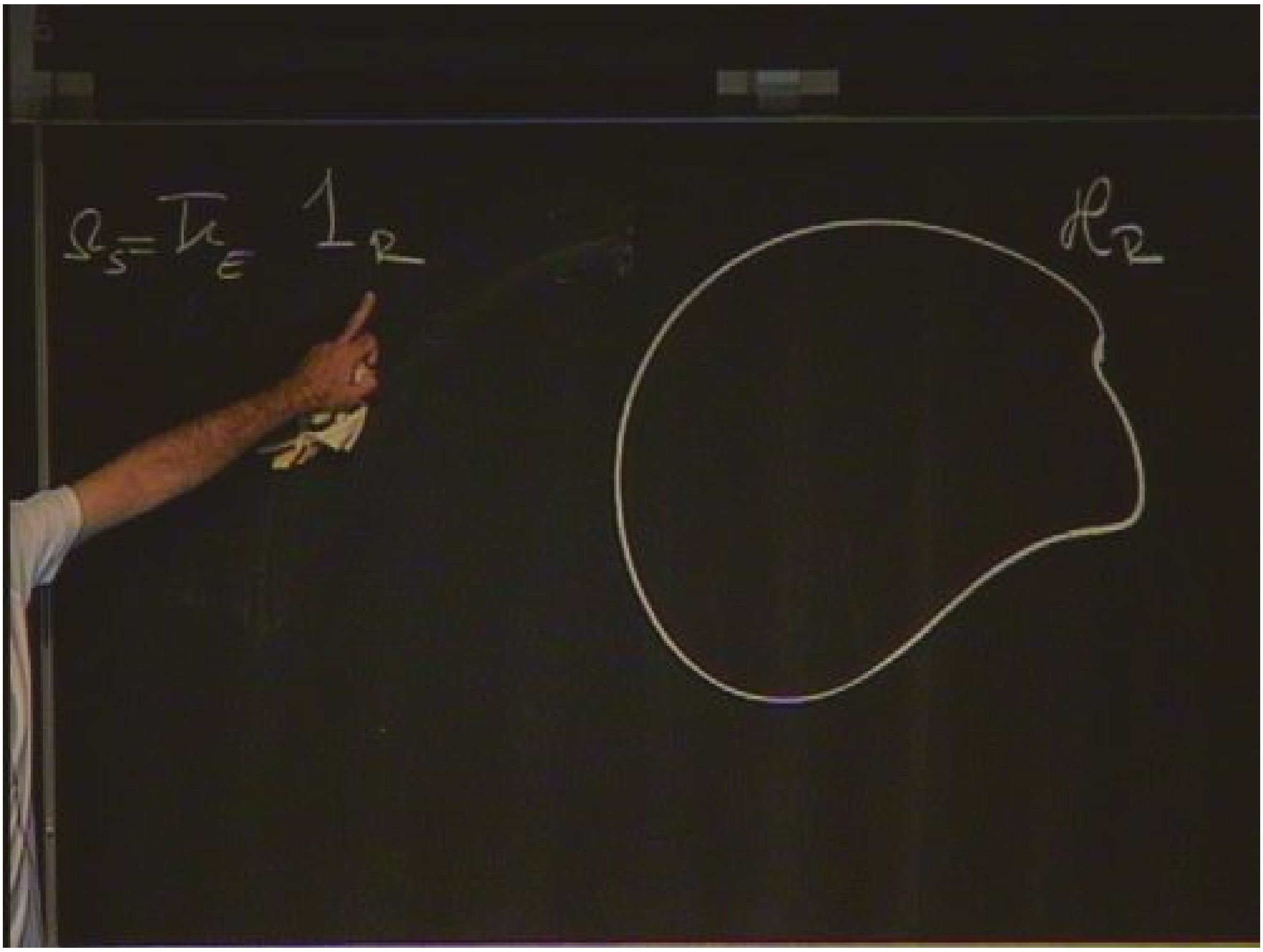


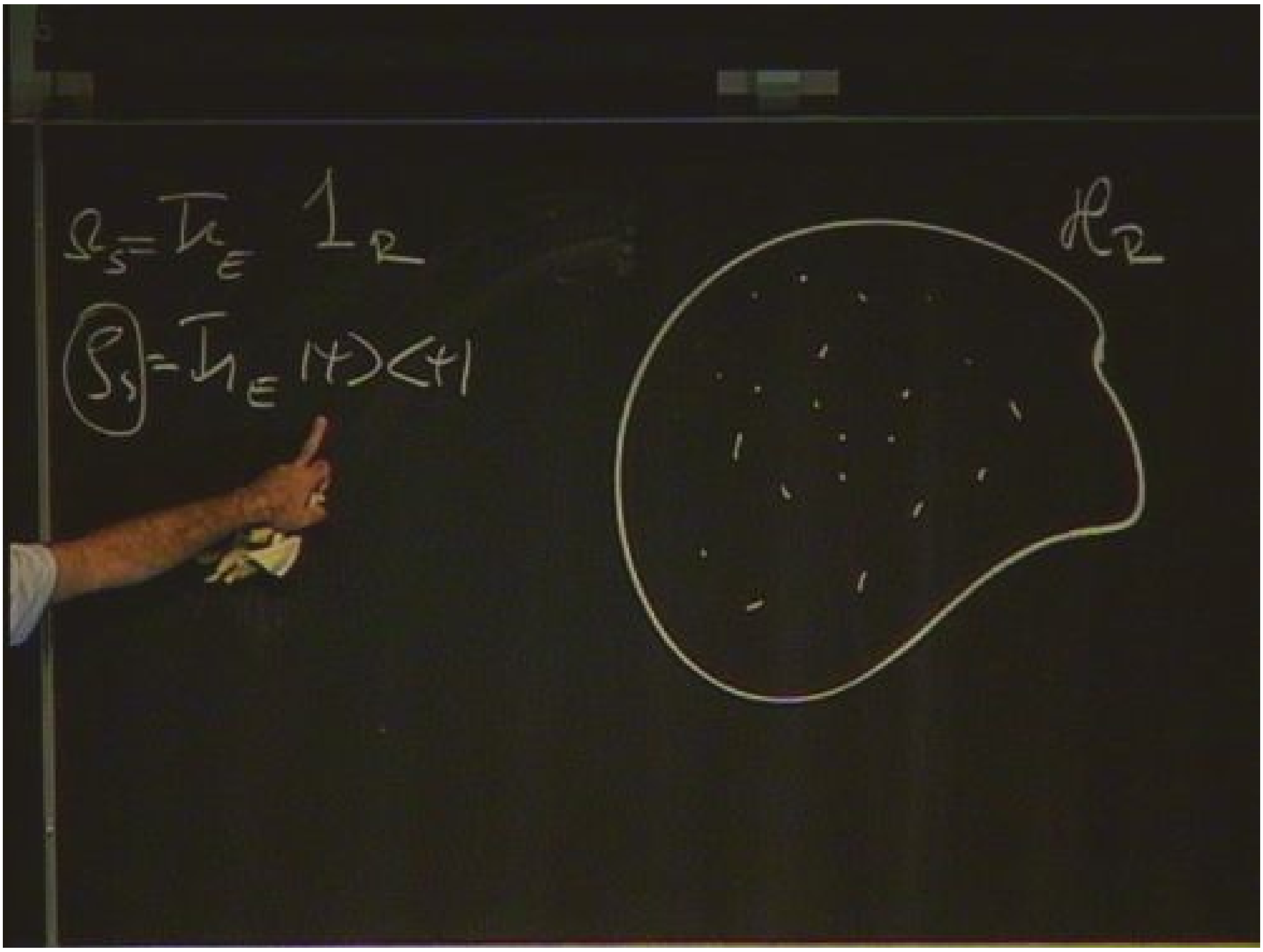




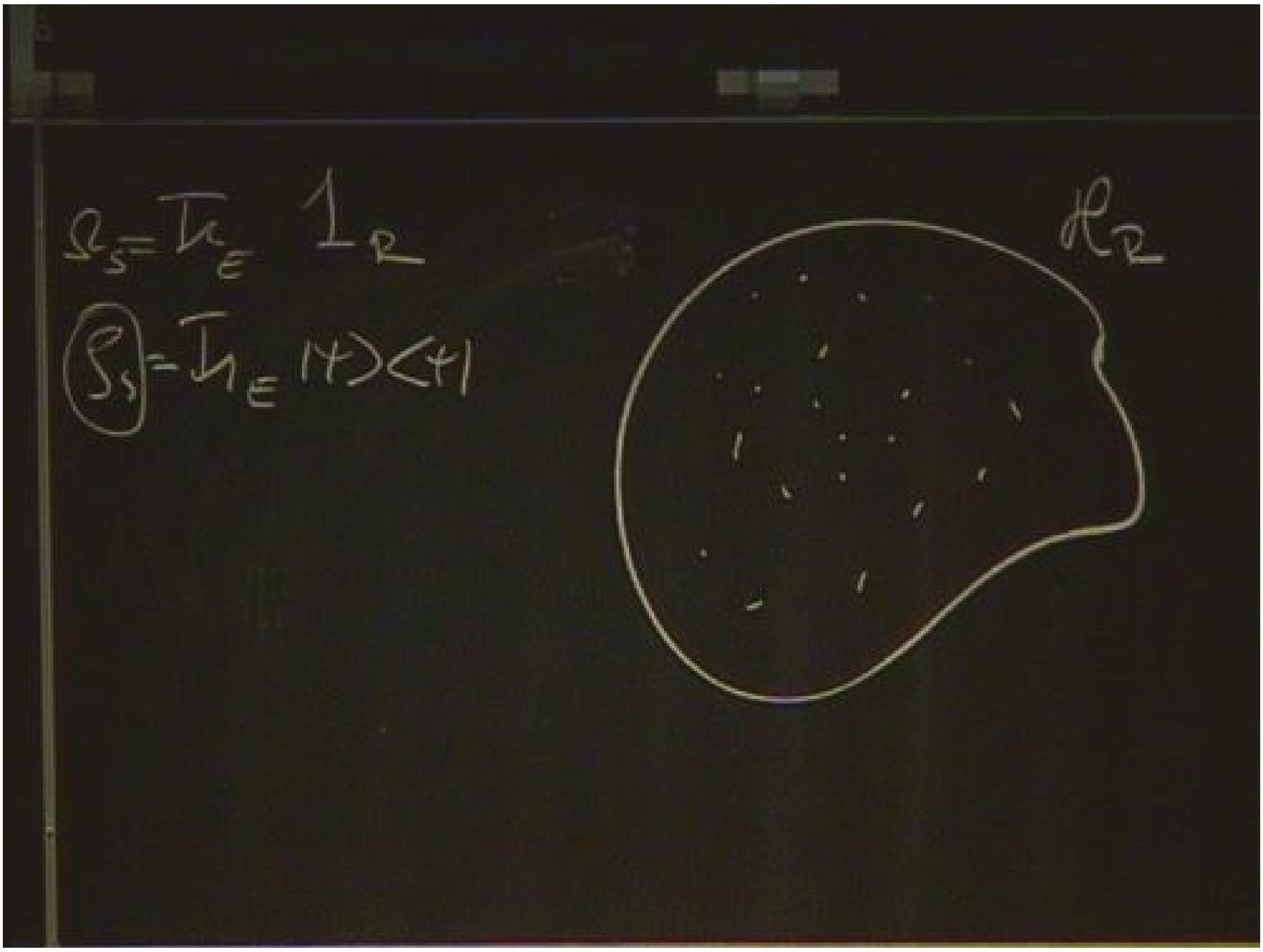


ε
s





$\rho_S = \mathrm{Tr}_E \, \mathbb{1}_R$
$\rho_S = \mathrm{Tr}_E \, |\psi\rangle\langle\psi|$
$\mathcal{H}_R$


$\rho_S = \mathrm{Tr}_E \, 1_R$
$\rho_S = \mathrm{Tr}_E |\psi\rangle\langle\psi|$
$\mathcal{H}_R$